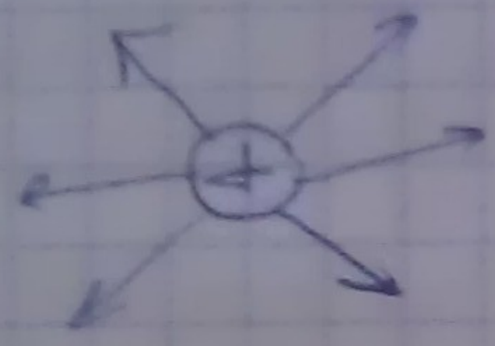
Физика

Ольга Дмитриевна

Урок. Электричество и магнетизм

Электрическое поле.

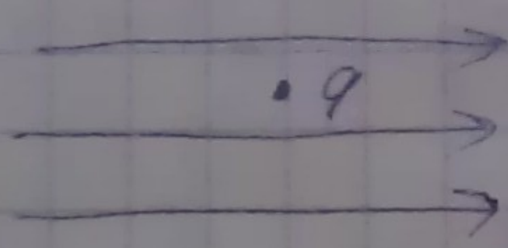
Электромагнитное поле – поле, создаваемое неподвижными зарядами.



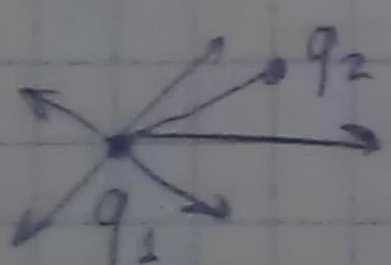


З-н Кулона: $\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^3}\vec{r}$

Напряжённость эл. поля: $\vec{E} = \frac{\vec{F}}{q}$ → заряд, на кот. действует поле.



Хар. силовой характеристикой поля.

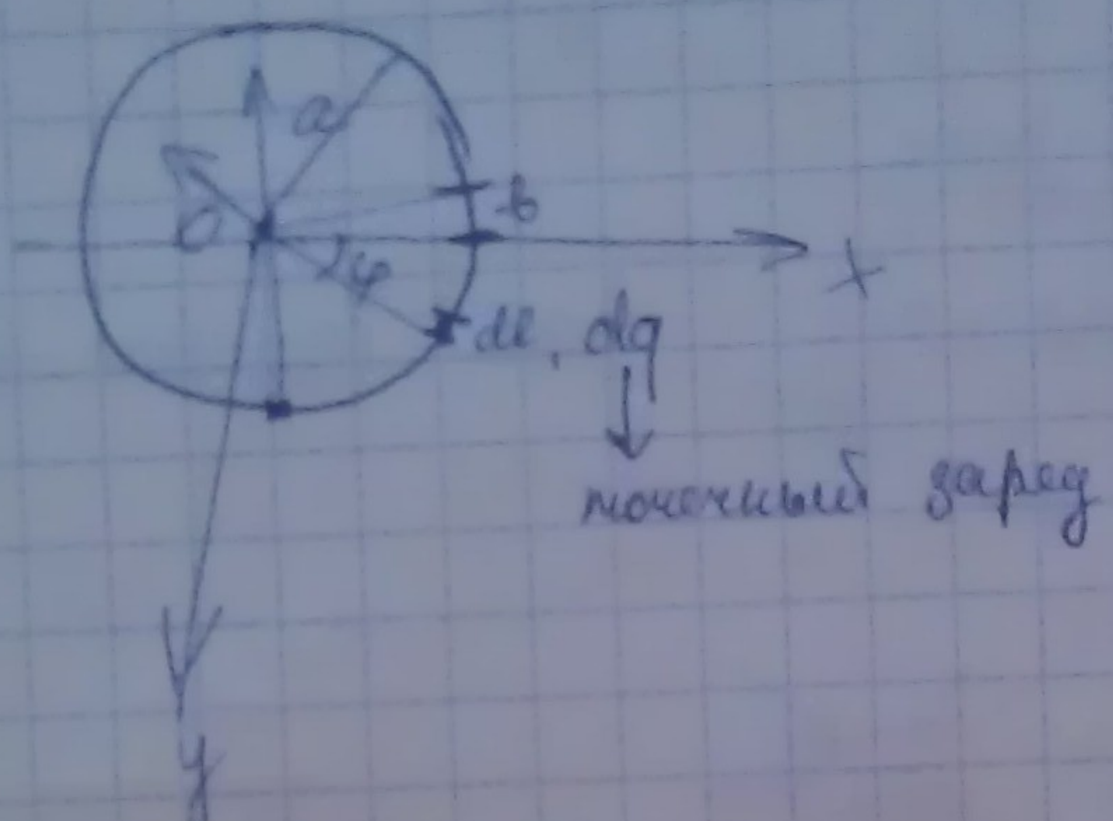


Ж.Ч. $\vec{E} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3 \cdot q_2} = \frac{q_1}{4\pi\varepsilon_0 r^3}\cdot\vec{r}$

Материя → вещество
→ поле
физ. вакуум

Материя = всё, что обладает энергией

№ 1.2 Дано: a, $b \ll a$, q $\vec{E}$ - ?



$$|d\vec{E}| = \frac{dq}{4\pi\varepsilon_0 a^2}$$

$dE_x = \frac{dq}{4\pi\varepsilon_0 a^2} \cdot \cos\varphi$ - проекция на Ox

$dE_y = \frac{dq}{4\pi\varepsilon_0 a^2} \cdot \sin\varphi$ - на Oy

$$dq = \frac{q \cdot dl}{(2\pi a - b)} \quad \text{т.к.} \quad \frac{dq}{q} = \frac{dl}{(2\pi a - b)}$$

$dl = a\,d\varphi$

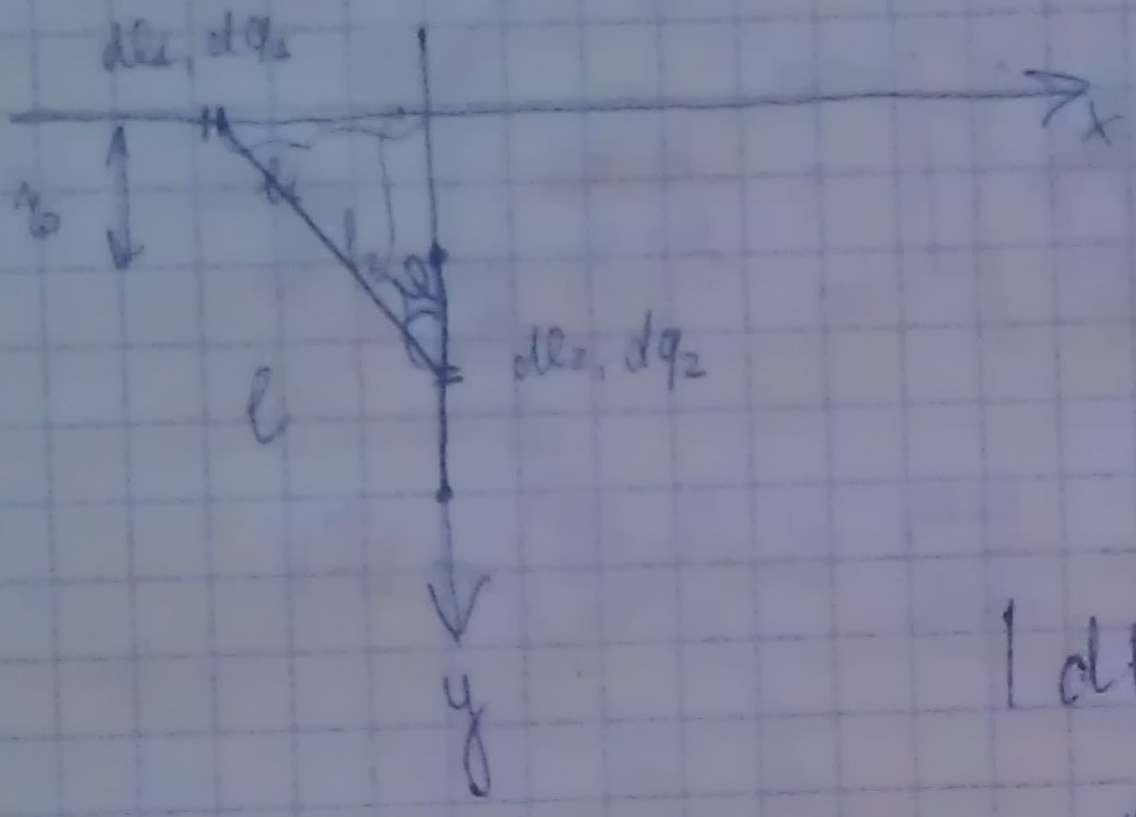
$$E_x = \int_0^{2\pi - b/a} \frac{q a\, d\varphi}{4\pi\varepsilon_0 a^2 (2\pi a - b)} \cos\varphi = \frac{q\left(-\sin\frac{b}{a}\right)}{4\pi\varepsilon_0 a (2\pi a - b)} =$$

$$E_y = \int_0^{2\pi - b/a} \frac{q a \sin\varphi\, d\varphi}{4\pi\varepsilon_0 a^2 (2\pi a - b)} = \frac{-q\left(1 - \cos\frac{b}{a}\right)}{4\pi\varepsilon_0 a (2\pi a - b)}$$

$\frac{b}{a} \to 0 \Rightarrow \sin\frac{b}{a} \approx \frac{b}{a} \Rightarrow E_y \to 0$

$\cos\frac{b}{a} \approx 1 - \frac{b^2}{2a^2}$

№ 1.6 Дано: $\kappa_1, l, \kappa_2, r_0$. $\vec{F}$ - ?



$dq_2 = \kappa_2 dl_2$

$dq_2 = \kappa_1 dl_1$

$|dF| = \frac{1}{4\pi\varepsilon_0} \frac{\kappa_2 \kappa_1 dl_2 dl_1}{r^2} =$

$= \frac{1}{4\pi\varepsilon_0} \frac{\kappa_2 \kappa_1 dl_2 dl_1}{l_1^2 + l_2^2}$

$dF_x = \frac{1}{4\pi\varepsilon_0} \frac{\kappa_2 \kappa_1 dl_2 dl_1}{l_1^2 + l_2^2} \sin\varphi$

$dF_y = dF\cos\varphi$

$\sin\varphi = \frac{l_1}{\sqrt{l_1^2 + l_2^2}}$; $\cos\varphi = \frac{l_2}{\sqrt{l_1^2 + l_2^2}}$

$F_x = \frac{\kappa_1 \kappa_2}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} dl_1 \int_{r_0}^{r_0+l} \frac{l_1 dl_2}{(l_1^2 + l_2^2)^{3/2}} = \frac{\kappa_1 \kappa_2}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} l_1 dl_1 \int_{r_0}^{r_0+l} \frac{dl_2}{(l_1^2 + l_2^2)^{3/2}} =$

