Никольский Михаил Сергеевич

Дифференциальные игры.

$$\dot x = f(x,t)$$

$$x \in \mathbb{R}^n, \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$f(x,t) \in \mathbb{R}^n, \quad t \in \mathbb{R}^1$$

$$\dot x_i = f_i(x,t)$$

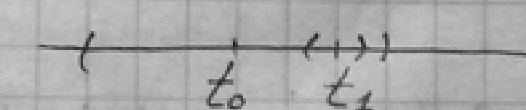
$$x(t_0) = x_0$$

$$x(t), \quad \underset{\delta > 0}{(t_0 - \delta, t_0 + \delta)}$$

$$\xi \in \mathbb{R}^n, \ \eta \in \mathbb{R}^1 \quad \forall (\xi, \eta) \to \rho(\xi,\eta) > 0:$$

$$|f(x',t) - f(x'',t)| \le L_{\xi\eta} |x' - x''|$$ лок. липшиц

$$\begin{pmatrix} x' \\ t \end{pmatrix}, \begin{pmatrix} x'' \\ t \end{pmatrix} \in S_{\rho(\xi,\eta)}(\xi,\eta)$$



пример: $\dot x = x^2$

$x(0) = x_0$

$x_0 = 0$ $\quad \cancel{x = \frac{x^3}{3}} \quad \cancel{x^2 = 3}$ $\quad x = \pm\sqrt{3}$

$\frac{dx}{x^2} = dt$

$-\frac{x^{-1}}{1} = t + C$

$\frac{1}{x} = -t + C$

$x(t) \equiv 0$

$x = \mp \frac{1}{t \mp C}$

$\frac{1}{x_0} = C$

$x_0 = \frac{1}{C}$

$x = -\frac{1}{t \mp \frac{1}{x_0}} = \mp \frac{x_0}{1 \mp t x_0}$

$t \in [0; \frac{1}{x_0})$

$\dot{x} = f(x, u) \qquad x(0) = x_0$

$x \in \mathbb{R}^n$

$u \in V \subset R^r$, $u = u(t) \in V$, где $t \geq 0$

$\dot{x} = f(x, u(t)) = g(x, t)$, $x(0) = x_0$

V – компакт

$V = S_1(0) \subset R^r$

$\dot{x}(t) = f(x(t), u(t))$

Мин. теория диф. ур-ний.

$\dot{x} = Ax + f(t)$

$f(t)$ непр. на R^1

$e^{tA} = E + tA + \ldots + \frac{t^k}{k!} A^k + \ldots$

$\| e^{tA} - \sigma_k \| \to 0$

при $k \to \infty$

$A \to \sum_{i,j=1}^{n} |a_{ij}|$.

$\dot{x} = Ax + f(t)$

$\dot{x} = Ax + Bu$, $u \in V$, $x(0) = x_0$

$x(t) = e^{tA} x_0 + \int_0^t e^{(t-s)A} Bu(s) ds$

$\dot{x} = Ax$

1) $\dot{x} = u$, $x \in \mathbb{R}^n$, $u \in V \subset R^n$

$A = 0$, $e^{A} = E$

$x(t) = x_0 + \int_0^t Bu(s) ds$

2) $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases}$ $\quad A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$e^{tA} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$

3) $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = u \end{cases}$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$e^{tA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & t & 0 \\ 0 & 0 & t \\ 0 & 0 & 0 \end{pmatrix} + (\dots$$

$$= \begin{pmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$$

15.09.08

$$\dot{x} = Ax + Bu$$

$$x \in \mathbb{R}^n, \ u \in V \subset \mathbb{R}^r$$

$$x(0) = x_0, \ u(t) \in V, \ t \geq 0 \ [0, T]$$

$$x(t) = e^{tA} x_0 + \int_0^t e^{(t-s)A} Bu(s)\,ds$$

$$\dot{x} = u$$

$$x(t) = x_0 + \int_0^t u(s)\,ds$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$e^{tA} = E + tA + \dots + \frac{t^k}{k!} A^k + \dots$$

Теория матриц

Гантмахер

$$\dot{x} = Ax$$

$$f_1(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ f_2(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \dots$$

$$e^{tA} = (f_1(t), \dots, f_n(t))$$

$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha x_2 + u \end{cases}$ $\alpha \neq 0$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ – нач. усл.

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$e^{tA} = \begin{pmatrix} 1 & \frac{1 - e^{-\alpha t}}{\alpha} \\ 0 & e^{-\alpha t} \end{pmatrix}$$

$\cancel{\dot{x}_2 = -\frac{\alpha x_2}{2}}$

$$\frac{dx_2}{x_2} = -\alpha dt \to \ln x_2 = -\alpha t + \ln C$$

$$x_2 = e^{-\alpha t} \cdot C$$

пример

$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + u \end{cases}$ $|u| \leq 1$

$\ddot{x} + x = 0$

$\begin{cases} x(0) = 1 \\ \dot{x}(0) = 0 \end{cases}$

$$e^{tA} = \begin{pmatrix} \cos t, & \sin t \\ -\sin t, & \cos t \end{pmatrix}$$ – матр. орт. поворота

Дифференциальные игры

– Д.И. преследования

пример 1 ДИ с простой динамикой

догоняет $\dot{x} = u$, $x, u \in \mathbb{R}^k$

убегает $\dot{y} = v$, $y, v \in \mathbb{R}^k$

$u \in P \subset \mathbb{R}^k$ $x(0) = x_0$

$v \in Q \subset \mathbb{R}^k$ $y(0) = y_0$

$|x(t_1) - y(t_1)| \leq l$

пример 2 (крокодил и мальчик)

крокодил $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases}$ $x_1, x_2, u \in \mathbb{R}^k$

$\mathbb{R}^{2k}$

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^{2k}$

мальчик $\dot{y} = v$ $|x_1(t_1) - y(t_1)| \leq l$

3.

$$\begin{cases} \dot{x} = u \\ \dot{y}_1 = y_2 \\ \dot{y}_2 = v \end{cases} \quad |x(t_1) - y(t)| \leq l.$$

4. (о 2 крокодилах)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \quad \begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = v \end{cases}$$

$u \in P$, $v \in Q$

5. $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha x_2 + u \end{cases}$ $\alpha > 0$

$$\underbrace{\ddot{x} + \alpha \dot{x} = u}_{\alpha > 0} \qquad \begin{cases} x_1 = x \\ x_2 = \dot{x} \end{cases}$$

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -\beta y_2 + v \end{cases} \quad |x_1(t_1) - y_1(t_1)| \leq l$$