$= \frac{\kappa_1 \kappa_2}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} l_1 dl_1 \frac{l_2}{l_1^2 \sqrt{l_1^2 + l_2^2}} \Big|_{r_0}^{r_0+l} = \frac{\kappa_1 \kappa_2}{4\pi\varepsilon_0} \int_{r_0}^{r_0+l} \left(\frac{1}{\sqrt{l_1^2 + l_2^2}} \right) \Big|_{-\infty}^{+\infty} dl_2 = 0$

$$F_y = \int_{-\infty}^{+\infty} dl_1 \int_{x_0}^{x_0+l} \frac{k_1 k_2}{4\pi\varepsilon_0} \frac{l_2\, dl_2}{(l_1^2+l_2^2)^{3/2}} =$$

$$= \frac{k_1 k_2}{4\pi\varepsilon_0} \int_{x_0}^{x_0+l} dl_2\, l_2 \int_{-\infty}^{+\infty} \frac{dl_1}{(l_1^2+l_2^2)^{3/2}} = \frac{k_1 k_2}{4\pi\varepsilon_0} \int_{x_0}^{x_0+l} l_2\, dl_2 \left(\frac{l_1}{l_2^2\sqrt{l_1^2+l_2^2}} \right) \Bigg|_{-\infty}^{+\infty}$$

$$= \frac{k_1 k_2}{4\pi\varepsilon_0} \int_{x_0}^{x_0+l} \frac{dl_2}{l_2} \left(\frac{l_1}{l_2\sqrt{1+\left(\frac{l_2}{l_1}\right)^2}} \right) \Bigg|_{-\infty}^{+\infty} = \frac{k_1 k_2}{2\pi\varepsilon_0} \int_{x_0}^{x_0+l} \frac{dl_2}{l_2} =$$

$$= \frac{k_1 k_2}{2\pi\varepsilon_0} \ln l_2 \Big|_{x_0}^{x_0+l} = \frac{k_1 k_2}{2\pi\varepsilon_0} \ln\left(\frac{x_0+l}{x_0}\right) = |\vec{F}|$$

№1.10 Дано: σ, R_1, R_2



$$dq = \sigma\, dS$$

$$\vec{dE} = \frac{dq\,\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$|dE| = dE_z = \frac{dq}{4\pi\varepsilon_0(r^2+z^2)} \cdot \cos\varphi =$$

$$= \frac{z\,dq}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} = \frac{z\,\sigma\,dS}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

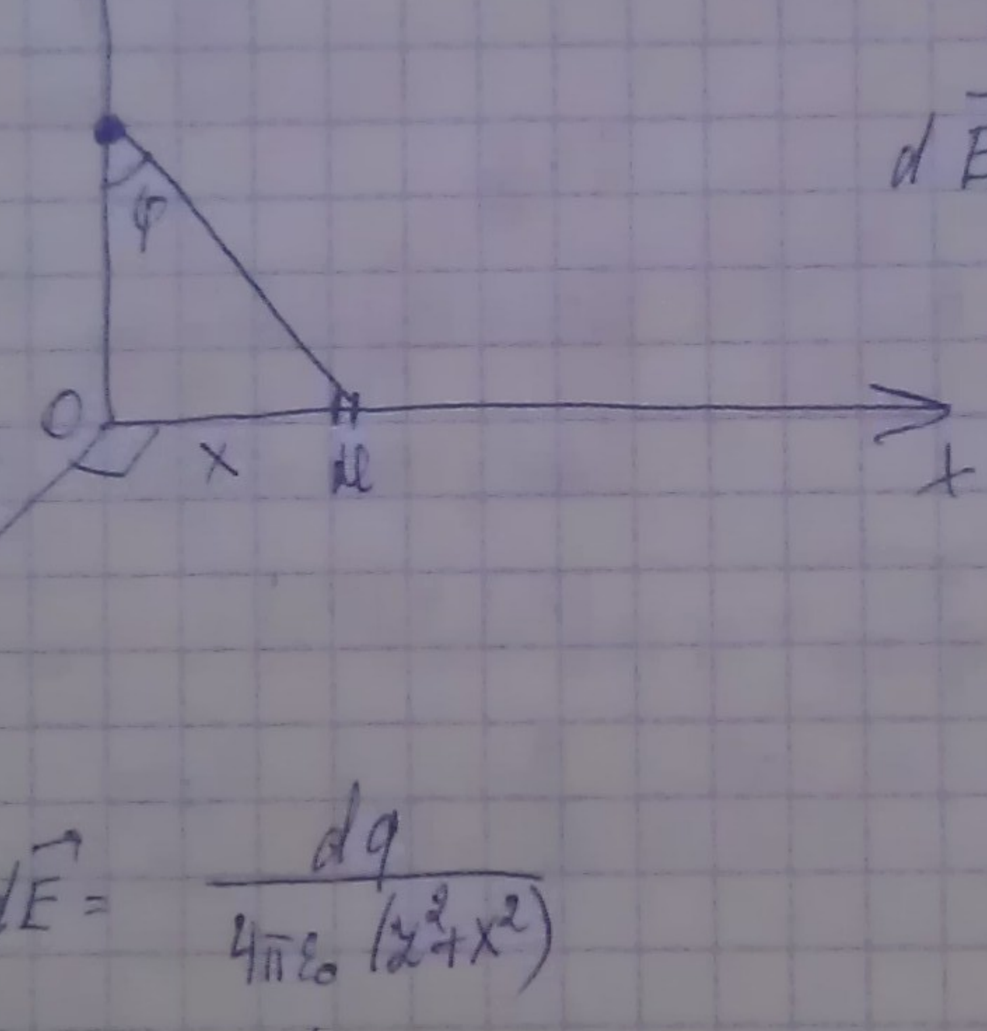
$$dS = r\,d\varphi\,dr$$

$$|dE| = \frac{z\sigma r\,d\varphi\,dr}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$E_z = \int_0^{2\pi} d\varphi \int_{R_1}^{R_2} \frac{z\sigma r\, dr}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} = \frac{z\sigma}{2\varepsilon_0} \int_{R_1}^{R_2} \frac{r\, dr}{(z^2+r^2)^{3/2}} =$$

$$= \frac{z\sigma}{2\varepsilon_0} \left(\frac{1}{\sqrt{R_1^2+z^2}} - \frac{1}{\sqrt{R_2^2+z^2}} \right)$$

№ 1.22 Дано κ, $\vec{E}$ - ?



$dq = \kappa\, dl$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^3}\vec{r}$$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 (z^2+x^2)}$$

$$dE_x = \frac{dq\, x}{4\pi\varepsilon_0 (z^2+x^2)^{3/2}} = \frac{x\kappa\, dx}{4\pi\varepsilon_0 (z^2+x^2)^{3/2}}$$

$$dE_z = \frac{dq\, z}{4\pi\varepsilon_0 (z^2+x^2)^{3/2}} = \frac{z\kappa\, dx}{4\pi\varepsilon_0 (z^2+x^2)^{3/2}}$$

$$E_x = \int_0^{+\infty} \frac{\kappa x\, dx}{4\pi\varepsilon_0 (x^2+z^2)} = \frac{\kappa}{4\pi\varepsilon_0} \left(-\frac{1}{\sqrt{x^2+z^2}} \right)\Big|_0^{+\infty} = \frac{\kappa}{4\pi\varepsilon_0 z}$$

$$E_z = \frac{\kappa z}{4\pi\varepsilon_0} \int_0^{+\infty} \frac{dx}{\sqrt{x^2+z^2}} = \frac{\kappa z}{4\pi\varepsilon_0} \frac{x}{z^2\sqrt{x^2+z^2}}\Big|_0^{+\infty} = \frac{\kappa}{4\pi\varepsilon_0 z}$$

$E_y = 0$

Обе сеи: $E_x = E_y = \frac{k}{4\pi\varepsilon_0 x}$

$$E_x = \frac{k}{2\pi\varepsilon_0 x}$$

$$|E| = \frac{k}{\pi\varepsilon_0 x}\sqrt{\frac{1}{16}+\frac{1}{16}+\frac{4}{16}} = \frac{k\sqrt{6}}{4\pi\varepsilon_0 x}$$

Дз: ~~1~~, ~~2~~, ~~3~~, ~~4~~, ~~7~~, ~~15~~, 21, ~~15~~

№ 1 1

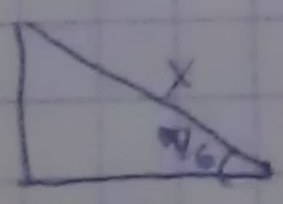


$\varphi = \frac{\pi}{6}$

$|AB| = |BC| = |AC| = a$

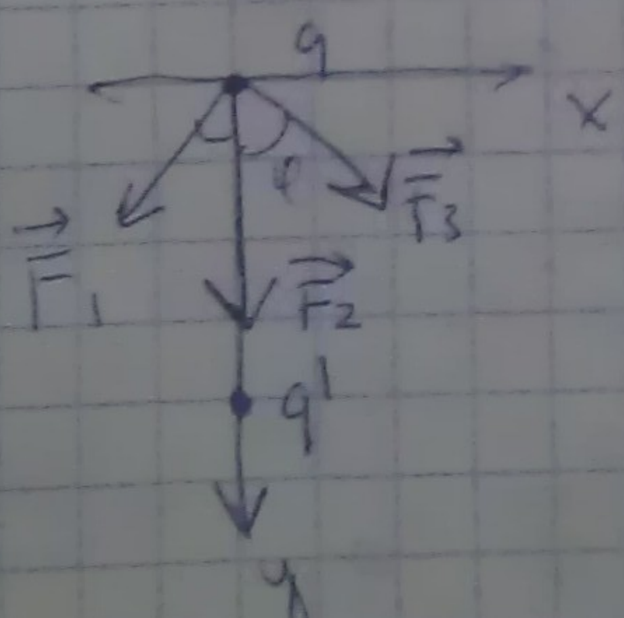
$|AO| = |BO| = |CO| = \frac{a\sqrt{3}}{3} = \frac{a}{\sqrt{3}}$

$|F_1| = |F_3|$



$\frac{\sqrt{3}}{2} = \frac{a}{2x}$ $\quad x = \frac{a\sqrt{3}}{3}$

$F_{1x} + F_{2x} + F_{3x} = 0$



$F_{1y} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\cdot\frac{\sqrt{3}}{2}$

$F_{3y} = F_{1y}$

$F_{2y} = \frac{1}{4\pi\varepsilon_0}\frac{3qq'}{a^2}$

$$\frac{\sqrt{3}qq'}{a^2} + \frac{q^2\sqrt{3}}{a^2} = 0 \Rightarrow \boxed{q' = -\frac{q}{\sqrt{3}}}$$

№ 1.3 Дано: $m = 9,1 \cdot 10^{-31}$ кг

$e = 1,6 \cdot 10^{-19}$ Кл

$G = 6,67 \cdot 10^{-11} \frac{Н \cdot м^2}{кг^2}$

$\frac{F_e}{F_г} - ?$

$|F_г| = G \frac{m^2}{r^2}$

$|F_e| = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$

$\frac{|F_e|}{|F_г|} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{m^2} \cdot \frac{1}{G} = \boxed{\left(\frac{e}{m}\right)^2 \cdot \frac{1}{4\pi\varepsilon_0 G}} =$

$= \left(\frac{1,6 \cdot 10^{-19}}{9,1 \cdot 10^{-31}}\right)^2 \cdot \frac{1}{4 \cdot 3,14 \cdot 8,85 \cdot 10^{-12} \cdot 6,67 \cdot 10^{-11}} =$

$= \frac{(0,17 \cdot 10^{12})^2}{741,4 \cdot 10^{-22}} = 3,89 \cdot 10^{-5} \cdot 10^{46} = 3,89 \cdot 10^{41}$

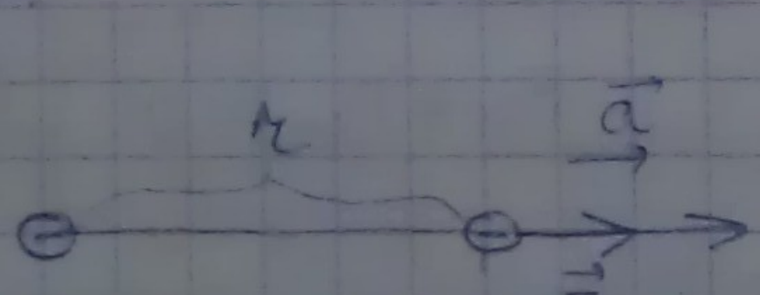
№ 1.4 Дано: $e = 1,6 \cdot 10^{-19}$ Кл

$r = 1$ мм $= 0,001$ м $= 10^{-3}$ м

$m = 9,1 \cdot 10^{-31}$ кг

$a - ?$

$\vec{F} = m\vec{a}$



Ox: $\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} = ma_x$

Oy: $a_y = 0$

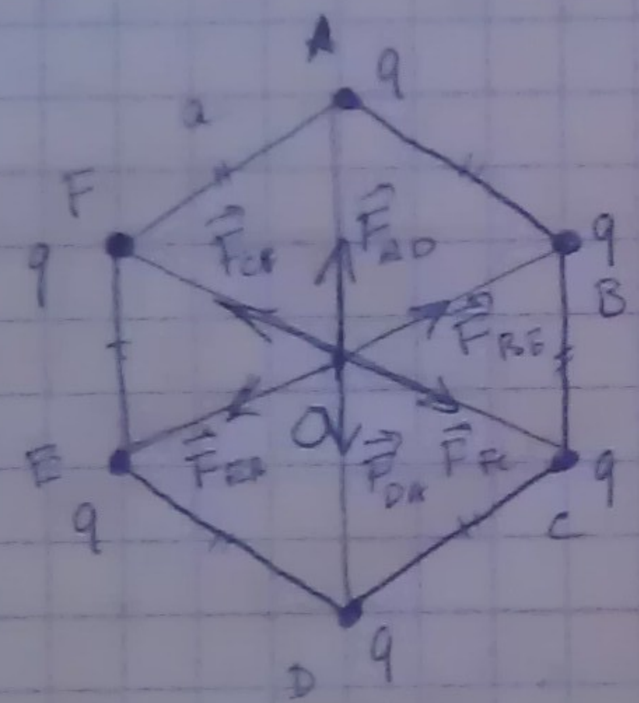
$\boxed{a_x = \frac{e^2}{4\pi\varepsilon_0 r^2 m}}$

$$= \frac{(1,6\cdot 10^{-19})^2}{4\cdot 3,14\cdot 8,85\cdot 10^{-12}\cdot 10^{-6}\cdot 9,1\cdot 10^{-31}} = \frac{2,56\cdot 10^{-38}}{4\cdot 3,14\cdot 8,85\cdot 9,1\cdot 10^{-49}} =$$

$$= \frac{256\cdot 10^{-40}}{1011\cdot 10^{-45}} = 0,25\cdot 10^{5}\left(\frac{N}{C^2}\right) = 2,5\cdot 10^{8}\ \frac{N}{C^2}$$

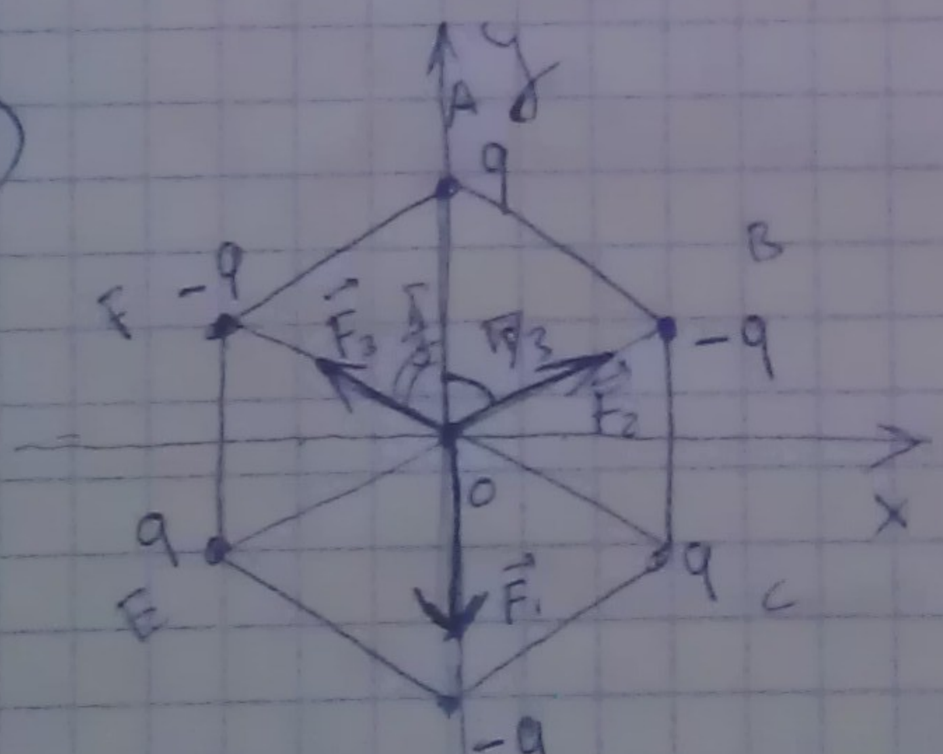
№ 1.7

а)



$$\vec{F}_{AD} = -\vec{F}_{DA}$$
$$\vec{F}_{BE} = \vec{F}_{EB}$$
$$\vec{F}_{CF} = -\vec{F}_{FC}$$
$$\Rightarrow \boxed{F = 0}$$

б)



$$\vec{F}_1 = \vec{F}_{AD} + \vec{F}_{DA}$$
$$\vec{F}_2 = \vec{F}_{EB} + \vec{F}_{BF}$$
$$\vec{F}_3 = \vec{F}_{CF} + \vec{F}_{FC}$$
$$|\vec{F}_1| = |\vec{F}_2| = |\vec{F}_3| = \frac{|\vec{F}|}{3}$$

Ox: $F_{1x} = 0$

$F_{2x} + F_{3x} = 0$

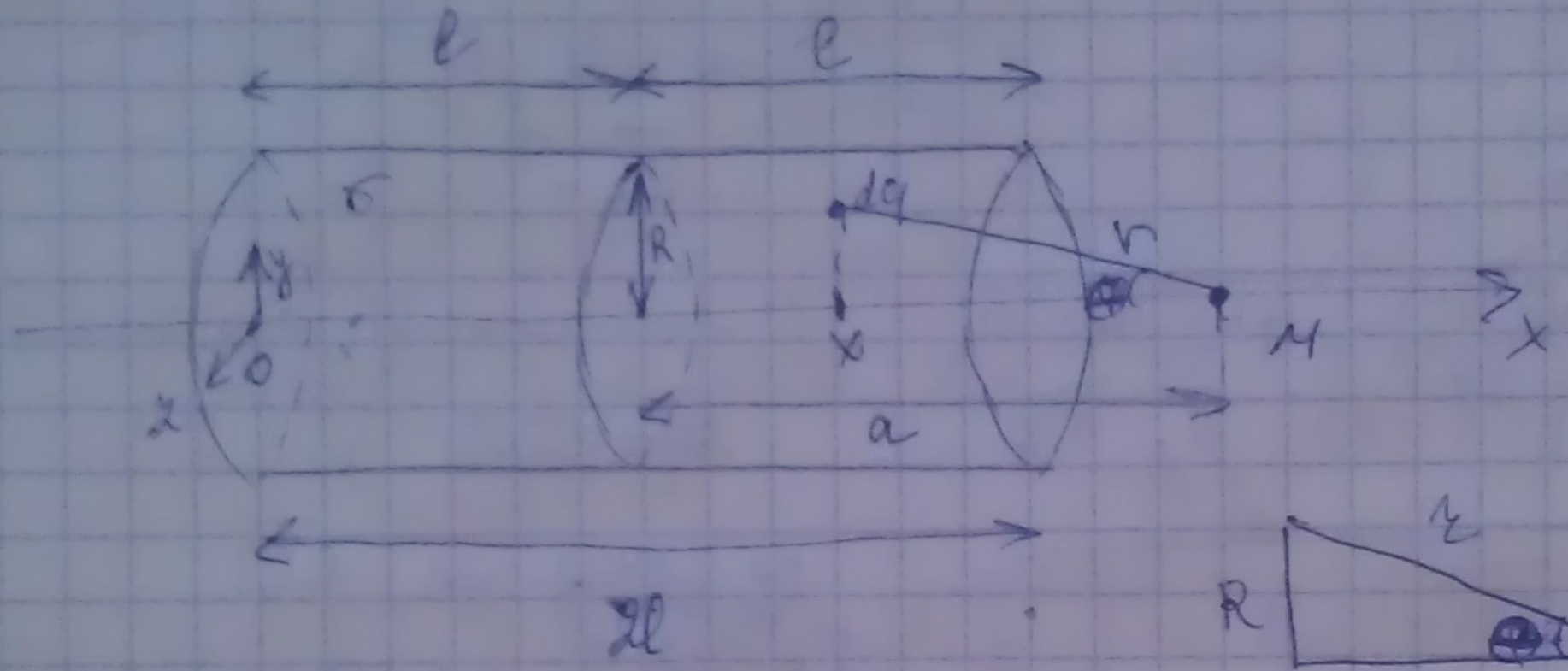
$\Rightarrow F_x = 0 \Rightarrow E_x = 0$

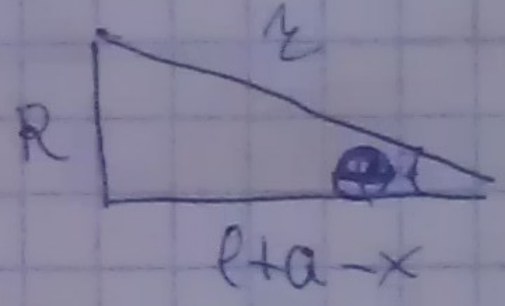
Oy: $F_{1y} = -|\vec{F}_1| = -\frac{|\vec{E}|}{3}$

$F_{2y} = F_{3y} = \frac{|\vec{E}|}{3}\cdot\cos 60^\circ = \frac{|\vec{E}|}{6}$

$F_y = -\frac{|\vec{E}|}{3} + 2\cdot\frac{|\vec{E}|}{6} = 0 \Rightarrow E_y = 0 \Rightarrow \boxed{E = 0}$

№ 1.16 Дано: $R, 2l, \sigma, a$





Реш: $E_y = 0 = E_z$

$$dE_x = \frac{dq}{4\pi\varepsilon_0 z^2} = \frac{\sigma dS}{4\pi\varepsilon_0 z^2} \qquad dS = R d\varphi dx !$$

$$z^2 = R^2 + (l+a-x)^2$$

$$\cos\alpha = \frac{l+a-x}{z} = \frac{l+a-x}{\sqrt{R^2+(l+a-x)^2}}$$

$$E_x = \int_0^{2\pi} d\varphi \int_0^{2l} \frac{R\sigma (l+a-x)\, dx}{4\pi\varepsilon_0 \left(R^2+(l+a-x)^2\right)^{3/2}} =$$