$\beta > 0$

6. более общий пример

$$\dot{x} = Ax + Du$$

$$\dot{y} = By + Fv$$

$x \in \mathbb{R}^k$, $y \in \mathbb{R}^l$, $u \in P \subset \mathbb{R}^p$, $v \in Q \subset \mathbb{R}^q$

$z = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^{k+l}$, M – термин. мн-во

$\begin{pmatrix} x(t_1) \\ y(t_1) \end{pmatrix} \in M$

$$z = \begin{pmatrix} x \\ y \end{pmatrix}$$

$z(t)$

$$\dot z = Cz + D_1 u + F_1 v$$

$$C = \begin{pmatrix} A & O_{k,e} \\ O_{e,k} & B \end{pmatrix} \qquad D_1 = \begin{pmatrix} D \\ O_{e,p} \end{pmatrix}$$

$$F_1 = \begin{pmatrix} O_{kq} \\ F \end{pmatrix} \qquad \begin{pmatrix} Du \\ 0 \end{pmatrix} = D_1 u$$

$z(0) = z_0$, M, $z(t_1) \in M$

Диф. игры

Р. Айзекс

$$\dot x = Ax + Bu$$

$x(0) = x_0$, M – термин. мн-во.

$x_0 \xrightarrow{\tilde u(t)} (M)$ $u(t) \in U$, $t \geq 0$

$\underline{u(x,t)}$

Н.Н. Красовский
А.И. Субботин
Позиционирование диф. игр.

Пример

$u \in S_\rho(0)$, $v \in S_\sigma(0)$

$\dot x = u$, $x, u \in \mathbb{R}^k$

$\dot y = v(t)$, $y, v \in \mathbb{R}^k$ $\boxed{\rho > \sigma}$

$x(0) = x_0$ $\qquad M = \{ z = \begin{pmatrix} x \\ y \end{pmatrix} : x(t_1) - y(t_1) = 0 \}$

$y(0) = y_0$

$u(x,y) = \rho \dfrac{y-x}{|y-x|}$, $y \neq x$

$x_0 \neq y_0$

$$z(t) = |x(t) - y(t)|^2$$

$$z(t) = \langle x(t) - y(t), x(t) - y(t) \rangle$$

$$\langle a, b \rangle = \sum a_i b_i$$

$$\dot z(t) = 2 \langle x(t) - y(t), \underset{u(x(t),\, y(t))}{\underset{\|}{u(t)}} - v(t) \rangle$$

$$\dot c(t) = 2 \langle a(t), \dot a(t) \rangle$$

$$c(t) = \langle a(t), a(t) \rangle$$

$u(x,y) = \rho \dfrac{y-x}{|y-x|}$, $y \neq x$

$\dot z(t) \le -2z^{1/2}(t) + 2\sigma z^{1/2}(t)$

$\langle x-y, \frac{y-x}{|y-x|}\rangle = -|y-x| = -z^{1/2}$

$\langle x-y, v\rangle \le z^{1/2}\sigma$

$\dot z(t) \le -2(\rho-\sigma) z^{1/2}(t)$, $\dot w = -2(\rho-\sigma) w^{1/2}$

$z(t) \le w(t)$ $\quad w(0) = z(0) = |x_0 - y_0|$

$$\frac{dw}{w^{1/2}} = -2(\rho-\sigma)\,dt$$

$$2w^{1/2} = -2(\rho-\sigma)t + \frac{C}{2}$$

$$\frac{C}{2} = \sqrt{|x_0 - y_0|}$$

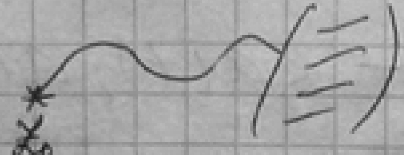
$$|x(t) - y(t)| \le |x_0 - y_0| - (\rho-\delta)t$$

$$t_1 \le \frac{|x_0 - y_0|}{\rho-\sigma}$$

$\dot z = Cz + D_1 u + F_1 v$

$z \in R^n$, $u \in P \subset R^p$, $v \in Q \subset R^q$

$z(0) = z_0$,



$u(t), v(t)$

Класс измеримых ф-ций.

Покрытие $S(E)$

6.10.08

$$C = A \overset{*}{-} B = \begin{cases} \{C : C + B \subset A\} \to C + B \subset A \\ \bigcap_{b \in B} (A - b) \end{cases}$$

$$S_\rho(0) \overset{*}{-} S_\sigma(0) \overset{?}{=} \begin{cases} ① \; S_{\rho-\sigma}(0) \\ ② \; \varnothing \end{cases} \quad ① \; 0 \le \sigma \le \rho \quad ② \; 0 \le \rho < \sigma$$

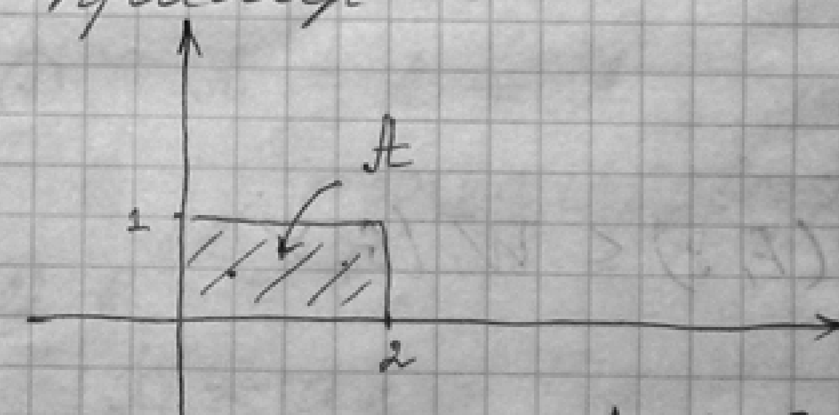
$\rho, \sigma \ge 0$

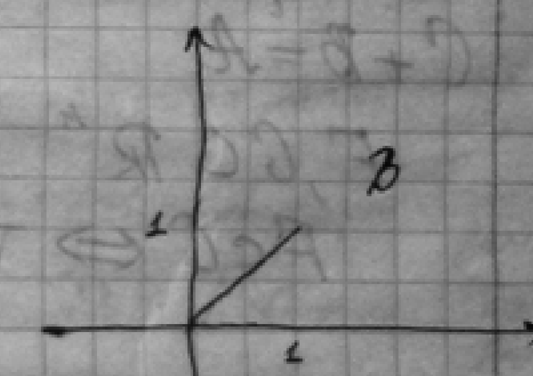
пример 2.