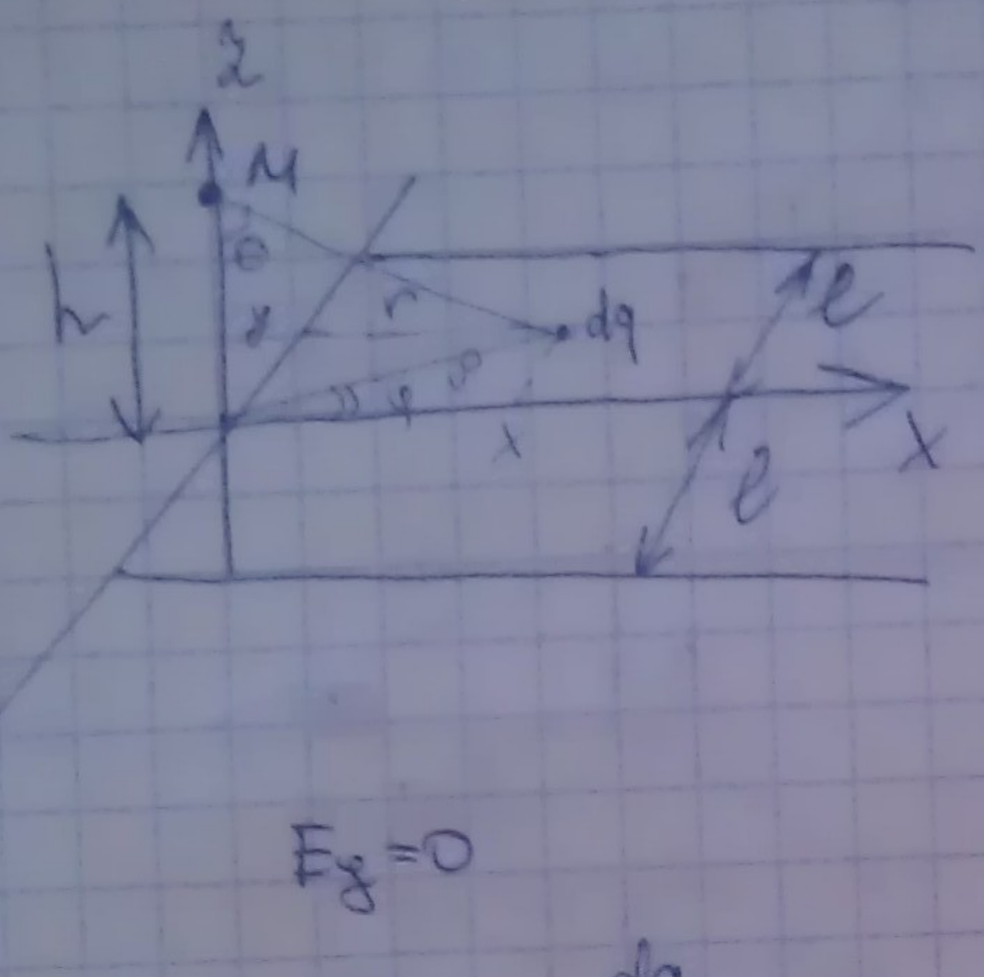
$$= \frac{-\sigma R}{2\varepsilon_0} \int_0^{2l} \frac{(l+a-x)\, d(l+a-x)}{\left(R^2+(l+a-x)^2\right)^{3/2}} = \frac{-\sigma R}{2\varepsilon_0} \int_{a+l}^{a-l} \frac{t\, dt}{(t^2+R^2)^{3/2}} =$$

$$= \left[\frac{\sigma R}{2\varepsilon_0} \left(\frac{1}{\sqrt{(a-l)^2+R^2}} - \frac{1}{\sqrt{(a+l)^2+R^2}} \right) \right]$$

№ 1.18

Дано: 2ℓ, σ, h

$\vec{E}_M$ - ?



$dq = \sigma dS$

$x \in [0, +\infty)$

$E_y = 0$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^3}\cdot\vec{r}$$

$$dE_x = \frac{dq}{4\pi\varepsilon_0 r^2}\cdot\frac{\sigma dS}{4\pi\varepsilon_0 r^2} \qquad dS = dx\,dy$$

$$r^2 = h^2 + \rho^2 = h^2 + x^2 + y^2$$

$$\cos\theta = \frac{h}{r} = \frac{h}{\sqrt{h^2+x^2+y^2}} \qquad \cos\varphi = \frac{x}{\sqrt{x^2+y^2}}$$

$$\sin\theta = \frac{\rho}{r} = \frac{\sqrt{x^2+y^2}}{\sqrt{h^2+x^2+y^2}} \qquad \sin\varphi = \frac{y}{\sqrt{x^2+y^2}}$$

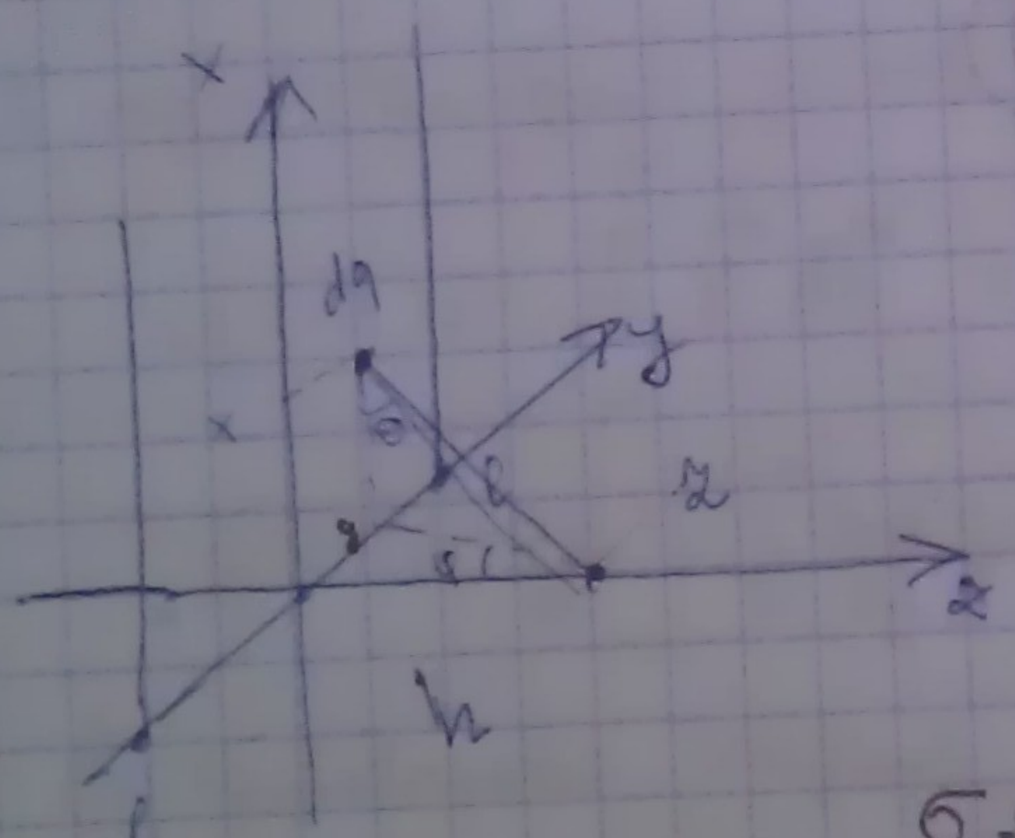
$$dE_x = \frac{\sigma\cdot\sin\theta\cos\varphi\,dx\,dy}{4\pi\varepsilon_0(h^2+x^2+y^2)} =$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\cdot\frac{\sqrt{x^2+y^2}}{\sqrt{h^2+x^2+y^2}}\cdot\frac{x}{\sqrt{x^2+y^2}}\cdot\frac{dx\,dy}{(h^2+x^2+y^2)} = \frac{\sigma}{4\pi\varepsilon_0}\cdot\frac{x\,dx\,dy}{(h^2+x^2+y^2)^{3/2}}$$

$$E_x = \int_0^{+\infty} dx \int_{-\ell}^{\ell} \frac{\sigma}{4\pi\varepsilon_0} \frac{h\,dy}{(h^2+x^2+y^2)^{3/2}} = \frac{\sigma}{4\pi\varepsilon_0} \int_{-\ell}^{\ell} dy \int_0^{+\infty} \frac{x\,dx}{(h^2+y^2+x^2)^{3/2}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_{-\ell}^{\ell} \left(\frac{-1}{\sqrt{x^2+h^2+y^2}} \Big|_0^{+\infty} \right) dy = \frac{\sigma}{4\pi\varepsilon_0} \int_{-\ell}^{\ell} \frac{dy}{\sqrt{h^2+y^2}} =$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \left(\ln\left(\ell + \sqrt{\ell^2+h^2}\right) - \ln\left(\sqrt{\ell^2+h^2} - \ell\right) \right)$$



$$dS = dx\,dy$$

$$dq = \sigma\,dS$$

$$\sin\theta = \frac{\sqrt{h^2+y^2}}{\sqrt{h^2+x^2+y^2}}$$

$$h^2 = x^2+y^2+h^2$$

$$dE_z = \frac{\sigma \sin\theta \cos\varphi\, dx\,dy}{4\pi\varepsilon_0 (h^2+y^2+x^2)}$$

$$\cos\varphi = \frac{h}{\sqrt{h^2+y^2}}$$

$$E_z = \int_0^{+\infty} dx \int_{-\ell}^{\ell} \frac{\sigma}{4\pi\varepsilon_0} \cdot \frac{\sqrt{h^2+y^2}}{\sqrt{h^2+x^2+y^2}} \cdot \frac{h}{\sqrt{h^2+y^2}} \cdot \frac{dx\,dy}{(h^2+y^2+x^2)} =$$

$$= \underbrace{\frac{\sigma h}{4\pi\varepsilon_0}}_{k} \int_0^{+\infty} dx \int_{-\ell}^{\ell} \frac{dy}{(h^2+x^2+y^2)^{3/2}} = k \int_{-\ell}^{\ell} dy \int_0^{+\infty} \frac{dx}{(h^2+x^2+y^2)^{3/2}} =$$

$$= k \int_{-\ell}^{\ell} \left(\frac{1}{h^2+y^2} \cdot \frac{x}{\sqrt{x^2+h^2+y^2}} \right) \Big|_0^{+\infty} dy = k \int_{-\ell}^{\ell} \frac{dy}{h^2+y^2} (1-0) =$$

$$= k \int_{-\ell}^{\ell} \frac{dy}{h^2+y^2} = \frac{1}{h} \operatorname{arctg} \frac{y}{h} \Big|_{-\ell}^{\ell} = \frac{k}{h} \left(\operatorname{arctg} \frac{\ell}{h} - \operatorname{arctg} \left(-\frac{\ell}{h}\right) \right)$$

$$= \frac{2k}{h} \operatorname{arctg} \frac{\ell}{h}, \text{ тогда}$$

$$\| ?$$

$$\frac{\sigma}{2\pi\varepsilon_0} \operatorname{arctg} \frac{\ell}{h} \cdot \operatorname{sgn}(z)$$

№ 1.21

Потенциал электрического поля.

Поле потенциально $\Leftrightarrow$ $\operatorname{rot}\vec{E}=0$ (создаётся неподв. зарядами).

$\vec{E} = -\operatorname{grad}\varphi$

φ - скаляр !!!