$S_\rho(a) \overset{*}{-} S_\sigma(b) = (a-b) + S_{\rho-\sigma}(0)$, если $\rho \ge \sigma > 0$

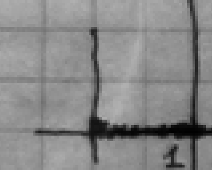
$= \varnothing$, если $\rho < \sigma$

пример 3:

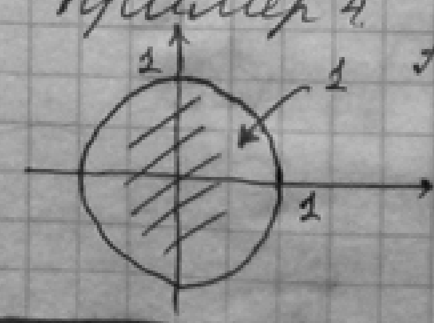


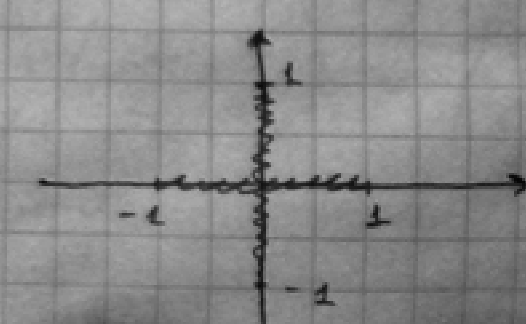


$A \overset{*}{-} B =$



пример 4





Лемма:

$A = B + D \Rightarrow C = D$

вып. компакт

$W(A, \psi) = \sup\limits_{a \in A} \langle a, \psi \rangle$

$\langle a, \psi \rangle \leq W(A, \psi)$, $|\psi| = 1$

$A + B = F$

$W(F, \psi) = W(A, \psi) + W(B, \psi)$

пример:

$S_\sigma(a) + S_{\rho - \sigma}(b - a) = S_\rho(b)$
$\rho \geq \sigma$

Опорная ф-я геометр. разности:

$C + B = A$

$F, G \subset \mathbb{R}^n$

$F \subset G \Leftrightarrow W(F, \psi) \leq W(G, \psi)$.

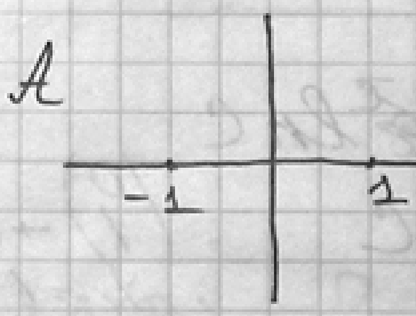
С/р.

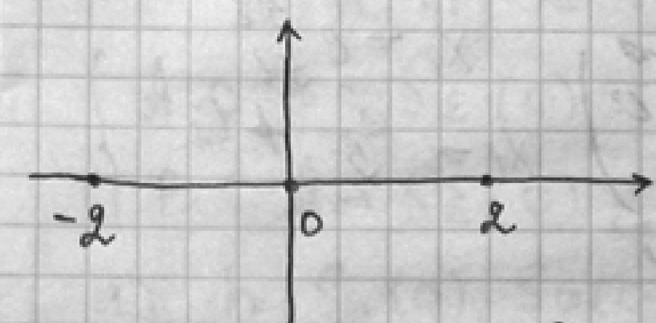
① $(-5) S_1(0) \overset{*}{-} 2 S_2(0)$

$\cancel{(\ldots)}\ S_2(0) = \cancel{1}\ S_1(0)$

② $S_3(a) \overset{*}{-} S_4(b) \quad \emptyset.$

③. A $\qquad A + A$





④ $A \overset{*}{-} A \overset{?}{=} A$ так может быть

точка 0, все пр-во.

⑤ $e^{tA} = ?$ $\qquad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad \frac{dx_2}{x_2} = 5dt$

$\ln x_2 = 5t + \ln c$

$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 5x_2 \end{cases}$ $\qquad A = \begin{pmatrix} 0 & 1 \\ 0 & 5 \end{pmatrix}$

$x_2 = e^{5t} c$

$x_2 = e^{5t}$

$dx_1 = e^{5t} dt$

$A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 0 & 25 \end{pmatrix} = 5A$ $\qquad x_1 = \frac{e^{5t}}{5} + c$

$A^3 = 5A^2 = 25A \qquad 5^{t-1} A.$

$-\frac{1}{5} = c$

$$e^{tA} = E + tA + \frac{t^2}{2}\cdot 5A + \frac{t^3}{6}\, 25A + \ldots + \frac{t^k}{k!}\, 5^{k-1} A$$

$$\sum_{n=1}^{k} \frac{t^n}{n!}\, 5^{n-1}$$

$$\frac{dx_2}{x_2} = 5\,dt$$

$$\ln x_2 = 5t + \ln C$$

$$x_2 = e^{5t}\cdot C \qquad \begin{pmatrix}0\\1\end{pmatrix} \to C = 1$$

$x_2 = e^{5t}$

$x_1 = e^{5t}$

$$e^{tA} = \begin{pmatrix} 1 & \frac{e^{5t}-1}{5} \\ 0 & e^{5t} \end{pmatrix} \qquad x_1 = \frac{e^{5t}}{5} + C_1$$