Опр. φ - работа, которую нужно совершить, чтобы переместить единичный заряд из данной точки в ту, где $\varphi=0$.

$$\varphi(M) = \int_M^0 \vec{F}\,d\vec{r} = \int_M^0 q\vec{E}\,d\vec{r} = \{q=1\} = \int_M^0 \vec{E}\,d\vec{r}.$$

$$\vec{E} = \vec{i}\frac{\partial\varphi}{\partial x} + \vec{j}\frac{\partial\varphi}{\partial y} + \vec{k}\frac{\partial\varphi}{\partial z}$$

Потенциальная энергия пробного заряда: $W=q\varphi$.

Зад. Дано: q, a; $\varphi(M)$ - ?



– беск. цепочка

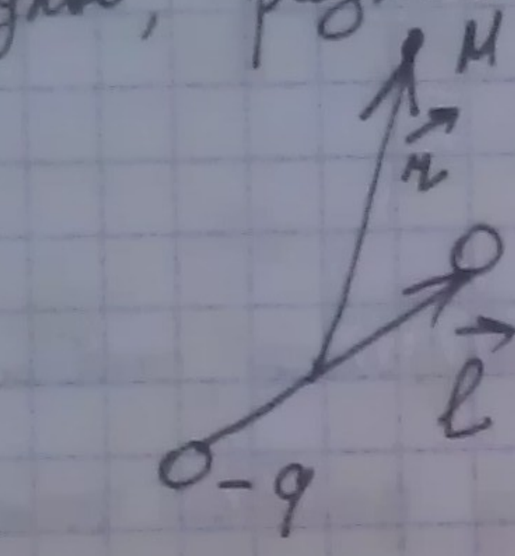
$$E = \frac{q}{4\pi\varepsilon_0 r^2}, \quad \varphi = \frac{q}{4\pi\varepsilon_0 r}$$

$$\varphi = \frac{-2q}{4\pi\varepsilon_0 a} + \frac{2q}{4\pi\varepsilon_0\cdot 2a} + \dots = \sum_{k=1}^{\infty}\frac{(-1)}{k}\cdot\frac{2q}{4\pi\varepsilon_0 a} =$$

$$= \frac{2q}{4\pi\varepsilon_0 a}\cdot\sum_{k=1}\frac{(-1)^k}{k} = \frac{q\ln 2}{2\pi\varepsilon_0 a}$$

№2.4 Дано: $\vec{p}, \vec{E}$. $W, \vec{M}, \vec{F} - ?$

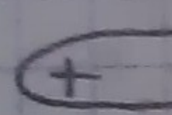
Диполь - система из двух зарядов, равных по модулю, разных по знаку.

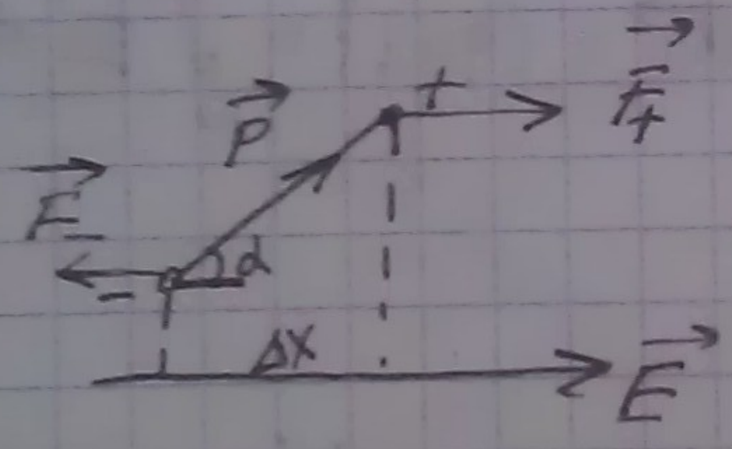


$\vec{p} = q\vec{l}$ – дипольный момент

Точечный диполь: $l \ll r$

NaCl - большинство молекул явл. диполем.





Поле однородно - величина не завис. от коорд.

1) $\vec{F} = \vec{F_+} - \vec{F_-}$

E - одн $\Rightarrow \vec{F} = 0$

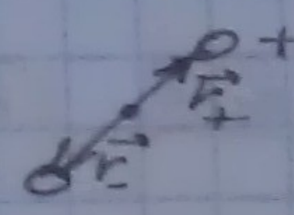
E - неодн $\Rightarrow \vec{F} = q\vec{E_+} - q\vec{E_-} = q(\vec{E_+} - \vec{E_-})$

$$F_x = q\frac{\Delta E_x}{\Delta x}\Delta x = q\frac{\partial E_x}{\partial x} l\cos\alpha = p\frac{\partial}{\partial x} \cdot E_x \cos\alpha =$$

$$= \frac{(\vec{p}, \vec{\nabla}) E_x}{}$$

$$\Rightarrow \boxed{\vec{F} = (\vec{p}, \vec{\nabla})\vec{E}}$$

2) $\vec{M} = [\vec{r}, \vec{F}] =$



$= [\vec{r}_+, \vec{F}_+] + [\vec{r}_-, \vec{F}_-] =$

$= [\vec{r}_+, q\vec{E}] + [\vec{r}_-, -q\vec{E}] =$

$= [\vec{r}_+, q\vec{E}] - [\vec{r}_-, q\vec{E}]$ – неодноор. поле

Если поле однородно: $\vec{M} = [q(\vec{r}_+ - \vec{r}_-), \vec{E}] =$

$= [q\vec{l}, \vec{E}] = [\vec{p}, \vec{E}]$

В вект. форм: $\boxed{\vec{M} = [\vec{p}, \vec{E}]}$

3) $W = q_+\varphi_+ + q_-\varphi_- = q(\varphi_+ - \varphi_-) = q\Delta\varphi =$

$= q\frac{\Delta\varphi}{\Delta x} \cdot \Delta x = q l \cos\alpha \cdot \frac{\partial\varphi}{\partial x} = -(\vec{p}, \vec{E})$

$\boxed{W = -(\vec{p}, \vec{E})}$

а) $\vec{E} \rightarrow$ $\uparrow \vec{p}$

$F = 0$

$M = pE$

$W = 0$

устойчивое равновесие

W_{min}

б) $\vec{p} \rightarrow$

$\vec{E} \rightarrow$

$F = 0$

$M = 0$

$W = -pE$

в) $\vec{E} \rightarrow$

$\vec{p} \leftarrow$

$F = 0$

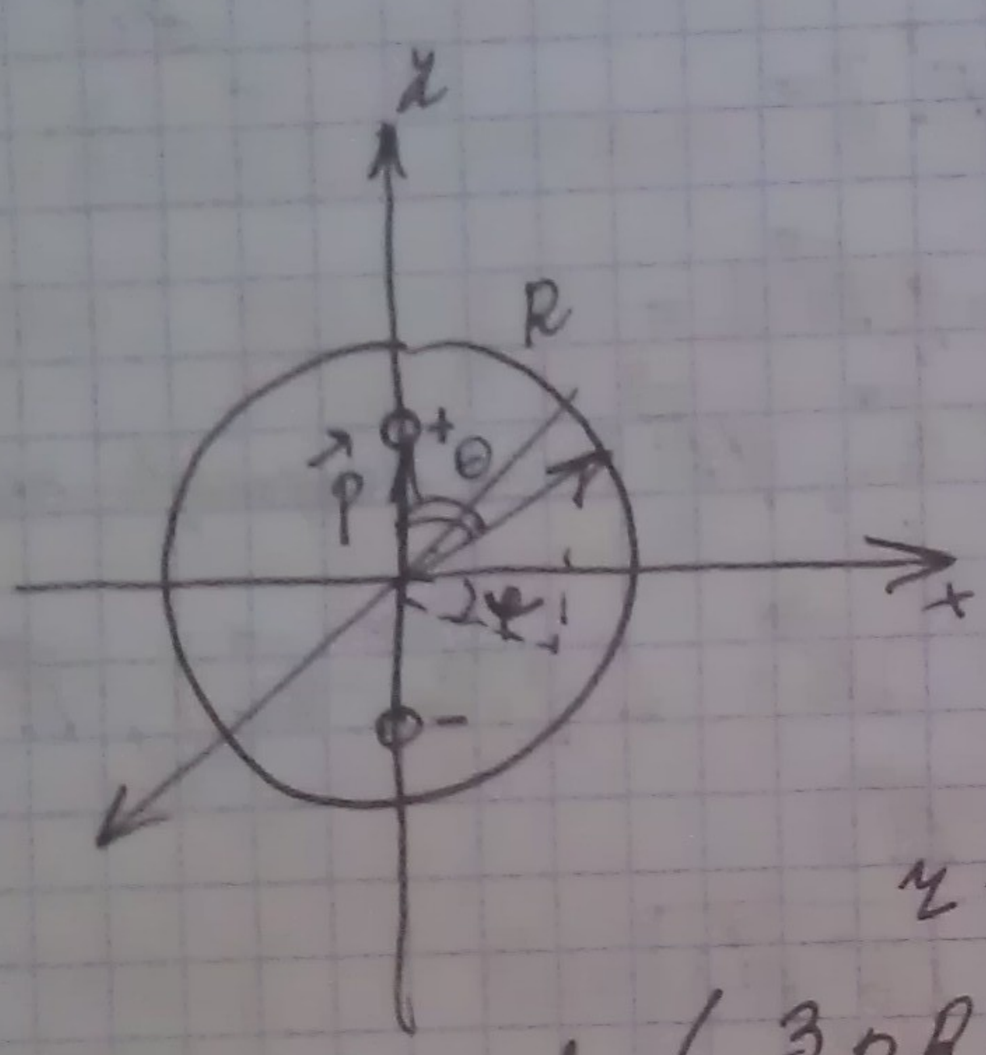
$M = 0$

$W = pE$

неустойчивое равновесие

W_{max}

№ 2.8. Дано: R, p. $|\vec{E}|(\varphi, \theta) - ?$



$$\varphi = \frac{(\vec{r}, \vec{p})}{4\pi\varepsilon_0 r^3}$$ – потенц. поля диполя

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec{p}, \vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right)$$

$r = R$

$$E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{3pR\cos\theta \cdot r\sin\theta\cos\varphi}{r^5} - 0\right) =$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{3pR\sin\theta\cos\theta\cos\varphi}{r^4}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3p\sin\theta\cos\theta\cos\varphi}{R^3}\right)$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\left(\frac{3pR\sin\theta\cos\theta\sin\varphi}{r^4}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3p\sin\theta\cos\theta\sin\varphi}{R^3}\right)$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\left(\frac{3pR\cos^2\theta \cdot R}{R^5} - \frac{p}{R^3}\right) =$$

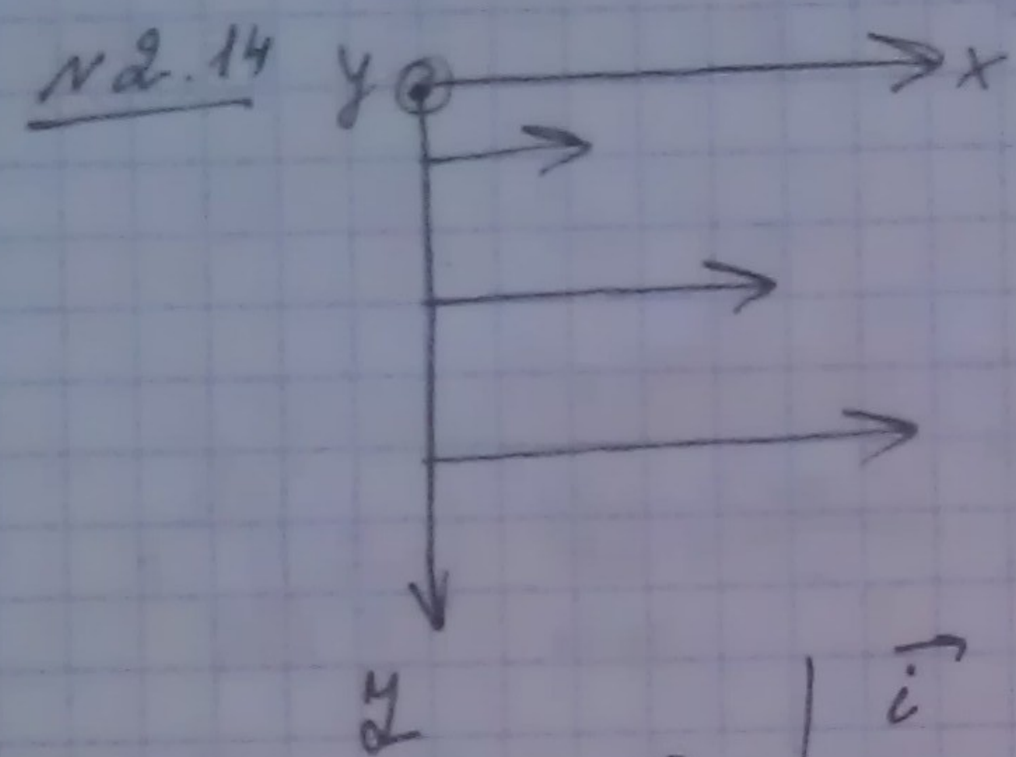
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{3p\cos^2\theta - p}{R^3}\right)$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\sqrt{\frac{9p^2\cos^2\theta\sin^2\theta}{R^6} + \left(\frac{3p\cos^2\theta - p}{R^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 R^3}\sqrt{9\cos^2\theta\sin^2\theta + 9\cos^4\theta - 6\cos^2\theta + 1} =$$

$$= \frac{p}{4\pi\varepsilon_0 R^3}\sqrt{3\cos^2\theta + 1}$$

№2.14



$\exists ?\ \vec{E}$

$rot\,\vec{E} = 0$?

$\vec{E} = \{kz,\ 0,\ 0\}$

$$rot\,\vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$rot\,\vec{E} = E_x\left(\vec{j}\cdot\frac{\partial}{\partial z} - \vec{k}\cdot\frac{\partial}{\partial y}\right) =$$

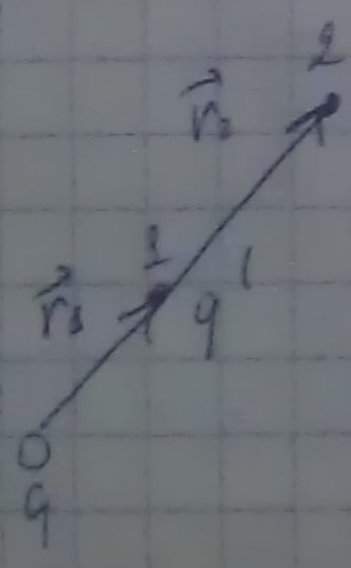
$$= \vec{j}\,\frac{\partial E_x}{\partial z} - \vec{k}\,\frac{\partial E_x}{\partial y}^{0} = \vec{j}\cdot k = k\vec{j} \neq 0$$

$\Rightarrow \nexists$ такое поле.

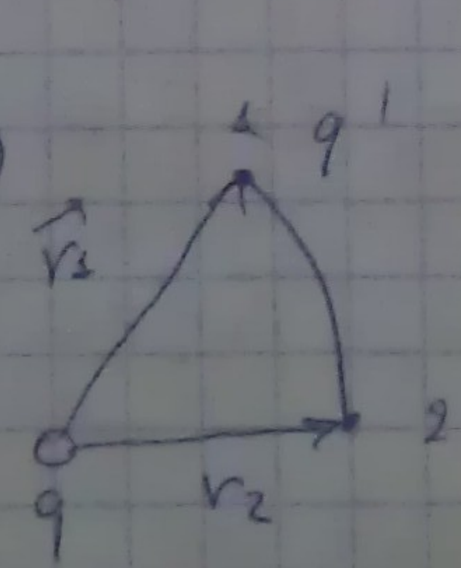
Д/з: ~~2~~, ~~5~~, ~~8~~, ~~6~~, 7, ~~9~~, ~~10~~, ~~15~~

№2.1 Дано: $q, q', \vec{r}_1, \vec{r}_2$ $A - ?$

а)

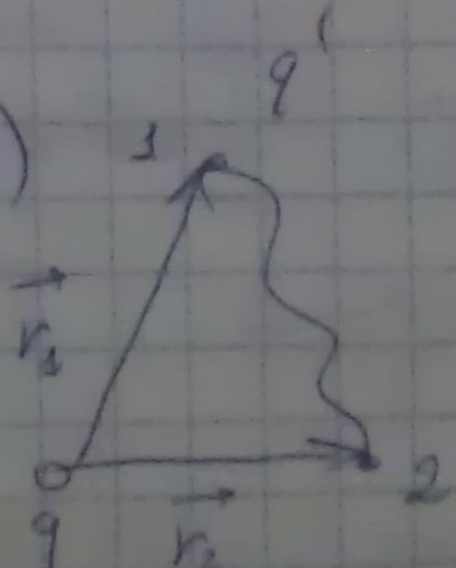


б)



$r_1 = r_2$

в)



$$dA = (\vec{F}, d\vec{r})$$

$$\vec{F} = \frac{qq' \vec{r}}{4\pi\varepsilon_0 r^3}$$

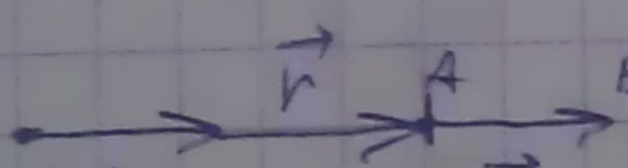
$$dA = \frac{qq'}{4\pi\varepsilon_0 r^3} (\vec{r}, d\vec{r})$$

$$A = \int_1^2 \frac{qq'}{4\pi\varepsilon_0 r^3} (\vec{r}, d\vec{r}) = \frac{qq'}{4\pi\varepsilon_0} \int_1^2 \frac{(\vec{r}, d\vec{r})}{r^3} = \frac{qq'}{4\pi\varepsilon_0} \left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} =$$

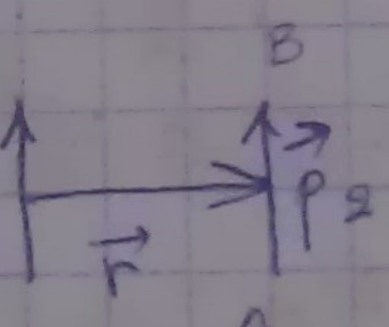
$$= \frac{qq'}{4\pi\varepsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

№25 Дано: $\vec{p}_1, \vec{p}_2, \vec{r}$. W – ?

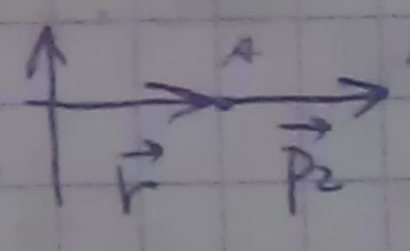
а)



б)



в)



] $\vec{p}_1 = q\vec{l}_1, \quad \vec{p}_2 = q\vec{l}_2$

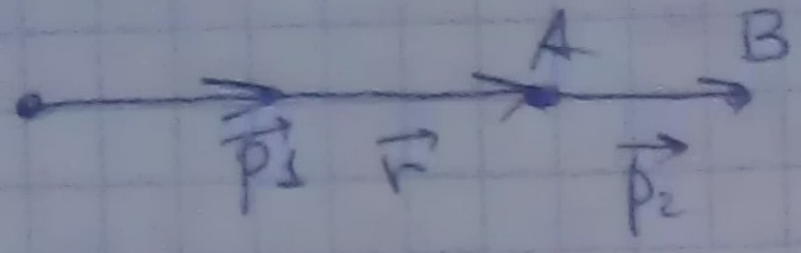
$$W = q_2(\varphi_A - \varphi_B)$$

Пример 24: $$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}_1 \vec{p}_2}{r_0^3} - \frac{3(\vec{r}_0, \vec{p}_1)(\vec{r}_0, \vec{p}_2)}{r_0^5}\right)$$

а) $\vec{r}_0 = \frac{\vec{l}_1}{2} + \vec{r} + \frac{\vec{l}_2}{2}$ б) $\vec{r}_0 = \vec{r}$, в) $\vec{r}_0 = \vec{r} + \frac{\vec{l}_2}{2}$

$\Rightarrow$ в) $W = 0$: $\vec{p}_1 \vec{p}_2 = 0, \quad \vec{p}_1 \vec{r}_0 = 0$

а)



$$\vec{r}_A = \frac{\vec{l}}{2} + \vec{r}$$

$$\vec{r}_B = \frac{\vec{l}}{2} + \vec{r} + \vec{l}_2$$

$$W = q_2(\varphi_B - \varphi_A)$$

$$\varphi(M) = \frac{q_1(\vec{r}_M, \vec{l}_1)}{4\pi\varepsilon_0 r_M^3} = \frac{(\vec{r}_M, \vec{p}_1)}{4\pi\varepsilon_0 r_M^3}$$

$$\varphi_A - \varphi_B = \frac{(\vec{r}_A, \vec{p}_1)}{4\pi\varepsilon_0 r_A^3} - \frac{(\vec{r}_B, \vec{p}_1)}{4\pi\varepsilon_0 r_B^3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{(\vec{r}_A, \vec{p}_1)}{r_A^3} - \frac{(\vec{r}_B, \vec{p}_1)}{r_B^3}\right)$$

$$(\vec{r}_A, \vec{p}_1) = \left(\frac{\vec{l}}{2} + \vec{r}, \vec{p}_1\right) = \left(\frac{\vec{l}}{2}, \vec{p}_1\right) + (\vec{r}, \vec{p}_1) =$$

$$= \frac{q_1 l^2}{2} + (\vec{r}, \vec{p}_1) = \frac{q_1 l^2}{2} + q(\vec{r}, \vec{l}_1) =$$

$$= \frac{q_1 l^2}{2} + q_1 r l_1 = q_1 l_1 \left(\frac{l_1}{2} + r\right)$$

$$r_A = \frac{l_1}{2} + r$$

$$(\vec{r}_B, \vec{p}_1) = \left(\frac{\vec{l}_1}{2}, \vec{p}_1\right) + (\vec{r}, \vec{p}_1) + (\vec{l}_2, \vec{p}_1) =$$

$$= (\vec{r}_A, \vec{p}_1) + (\vec{l}_2, \vec{p}_1) = q_1 l_1 \left(\frac{l_1}{2} + r\right) +$$

$$+ l_2 l_1 q_1 = l_1 q_1 \left(\frac{l_1}{2} + r + l_2\right)$$

$$r_B = \frac{l_1}{2} + r + l_2$$

$$(\vec{r}_A, \vec{p}_1) = q_1 l_1 r_A \qquad (\vec{r}_B, \vec{p}_1) = l_1 q_1 r_B = p_1 r_B$$

$$= p_1 r_A$$

$$\varphi_A - \varphi_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{p_1 r_A}{r_A^3} - \frac{p_1 r_B}{r_B^3}\right) =$$

$$= \frac{p_1}{4\pi\varepsilon_0}\left(\frac{1}{r_A^2} - \frac{1}{r_B^2}\right)$$

$$\frac{1}{r_A^2} - \frac{1}{r_B^2} = \frac{r_B^2 - r_A^2}{r_A^2 r_B^2} = \frac{(r_B - r_A)(r_B + r_A)}{r_A^2 r_B^2} =$$

$$= \frac{l_2\left(l_1 + 2r + l_2\right)}{\left(\frac{l_1}{2}+r\right)^2\left(\frac{l_1}{2}+r+l_2\right)^2}$$