$$\frac{dx_1}{dt} = e^{5t}\cdot C$$

$$dx_1 = dt\, e^{5t}\cdot C$$

$$x_1 + C_1 = \frac{e^{5t}}{5}\cdot C.$$

$$x_1(0) = 1$$

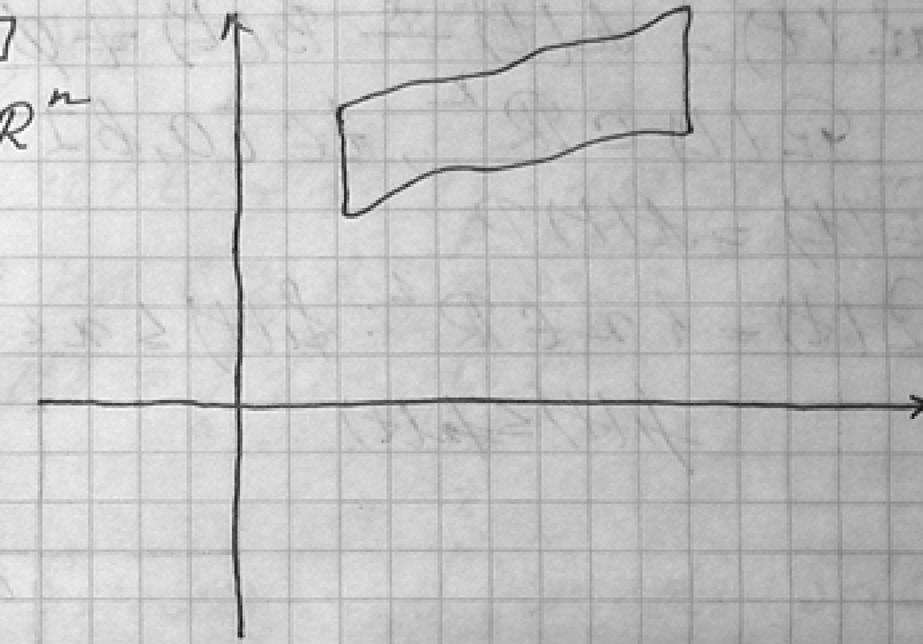
$$1 + C_1 = \frac{1}{5}\cdot C \qquad C = 0$$

$$C_1 = -1$$

13.10.08. Многозн. отображение

$t \in [a, b]$

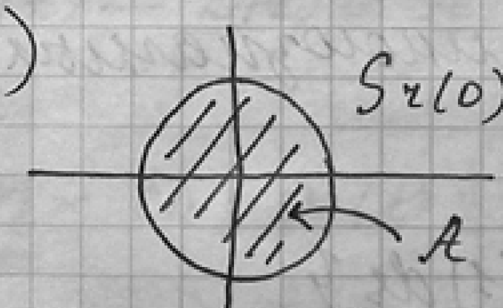
$\Omega(t) \subset R^n$



$$h(A, B) \overset{?}{=} h(B, A)$$

$$\begin{cases} A \subset B + S_\varepsilon(0) \\ B \subset A + S_\varepsilon(0) \end{cases}$$ это наим. ε при кот вып оба

1) $h(S_\rho(0), S_\sigma(0)) = \rho - \sigma$, $\rho \geq \sigma$

2) 

$\varepsilon = r.$

Лемма:

$h(\Omega(t), \Omega(t_0)) \to 0$ при $t \to t_0$, $t \in [a, b]$

Лемма. $A(t), B(t)$ на $[a, b]$

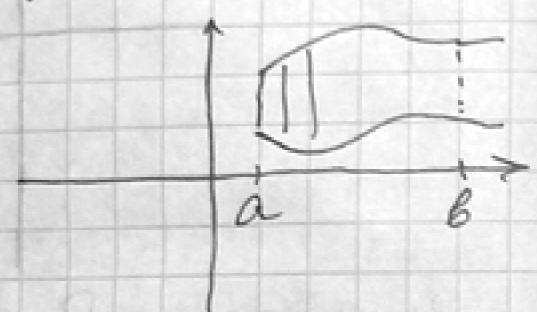
$\Omega(t) = A(t) \underline{*} B(t) \neq \emptyset$ на $[a,b]$

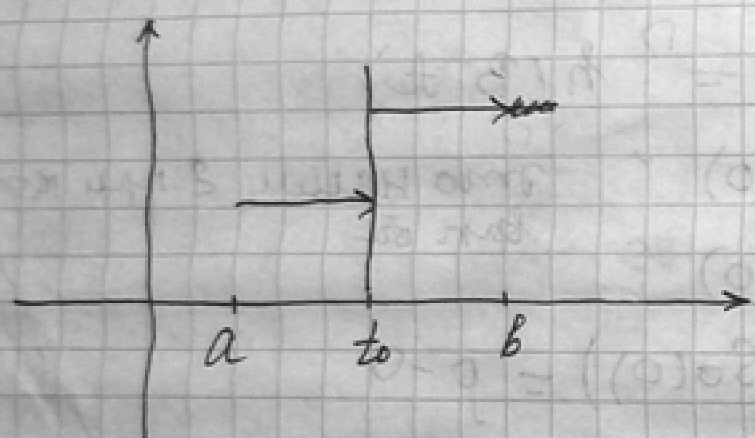
20.10.08 $\Omega(t) \subset \mathbb{R}^k$, $t \in [a, b]$

$\Omega(t) = \lambda(t) P$

$\Omega(t) = \{x \in \mathbb{R}^1 : f_1(t) \leq x \leq f_2(t)\}$

$f_1(t) \leq f_2(t)$.





Интегрирование многозначных отображений.

$$\int_a^b \Omega(s)\,ds = \left\{\int_a^b \omega(s)\,ds\right\}$$

$\omega(s) \in \Omega(s)$, $[a, b]$

$\omega(s)$ – суммир. по Лебегу ф-я

Теорема. $\Omega(t)$ – полунепр. сверху тогда $\int_a^b \Omega(s)\,ds$ – непустой вып. комп.

Далее,

$$W(\mathfrak{A}, \psi) = \int_a^b W(\Omega(s), \psi)\,ds, \quad \psi \in \mathbb{R}^k$$

Лемма. $\Omega(t) = f(t) P$, где $f(t) \geq 0$, P – вып. компакт, непр. ф-я на $[a, b]$, тогда

$$\int_a^b \Omega(s)\,ds = \int_a^b f(s)\,ds\, P.$$

пример:

$$\int_0^\pi \sin s\, S_5(0)\,ds = \int_0^\pi \sin s\,ds \cdot S_5(0)$$

$$= -\cos s \Big|_0^\pi S_5(0)$$

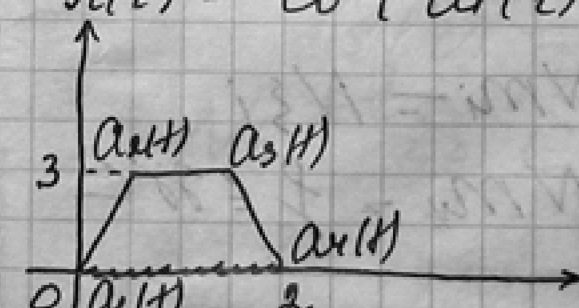
$$\int_0^{2\pi} \sin s \cdot S_5(0)\,ds$$

27.10.08

$A(t) \underline{*} B(t) = \Omega(t)$

$n = 2$

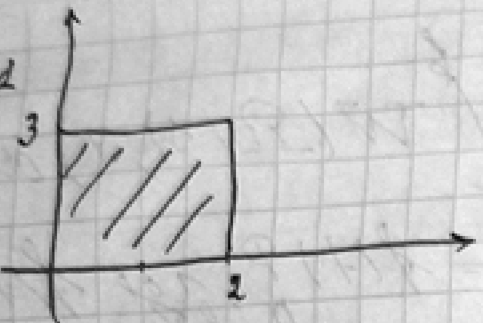
$A(t) = co\{a_1(t); \ldots; a_4(t)\}$



$a_2(t) = \begin{pmatrix} 1-t \\ 3 \end{pmatrix}$, $t \in [0, 1]$.

$a_3(t) = \begin{pmatrix} 1+t \\ 3 \end{pmatrix}$

$t=1$



$B(t) = \begin{Bmatrix} 0 \le x_1 \le 2 \\ 0 \end{Bmatrix}$

Дифференциальные игры.

$\dot{z} = Az - Bu + Cv$

$z \in \mathbb{R}^n$, $B - n \times r$; $u \in P$, $v \in Q$

$z(0) = z_0$ $C - n \times q$; $M \neq \mathbb{R}^n$

$z_0 \notin M$

$M = \mathfrak{M} + L$

$m_i \to m_* \in M$

M-замкн. мн-во ?!

$m_i = \underset{\mathfrak{M}}{\xi_i} + \underset{L}{\eta_i}$ $\quad Nm_i = N\xi_i$

$Nm_* = \xi \in \mathfrak{M}$

17.11.08

$\dot{z} = Az - Bu + Cv$

$u \in P \subset \mathbb{R}^p$, $v \in Q \subset \mathbb{R}^q$

$z(0) = z_0$, $M = \mathfrak{M} + L$

$L = \{Nz = 0\}$, $z_0 \notin M$

1. $\dot{z}_1 = -u$ $\quad \begin{pmatrix} E_k \\ O_k \end{pmatrix}$ $Ne^{\tau A}BP$

$\dot{z}_2 = -v$ $\quad Ne^{\tau A}CQ$

$z_1, z_2, u, v \in \mathbb{R}^k$

$M = \{z \in \mathbb{R}^{2k} : |z_1 - z_2| \le l\}$

$N = (E_k, -E_k)$ $\quad \begin{matrix} \dot{z}_1 = 0 \\ \dot{z}_2 = 0 \end{matrix}\}$ $\quad L = \{z_1 - z_2 = 0\}$

$e^{\tau A} = \begin{pmatrix} E_k, & O_k \\ O_k, & E_k \end{pmatrix} = Ne^{\tau A}\begin{pmatrix} u \\ 0 \end{pmatrix} = u$

$\mathfrak{M} = \{z \in \mathbb{R}^{2k} : |z_1| \le l, z_2 = 0\}$

$Ne^{\tau A}Cv = -v$

$\hat{w}(\tau) = P \overset{*}{-} (-Q) \neq \emptyset$ при всех $\tau \ge 0$

$Ne^{tA}z_0 \overset{?}{\in} N\mathfrak{M} + \int_0^t \hat{w}(\tau)d\tau$

$z_0 = \begin{pmatrix} z_1^0 \\ z_2^0 \end{pmatrix}$

$Ne^{tA}z_0 = z_1^0 - z_2^0$

$z_1^0 - z_2^0 \overset{?}{\in} S_\ell(0) + \underbrace{\int_0^t (P \overset{*}{-} (-Q))\,ds}_{\Omega - \text{выпукло}} = *$

$\int_0^t \Omega\, ds \overset{?}{=} t\Omega$

вып. компакт.

по лемме: $\int_a^b \underset{\geq 0}{f(s)}\, \underset{\text{вып. комп.}}{P}\, ds = \int_a^b f(s)\,ds\, P$

$* = S_\ell(0) + t\underbrace{(P \overset{*}{-} (-Q))}_{\Omega}$

$= S_\ell(0) + t S_\sigma(0)$

$0 \in \operatorname{int}\Omega \leftrightarrow S_\sigma(0) \subset \Omega$

$|z_1^0 - z_2^0| \leq \ell + t\sigma$

2. $\left.\begin{array}{l} \dot z_1 = z_2 \\ \dot z_2 = -u \\ \dot z_3 = v \end{array}\right\}$ $M = \{z \in \mathbb{R}^{3k} : |z_1 - z_3| \leq \ell\}$

$M = \mathfrak{M} + L$

$k = 1, 2, \ldots$ $L = \{z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} : z_1 - z_3 = 0\}$

$N = (E_k, O_k, -E_k)$, $\mathfrak{M} = \{z : |z_1| \leq \ell,\ z_2 = z_3 = 0\}$

$Ne^{\tau A} B \cancel{u} = P$, $Ne^{\tau A} CQ$

$Ne^{\tau A} \underbrace{Cv}_{\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}} = N\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} = -v$

$Ne^{\tau A} \underset{Pu}{\ } = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix} = N\begin{pmatrix} \tau u \\ u \\ 0 \end{pmatrix} = \tau u$

$\hat{W}(\tau) = \tau P \overset{*}{-} (-Q) = \tau S_\rho(0) \overset{*}{-} S_\sigma(0)$

$\rho\tau - \sigma \geq 0$ $\tau \in [0; \theta]$

$e^{\tau A} = \begin{pmatrix} e^{\tau A_1} & 0 \\ 0 & e^{\tau A_2} \end{pmatrix} = \begin{pmatrix} E_k; & O_k; & O_k \\ O_k; & E_k; & \tau E_k \\ O_k; & O_k; & E_k \end{pmatrix}$

$Ne^{\tau A} Bu = Ne^{\tau A}\begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix} = u$

$Ne^{\tau A} \underset{\begin{pmatrix} 0 \\ \ell \end{pmatrix}}{Cv} = -\tau v$

24.11.08

① $\left\{\begin{array}{l} \dot z_1 = -u \\ \dot z_2 = v \end{array}\right.$ $A = 0$

$M = \{z \in \mathbb{R}^{2k} : |z_1 - z_2| \leq \ell\}$

$e^{tA} = E_{2k} = \begin{pmatrix} E_k, & O_k \\ O_k, & E_k \end{pmatrix}$

$N = (E_k, -E_k)$, $L = \{z_1 - z_2 = 0\}$

$\begin{array}{l} \dot z_1 = \\ \dot z_2 = \end{array} \begin{pmatrix} -u \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ v \end{pmatrix}$

$\dot z = Az - Bu + Cv$

$Bu = \begin{pmatrix} u \\ 0 \end{pmatrix}$; $Cv = \begin{pmatrix} 0 \\ v \end{pmatrix}$