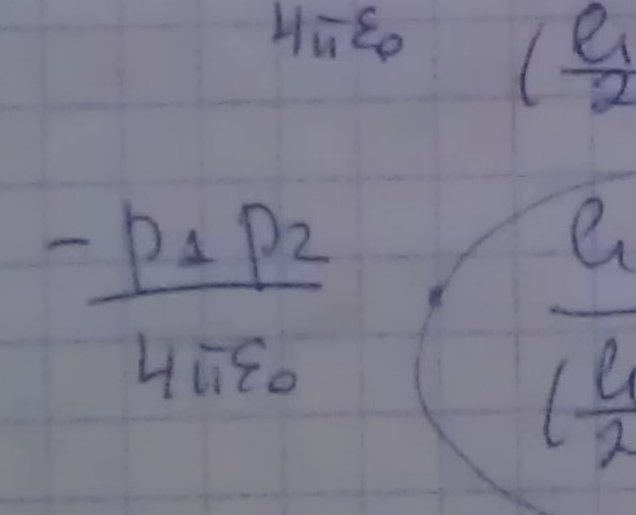
$$\varphi_B - \varphi_A = \frac{-p_1}{4\pi\varepsilon_0}\,\frac{l_2(l_1+2r+l_2)}{\left(\frac{l_1}{2}+r\right)^2\left(\frac{l_1}{2}+r+l_2\right)^2}$$

$$W = \frac{-p_1 p_2}{4\pi\varepsilon_0}\,\frac{l_1+2r+l_2}{\left(\frac{l_1}{2}+r\right)^2\left(\frac{l_1}{2}+r+l_2\right)^2} = \frac{4}{\pi r^3} ?$$

$l_{1,2} \ll r$

$$\left(\frac{l_1^2}{4} + l_1 r + r^2\right)\left(\frac{l_1^2}{4} + r^2 + l_2^2 + l_1 r + l_1 l_2 + 2 r l_2\right)$$

б)



$$W = q_2(\varphi_B - \varphi_A)$$

$$\vec{r}_B = \vec{r} + \frac{\vec{l}_2}{2}, \qquad \vec{r}_A = \vec{r} - \frac{\vec{l}_2}{2}$$

$$r_B = \sqrt{r^2 + \frac{l_2^2}{4}}, \qquad r_A = r_B$$

$$\varphi_B - \varphi_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{(\vec{r}_B, \vec{p}_2)}{r_B^3} - \frac{(\vec{r}_A, \vec{p}_1)}{r_A^3}\right)$$

$$(\vec{r}_B, \vec{p}_1) = (\vec{r} + \frac{\vec{l}_2}{2}, \vec{p}_1) = (\vec{r}, \vec{p}_1) + \frac{1}{2}(\vec{l}_2, \vec{p}_1) = \frac{l_2 p_1}{2}$$

$$(\vec{r}_A, \vec{p}_2) = (\vec{r} - \frac{\vec{l}_2}{2}, \vec{p}_1) = -\frac{1}{2} l_2 p_1$$

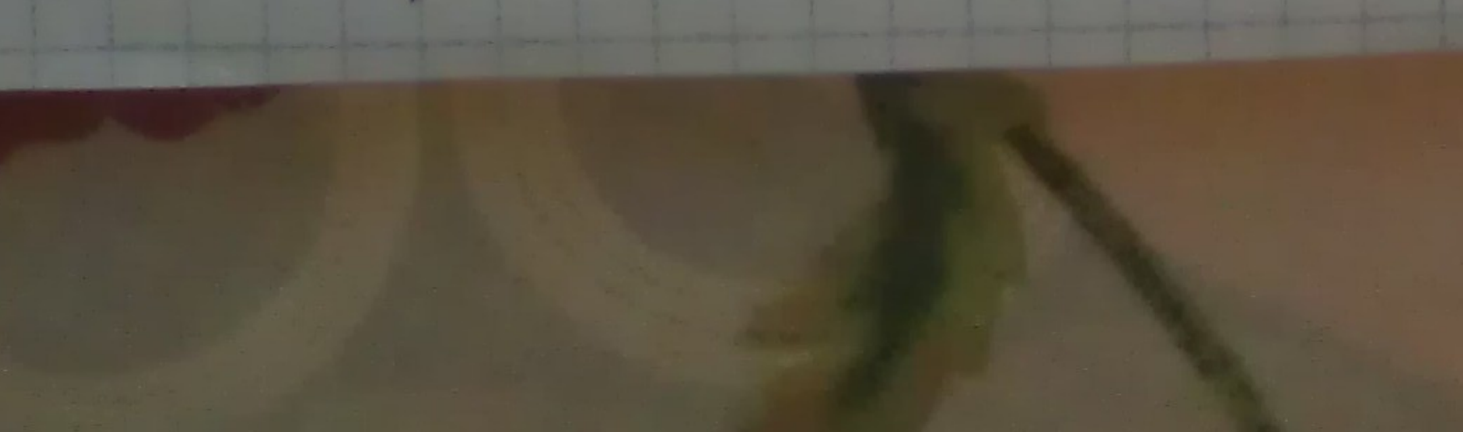
$$\varphi_B - \varphi_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{\frac{l_2 p_1}{2}}{r_A^3} + \frac{\frac{l_2 p_1}{2}}{r_A^3}\right) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{l_2 p_1}{r_A^3}$$

$$W = \frac{1}{4\pi\varepsilon_0}\frac{p_1 p_2}{r_A^3}$$

$$r_A^3 = \left(r^2 + \frac{l_2^2}{4}\right)^{3/2} = r^3\left(1 + \frac{l_2^2}{r^2 4}\right)^{3/2} \overset{?}{\approx} \frac{r^3}{2}$$

$$\Rightarrow W = \frac{p_1 p_2}{2\pi\varepsilon_0 r^3}$$

№2.6 Дано: $\vec{p_e}, \vec{E}, m, l$, $\vec{E}$ - однор. $\omega, \vec{F}$ - ?



1) $W = 0$

$F = 0$

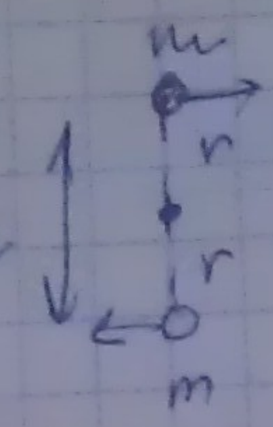
$M = pE$

2) $W = -pE$

$F = 0$

$M = 0$

$$W = +p_e E = \frac{I\omega^2}{2} = \frac{ml^2\omega^2}{24}$$



$$r = l/2$$

$$I = \sum_{i=1}^{N} m_i r_i^2$$

$$I = \frac{ml^2}{4} + \frac{ml^2}{4} = \frac{ml^2}{2}$$

$$pE = \frac{ml^2\omega^2}{4}$$

$$\omega^2 = \frac{4p_eE}{ml^2} \Rightarrow \boxed{\omega = \frac{2}{l}\sqrt{\frac{p_eE}{m}}}$$

$$\vec{M} = \frac{d\vec{N}}{dt}, \quad \vec{N} = I\vec{\omega}$$

$$\vec{M} = I\frac{d\vec{\omega}}{dt} = I\vec{\beta}$$

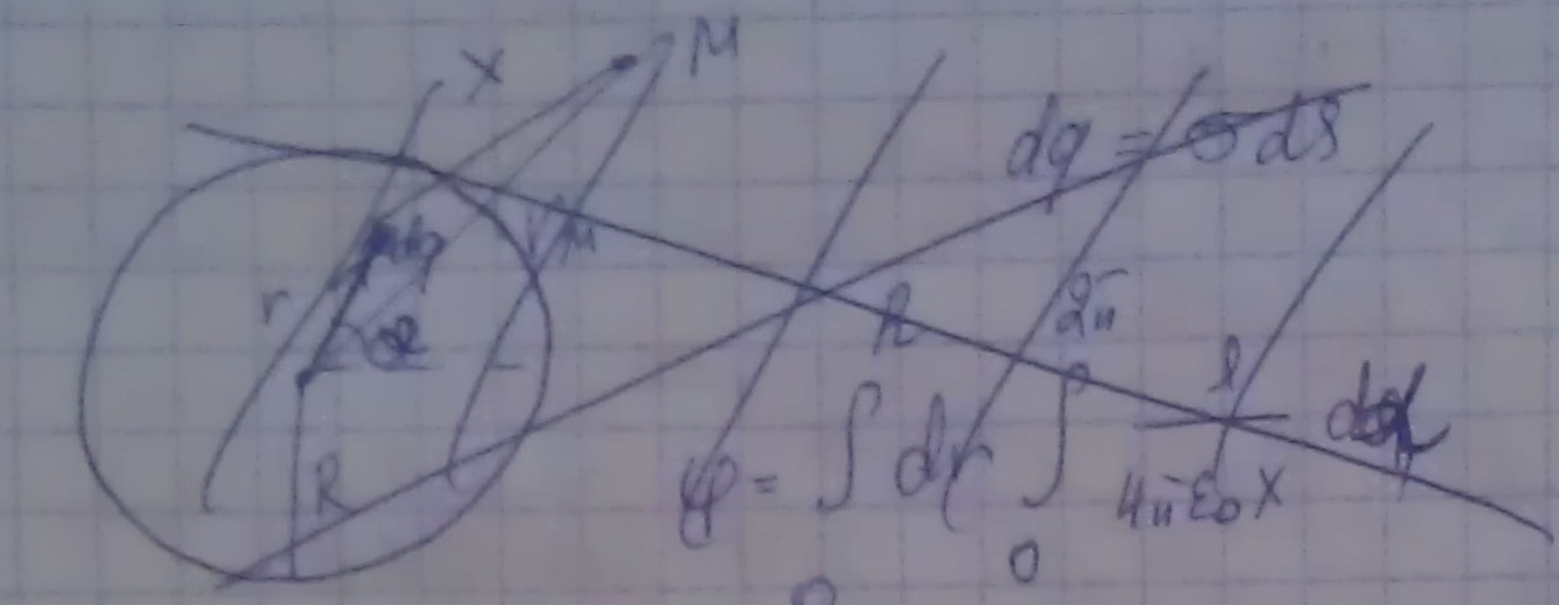
В нач. момент. времени $M = p_eE$

$$\Rightarrow p_eE = I\vec{\beta} = M$$

$$p_eE = \frac{ml^2}{2}\cdot\vec{\beta}$$

$$\Rightarrow \boxed{\vec{\beta} = \frac{2p_eE}{ml^2}}$$

№ 2.15 Дано: R, σ φ - ?