$B = \begin{pmatrix} E_k \\ O_k \end{pmatrix} \to C = \begin{pmatrix} O_k \\ E_k \end{pmatrix}$

$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$

$e^{tA} = \begin{pmatrix} e^{tA_1}, & 0 \\ 0, & e^{tA_2} \end{pmatrix}$

(2) $\dot z_1 = z_2$

$\dot z_2 = -u$

$\dot z_3 = v$

$E_k \approx \hat 1$
$O_k \approx \hat 0$
$\alpha E_k \approx \hat \alpha$

$M = \{ |z_1 - z_2| \le \ell$

$N = (E_k, O_k, -E_k)$

$L = \{ z_2 - z_3 = 0 \}$

$$A = \left(\begin{array}{cc|c} O_k, & E_k, & O_k \\ O_k, & O_k, & O_k \\ \hline O_k, & O_k, & O_k \end{array} \right)$$

$e^{tA_1} = \begin{pmatrix} E_k & tE_k \\ O_k & E_k \end{pmatrix}$

$$\begin{aligned} \dot z_1 &= z_2 - 0 + 0 \\ \dot z_2 &= -u + 0 \\ \dot z_3 &= -0 + v \end{aligned}$$

$Ne^{\tau A}BP \overset{*}{-} Ne^{\tau A}CQ = \tau P \overset{*}{-} (-1)Q$

$Q = S_\sigma(0), \sigma > 0$

(3) $\begin{cases} \dot z_1 = -u \\ \dot z_2 = z_3 \\ \dot z_3 = v \end{cases}$

$M = \{ |z_1 - z_2| \le \ell \}$

$N = (E_k, -E_k, O_k)$

$L = \{ z_1 - z_2 = 0 \}$

$$e^{tA} = \left(\begin{array}{c|c} E_k & 0 \\ \hline 0 & \begin{matrix} E_k, & tE_k \\ O_k, & E_k \end{matrix} \end{array} \right)$$

$Bu = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}$ $Cv = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$

$Ne^{\tau A}Bu = u \longrightarrow P$

$Ne^{\tau A}Cv = -\tau v \longrightarrow (-\tau)Q.$

$\hat W(\tau) = P \overset{*}{-} (-\tau)Q = \{ P = S_\rho(0), \rho > 0$
$Q = S_\sigma(0), \sigma > 0 \}$
$= S_\rho(0) \overset{*}{-} (-\tau)S_\sigma(0) = S_{(\rho - \tau\sigma)}(0) = (\rho - \tau\sigma) \cdot S_1(0)$

$\rho > \tau\sigma$

$[0, \frac{\rho}{\sigma}]$.

$$N e^{tA} z_0 \overset{?}{\in} N\mathfrak{M} + \int_0^t (\rho - \tau\sigma) S_1(0)\,ds \quad t \in [0; \frac{\rho}{\sigma}]$$

$$z_0 = \begin{pmatrix} z_{10} \\ z_{20} \\ z_{30} \end{pmatrix} \quad (E_k, -E_k, -tE_k)\, z_0 =$$

$= z_{10} - z_{20} - t z_{30}$ – л. часть.

$$N\mathfrak{M} = S_l(0).$$

$$\mathfrak{M} = \left\{ \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} : |z_1| \le l,\ z_2 = z_3 = 0 \right\}$$

$$\int_0^t (\rho - \tau\sigma) S_1(0)\,d\tau = \int_0^t (\rho - \tau\sigma)\,d\tau\, S_1(0)$$

$$= (\rho t - \sigma \frac{t^2}{2}) S_1(0)$$

$$z_{10} - z_{20} - t z_{30} \overset{?}{\in} (l + \rho t - \frac{\sigma t^2}{2}) S_1(0)$$

$$t \in [0; \frac{\rho}{\sigma}]$$

$$|z_{10} - z_{20} - t z_{30}| \le l + \rho t - \frac{\sigma t^2}{2}$$

④ (2 проходила.)

$$\begin{cases} \dot z_1 = z_2 \\ \dot z_2 = -u \\ \dot z_3 = z_4 \\ \dot z_4 = v \end{cases} \qquad \begin{aligned} &M = \{z \in \mathbb{R}^{4k} : |z_1 - z_3| \le l\} \\ &N = (E_k, O_k, -E_k, O_k) \\ &\mathfrak{M} = \{z \in \mathbb{R}^{4k} : |z| \le l, z_2 = z_3 = z_4 = 0\} \end{aligned}$$

$$e^{tA} = \begin{pmatrix} e^{tA_1} & 0 \\ 0 & e^{tA_1} \end{pmatrix} = \left(\begin{array}{c|c} \begin{matrix} E_k & tE_k \\ O_k & E_k \end{matrix} & O_{2k} \\ \hline O_{2k} & \begin{matrix} E_k & tE_k \\ O_k & E_k \end{matrix} \end{array} \right)$$

$$Bu = \begin{pmatrix} 0 \\ u \\ 0 \\ 0 \end{pmatrix} \qquad Cv = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

$$N e^{\tau A} Bu = (E_k, O_k, -E_k, O_k) \begin{pmatrix} \tau u \\ u \\ 0 \\ 0 \end{pmatrix} = \tau u$$

$$N e^{\tau A} Cv = (-\tau) v$$

$$N e^{\tau A} BP = \tau P, \quad N e^{\tau A} CQ = (-\tau) Q$$

$$N e^{\tau A} BP \overset{*}{-} N e^{\tau A} CQ = \tau P \overset{*}{-} (-\tau) Q$$

$= \{ P = S_\rho(0);$

$Q = S_\sigma(0),\ \rho, \sigma > 0\} =$

$$= \tau S_\rho(0) \overset{*}{-} (\tau) S_\sigma(0) = \tau S_{\rho - \sigma}(0), \quad \tau\rho \ge \tau\sigma, \quad \rho \ge \sigma$$

$$N e^{tA} z_0 \overset{?}{\in} N\mathfrak{M} + \int_0^t \hat w(\tau)\,d\tau$$

$$z_0 \to \begin{pmatrix} z_{10} \\ z_{20} \\ z_{30} \\ z_{40} \end{pmatrix} \qquad N\mathfrak{M} = S_l(0)$$

$$\int_0^t (\rho\tau - \sigma\tau) S_1(0)\,d\tau$$

$$O_k \quad tE_k \quad -E_k \quad -tE_k$$

$$\text{л.ч.} \Rightarrow z_{10} + t z_{20} - z_{30} - t z_{40} \overset{?}{\in} S_l(0) + \int_0^t \hat w(\tau)\,d\tau$$

$$= S_l(0) + (\frac{\rho t^2}{2} - \sigma \frac{t^2}{2}) S_1(0) = (l + (\rho - \sigma)\frac{t^2}{2}) S_1(0).$$

если $\rho=\sigma$, то

$z_{10}+tz_{20}-z_{30}-tz_{40}$

1.12.08 Контрольный пример Понтрягина

$$\begin{cases}\dot z_1=z_2\\ \dot z_2=-\alpha z_2-u\\ \dot z_3=z_4\\ \dot z_4=-\beta z_4+v\end{cases}$$

$\ddot x+\alpha\dot x=-u$

$\ddot y+\beta\dot y=v$

$M=\{z\in\mathbb{R}^{4k}: |z_1-z_3|\le l\}$

$N=(E_k,O_k,-E_k,O_k)=(\hat 1,\hat 0,-\hat 1,\hat 0)$

$L=\{z: z_1-z_3=0\}$

$$A=\left(\begin{array}{cc|cc} O_k & E_k & O_k & O_k\\ O_k & -\alpha E_k & O_k & O_k\\ \hline O_k & O_k & O_k & E_k\\ O_k & O_k & O_k & -\beta E_k\end{array}\right)$$

$A_1=\begin{pmatrix}O_k & E_k\\ O_k & -\alpha E_k\end{pmatrix}$

$e^{tA_1}=E+tA_1+\frac{t^2}{2!}A_1^2+\dots$

$\begin{cases}\dot y_1=y_2\\ \dot y_2=-\alpha y_2\end{cases}$ $\frac{dy_2}{y_2}=-\alpha dt$ $\ln y_2=-\alpha t+\ln C$ $y_2=e^{-\alpha t}\cdot C$

$\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}$

$$e^{tA_1}=\begin{pmatrix}1 & \frac{1-e^{-\alpha t}}{\alpha}\\ 0 & e^{-\alpha t}\end{pmatrix}$$

$$e^{tA_2}=\begin{pmatrix}1 & \frac{1-e^{-\beta t}}{\beta}\\ 0 & e^{-\beta t}\end{pmatrix}$$

$$e^{tA}=\begin{pmatrix}E_k & \frac{1-e^{-\alpha t}}{\alpha}E_k & O_k & \\ O_k & e^{-\alpha t}E_k & & \\ & & E_k & \frac{1-e^{-\beta t}}{\beta}E_k\\ O_k & & O_k & e^{-\beta t}E_k\end{pmatrix}\begin{pmatrix}0\\u\\0\\0\end{pmatrix}=$$

$$=N\begin{pmatrix}\frac{1-e^{-\alpha t}}{\alpha}u\\ e^{-\alpha t}u\\ 0\\ 0\end{pmatrix}=\frac{1-e^{-\alpha t}}{\alpha}u \qquad Bu=\begin{pmatrix}0\\u\\0\\0\end{pmatrix}$$

$Ne^{\tau A}BS_\rho(0)=\rho\frac{1-e^{-\alpha\tau}}{\alpha}S_1(0)$

$Ne^{\tau A}CS_\sigma(0)=\sigma\frac{1-e^{-\beta\tau}}{\beta}S_1(0)$

$\hat w(\tau)=\rho\frac{1-e^{-\alpha\tau}}{\alpha}S_1(0)\overset{*}{-}\sigma\frac{1-e^{-\beta\tau}}{\beta}S_1(0)$

$\rho\frac{1-e^{-\alpha\tau}}{\alpha}\geqslant\sigma\frac{1-e^{-\beta\tau}}{\beta}$

если $\rho>\sigma$, то

$$\rho \cdot \frac{\alpha\tau + O(\tau^2)}{\alpha} \overset{?}{\geqslant} \sigma \frac{\beta\tau + O(\tau^2)}{\beta}$$

$$\rho\,(\tau + O(\tau^2)) \geqslant \sigma\,(\tau + O(\tau^2)).$$

$$\rho\tau + O(\tau^2)(\rho - \sigma) \geqslant \sigma\tau$$

$$(\rho-\sigma)\tau + O(\tau^2)(\rho-\sigma) \geqslant 0$$

$$\rho\,\frac{1-e^{-\alpha\tau}}{\alpha} \geqslant \sigma\,\frac{1-e^{-\beta\tau}}{\beta}$$

$$\frac{\rho}{\alpha}\cdot\frac{\beta}{\sigma}\,\frac{1-e^{-\alpha\tau}}{1-e^{-\beta\tau}} \geqslant 1$$

$$f(\tau) = \frac{1-e^{-\alpha\tau}}{1-e^{-\beta\tau}}$$

$$f(0) = \frac{\alpha}{\beta}.$$

$$f(+\infty) = 1$$

1 случай $\alpha > \beta$ $\rho > \sigma$

$f(\tau) \in [1;\ \frac{\alpha}{\beta}]$.

1 $\frac{\alpha}{\beta}$ $f(\tau_1) = \frac{\alpha}{\beta}$

$$h(\tau) = \frac{1-e^{-\alpha\tau}}{\alpha} - \frac{1-e^{-\beta\tau}}{\beta}$$

$$h(0) = 0 \quad h(\tau_1) = 0.$$

$$h'(\tau) = e^{-\alpha\tau} - e^{-\beta\tau}, \quad e^{-\alpha\tau_2} - e^{-\beta\tau_2} = 0$$

2 случай $\beta > \alpha$

$$f(\tau) = \frac{1-e^{-\alpha\tau}}{1-e^{-\beta\tau}}, \quad g(\tau) = \frac{1-e^{-\beta\tau}}{1-e^{-\alpha\tau}}$$

$$g(\tau) \in [1;\ \frac{\beta}{\alpha}]$$

$$f(\tau) \in [\frac{\alpha}{\beta},\ 1]$$

$$\gamma(\tau) = \frac{\rho}{\alpha}\cdot\frac{\sigma}{\beta}, \quad f(\tau) \overset{?}{\geqslant} 1$$

$$\gamma(\tau) \in co\left\{\frac{\rho}{\alpha}\cdot\frac{\sigma}{\beta};\ \frac{\rho}{\sigma}\right\}.$$

$$\left.\begin{aligned} &\frac{\rho}{\alpha}\cdot\frac{1}{\frac{\sigma}{\beta}} \geqslant 1 \\ &\frac{\rho}{\sigma} \geqslant 1 \end{aligned}\right\} \qquad \begin{aligned} &\frac{\rho}{\alpha} \geqslant \frac{\sigma}{\beta} \\ &\rho \geqslant \sigma \end{aligned}$$

$$z_1^0 + \frac{1-e^{-\alpha t}}{\alpha} z_2^0 - z_3^0 - \frac{1-e^{-\beta t}}{\beta} z_4^0 \overset{?}{\in} S_1(0) +$$

$$+ \int_0^t \left(\rho\cdot\frac{1-e^{-\alpha\tau}}{\alpha} - \sigma\,\frac{1-e^{-\beta\tau}}{\beta}\right) d\tau \cdot S_1(0)$$

$$\rho\cdot\frac{1-e^{-\alpha\tau}}{\alpha} - \sigma\,\frac{1-e^{-\beta\tau}}{\beta} = \mu(\tau)$$

$$\mu(\tau) \approx \frac{\rho}{\alpha} - \frac{\sigma}{\beta} > 0$$

15.15 – контр. работа.

7 12.08.

$$\dot z = Az - Bu + Cv \qquad [0, \theta]$$

$$N e^{tA} z_0 \in N\mathcal{W}_\tau + \int_0^t \hat{w}(\tau)\,d\tau$$

(это должно быть выполнено, надо проверить)

$N \cdot \gamma$ $\boxed{}$

как решить $Ny = b \in NR^n$

$$y = \underbrace{N^+ b}_{\text{псевдообратная матрица}} + L$$

$$e^{tA} x_0 \in N^+\left(N\mathfrak{M} + \int_0^t \hat{w}(\tau)\,d\tau\right) + L$$

$$\Rightarrow \quad x_0 \in e^{-tA}\left(N^+\left(N\mathfrak{M} + \int_0^t \hat{w}(\tau)\,d\tau\right) + L\right)$$

$$\bigcup_{t\in[0;\theta]} e^{-tA}(.(.).) \setminus M$$

$$Ne^{\tau A} x_0 = \xi + \int_0^\tau \tilde{w}(\tau)\,ds$$

$$Ne^{\tau A} x_0 \in N\mathfrak{M} + \int_0^\tau \hat{w}(\tau)\,d\tau$$

$\eta \in$ гр. $\int_0^\tau \hat{w}(\tau)\,d\tau$?

$$\dot{x} = Ax + Bu \qquad u \in P$$

$$x(0) = x_0, \quad x(t_1) = x_1$$

Допустим, что $\eta \in \operatorname{int} \int_0^\tau \hat{w}(\tau)\,d\tau$

$$\eta + S_{2\varepsilon}(0) \subset \int_0^\tau \hat{w}(\tau)\,d\tau$$

$$Ne^{\tau A} x_0 + S_{2\varepsilon}(0) \subset \xi + \int_0^\tau \hat{w}(\tau)\,d\tau$$

$$\{ Ne^{\tau A} x_0 = \xi + \eta \}$$

$Ne^{tA} x_0 + S_\varepsilon(0) \subset \xi + \int_0^t \hat{w}(\tau)\,d\tau + S_\varepsilon(0)$, где $t < \tau$ и t близок к τ.

$$Ne^{\tau A} x_0 \in Ne^{tA} x_0 + S_\varepsilon(0)$$

$$\underset{\text{вып. комп.}}{U} + S_\varepsilon(0) \subset \underset{\text{вып. комп.}}{V} + S_\varepsilon(0).$$

$U \subset V$ — ?

$$W(U,\psi) \le W(V,\psi)$$

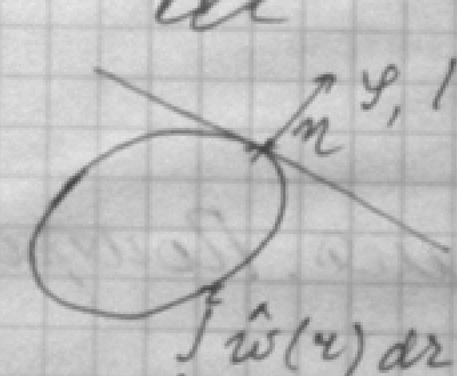
$$\Downarrow$$

$$U \subset V$$

$$\Rightarrow \quad Ne^{tA} x_0 \in \xi + \int_0^t \hat{w}(\tau)\,d\tau \in N\mathfrak{M} + \int_0^t \hat{w}(\tau)\,d\tau$$

~~какая~~

$\cancel{e^{tA}}$



$$\int_0^\tau W(\hat{w}(\tau),\psi)\,ds = \int_0^\tau \langle \tilde{w}(\tau),\psi\rangle\,d\tau$$

$$\langle \tilde{w}(\tau),\psi\rangle \overset{\text{п.в.}}{=} W(\hat{w}(\tau),\psi)$$

на мн-ве пол. меры

$$W(\hat{w}(\tau),\psi) = \max_{\xi \in \hat{w}(\tau)} \langle \xi,\psi\rangle$$

$$\dot{x} = Ax + Bu$$

$$\dot{\psi} = -A^*\psi$$

$$\max_{u\in P} \langle \tilde{\psi}(t), Bu\rangle \overset{\text{п.в.}}{=} \langle \tilde{\psi}(t), B\tilde{u}(t)\rangle$$

$$Ne^{t_1 A} z_0 \in \underbrace{N\mathfrak{M}}_{\text{вып. комп.}} + \int_0^{t_1} \hat{w}(\tau)\,d\tau \leftarrow \text{вып. комп.}$$

$$\langle Ne^{t_1 A} z_0, \psi \rangle \le W(N\mathfrak{M}, \psi) + \int_0^{t_1} W(\hat{w}(\tau), \psi)\,d\tau$$

$\forall \psi.$

$$0 \le \min_{|\psi|=1} \left(W(N\mathfrak{M}, \psi) + \int_0^{t_1} W(\hat{w}(\tau), \psi)\,d\tau - \langle Ne^{t_1 A} z_0, \psi \rangle \right)$$

$$\lambda(t, z_0) = \min_{|\psi|=1} \left(W(N\mathfrak{M}, \psi) + \int_0^{t} W(\hat{w}(\tau), \psi)\,d\tau - \langle Ne^{tA}, z_0 \rangle \right)$$

$$\lambda(0, z_0) < 0.$$

К/р.

2 вар.

Вычисл по 1-му мет. Понтрягина

Ne^{tA} BP

Ne^{tA} CQ, раб. мет. Понтр.?

Пятница 11^{00} у каср.