$$dq = \frac{q}{2\pi} d\alpha$$

$$\varphi_0 = \int_0^{2\pi} \frac{q\,d\alpha}{4\pi\varepsilon_0 \cdot 2\pi \cdot X} =$$

$$= \int_0^{2\pi} \frac{q\,d\alpha}{2\pi^2 \varepsilon_0 X}$$

т. cos: $X^2 = r^2 + r_M^2 - 2 r r_M \cos\alpha$

$$\varphi_0 = \int_0^{2\pi} \frac{q\,d\alpha}{2\pi^2 \varepsilon_0 \sqrt{r^2 + r_M^2 - 2 r r_M \cos\alpha}}, \text{ крайн} \Rightarrow r_M = R$$

$$\varphi = \int_0^R 2\pi r \sigma\,dr \int_0^{2\pi} \frac{d\alpha}{2\pi^2 \varepsilon_0 \sqrt{r^2 + R^2 - 2 r R \cos\alpha}} =$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_0^R dr \int_0^{2\pi} \frac{2\,d\alpha}{\sqrt{r^2 + R^2 - 2 r R \cos\alpha}} = \int_0^{2\pi} d\alpha \int_0^R \frac{r\,dr}{\sqrt{r^2 + R^2 - 2 r R \cos\alpha}}$$

$$\int_0^R \frac{r\,dr}{\sqrt{r^2 + R^2 - 2 r R \cos\alpha}} = \int_0^R \frac{r\,dr}{\sqrt{r^2 - 2 r R \cos\alpha + R^2 \cos^2\alpha + R^2 - R^2 \cos^2\alpha}} =$$

$$- \int_0^R \ln\left(r - R\cos\alpha + \sqrt{(r - R\cos\alpha)^2 + R^2\sin^2\alpha}\right) dr$$

$$\varphi = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} d\alpha \int_0^R \frac{r\,dr}{\sqrt{r^2 + R^2 - 2rR\cos\alpha}}$$

$$\int_0^R \frac{r\,dr}{\sqrt{r^2 + R^2 - 2rR\cos\alpha}} = \int_0^R \frac{r\,dr}{\sqrt{(r - R\cos\alpha)^2 + R^2\sin^2\alpha}} =$$

$$= \int_0^R \frac{(r - R\cos\alpha)\,dr}{\sqrt{(r - R\cos\alpha)^2 + R^2\sin^2\alpha}} + \int_0^R \frac{R\cos\alpha\,dr}{\sqrt{(r - R\cos\alpha)^2 + R^2\sin^2\alpha}} =$$

$$= \int_0^R \frac{(r - R\cos\alpha)\,d(r - R\cos\alpha)}{\sqrt{(r - R\cos\alpha)^2 + R^2\sin^2\alpha}} + R\cos\alpha \int_0^R \frac{d(r - R\cos\alpha)}{\sqrt{(r - R\cos\alpha)^2 + R^2\sin^2\alpha}} =$$

$$= \sqrt{(2-R\cos\alpha)^2 + R^2\sin^2\alpha}\Big|_0^R +$$

$$+ R\cos\alpha \ln\left(2 - R\cos\alpha + \sqrt{(2-R\cos\alpha)^2 + R^2\sin^2\alpha}\right)\Big|_0^R =$$

$$= \sqrt{R^2(1-\cos\alpha)^2 + R^2\sin^2\alpha} - \sqrt{R^2\cos^2\alpha + R^2\sin^2\alpha} +$$

$$+ R\cos\alpha \ln\left(R - R\cos\alpha + \sqrt{R^2(1-\cos\alpha)^2 + R^2\sin^2\alpha}\right) -$$

$$- R\cos\alpha \ln\left(-R\cos\alpha + \sqrt{R^2\cos^2\alpha + R^2\sin^2\alpha}\right) =$$

$$= R\sqrt{1-2\cos\alpha+1} - R + R\cos\alpha \ln\left(R(1-\cos\alpha) + \right.$$

$$\left. + R\sqrt{2-2\cos\alpha}\right) - R\cos\alpha \ln\left(-R\cos\alpha + R\right) =$$

$$= R\sqrt{2-2\cos\alpha} - R + R\cos\alpha \ln\left(R(1-\cos\alpha) + R\sqrt{2-2\cos\alpha}\right)$$

$$- R\cos\alpha \ln\left(R(1-\cos\alpha)\right) =$$

$$= R\left(\sqrt{2}\cdot\sqrt{1-\cos\alpha} - 1 + \cos\alpha \ln\left(R(1-\cos\alpha) + R\sqrt{2}\cdot\sqrt{1-\cos\alpha}\right)\right.$$

$$\left. - \cos\alpha \ln\left(R(1-\cos\alpha)\right)\right)$$

$$\ln R\left((1-\cos\alpha) + \sqrt{1-\cos\alpha}\cdot\sqrt{2}\right) - \ln R(1-\cos\alpha) =$$

$$= \ln \frac{1-\cos\alpha + \sqrt{2}\cdot\sqrt{1-\cos\alpha}}{1-\cos\alpha} = \ln\left(1 + \frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right)$$

$$\int_0^{\pi} \ldots = R\left(\sqrt{2}\cdot\sqrt{1-\cos\alpha} - 1 + \cos\alpha \ln\left(1 + \frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right)\right.$$

$$\varphi = \frac{R\sigma}{4\pi\varepsilon_0}\left(\int_0^{2\pi} \cos\alpha \ln\left(1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right)d\alpha + \int_0^{2\pi}\sqrt{2}\sqrt{1-\cos\alpha}\,d\alpha - 2\pi\right)$$

$$\int_0^{2\pi}\sqrt{1-\cos\alpha}\,d\alpha = \int_0^{2\pi}\frac{\sqrt{1-\cos^2\alpha}}{\sqrt{1+\cos\alpha}}d\alpha = \int_0^{2\pi}\frac{|\sin\alpha|}{\sqrt{1+\cos\alpha}}d\alpha =$$

$$= \int_0^{\pi}\frac{\sin\alpha\,d\alpha}{\sqrt{1+\cos\alpha}} + \int_{\pi}^{2\pi}\frac{-\sin\alpha\,d\alpha}{\sqrt{1+\cos\alpha}} = \int_0^{\pi}\frac{d(\cos\alpha)}{\sqrt{1+\cos\alpha}} - \int_{\pi}^{2\pi}\frac{d(\cos\alpha)}{\sqrt{1+\cos\alpha}} =$$

$$= 2(1+\cos\alpha)^{1/2}\Big|_{\pi}^{2\pi} - 2(1+\cos\alpha)^{1/2}\Big|_0^{\pi} =$$

$$= 2\sqrt{2} - 0 - 0 + 2\sqrt{2} = 4\sqrt{2}$$

$$\varphi = \frac{R\sigma}{4\pi\varepsilon_0}\left(\int_0^{4\pi}\cos\alpha\ln\left(1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right)d\alpha + 8 - 2\pi\right)$$

$$I = \int_0^{4\pi}\cos\alpha\ln\left(1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right)d\alpha$$

$$f(\alpha) = \cos\alpha,\quad f'(\alpha) = \sin\alpha$$

$$g(\alpha) = \ln\left(1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right),\quad g'(\alpha) = \frac{\left(\sqrt{1-\cos\alpha}\right)'}{1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}} =$$

$$= \sqrt{2}\,\frac{\frac{\sin\alpha\cdot(-1/2)}{(1-\cos\alpha)^{3/2}}}{1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}} = -\frac{\sqrt{2}}{2}\cdot\frac{\sin\alpha}{(1-\cos\alpha)^{3/2}+\sqrt{2}\,(1-\cos\alpha)} =$$

$$= -\frac{\sqrt{2}}{2}\,\frac{\sin\alpha}{(1-\cos\alpha)(\sqrt{1-\cos\alpha}+\sqrt{2})}$$

$$I = \int_0^{2\pi} \sin\alpha \ln\left(1+\frac{\sqrt{2}}{\sqrt{1-\cos\alpha}}\right)\Big|_0^{2\pi} - \int_0^{2\pi} \frac{-\frac{\sqrt{2}}{2}\sin^2\alpha\,d\alpha}{(1-\cos\alpha)(\sqrt{1-\cos\alpha}+\sqrt{2})} =$$

$$= \frac{\sqrt{2}}{2}\int_0^{2\pi} \frac{(1-\cos\alpha)(1+\cos\alpha)\,d\alpha}{(1-\cos\alpha)(\sqrt{1-\cos\alpha}+\sqrt{2})} = \frac{\sqrt{2}}{2}\int_0^{2\pi} \frac{(1+\cos\alpha)\,d\alpha}{\sqrt{1-\cos\alpha}+\sqrt{2}}$$

$$\int_0^{2\pi} \frac{d\alpha}{\sqrt{1-\cos\alpha}+\sqrt{2}} = \int_0^{2\pi} \sqrt{1-\cos\alpha}+\sqrt{2}\,d\alpha$$

$$\int_0^{2\pi} \frac{(1+\cos\alpha)\,d\alpha}{\sqrt{1-\cos\alpha}+\sqrt{2}} = \int_0^{2\pi} \frac{(1+\cos\alpha)(\sqrt{1-\cos\alpha}-\sqrt{2})\,d\alpha}{1+\cos\alpha-2} =$$

$$= -\int_0^{2\pi} (\sqrt{1-\cos\alpha}-\sqrt{2})\,d\alpha = \int_0^{2\pi} (\sqrt{2}-\sqrt{1-\cos\alpha})\,d\alpha =$$

$$= 2\pi\sqrt{2} - \int_0^{2\pi} \sqrt{1-\cos\alpha}\,d\alpha = 2\pi\sqrt{2} - 4\sqrt{2}$$

$$I = 2\pi - 4 \Rightarrow \varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\pi}{} - 4 + 8 - 2\pi\right) \Rightarrow \boxed{\varphi = \frac{4}{4\pi\varepsilon_0}}$$

№2.10 Дано: $r = 1\,см$, $\varphi = 3000\,В$, $m = 9{,}1 \cdot 10^{-31}\,кг$

$e = -1{,}6 \cdot 10^{-19}\,Кл$

$n - ?$

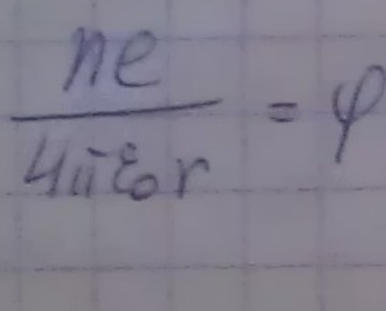
$$\varphi = \frac{q}{4\pi\varepsilon_0 r}, \quad q = ne$$

$$\frac{ne}{4\pi\varepsilon_0 r} = \varphi \Rightarrow n = \frac{4\pi\varepsilon_0 r \cdot \varphi}{e} = \frac{4 \cdot 3{,}14 \cdot \varepsilon_0 \cdot 0{,}01 \cdot 3 \cdot 10^3}{-1{,}6 \cdot 10^{-19}}$$

$$\Delta m = mn = \frac{m 4\pi\varepsilon_0 r \cdot \varphi}{e} = \frac{9{,}1 \cdot 10^{-31} \cdot 4 \cdot 3{,}14 \cdot \varepsilon_0 \cdot 0{,}01 \cdot 3 \cdot 10^3}{-1{,}6 \cdot 10^{-19}}$$

$$\approx \boxed{2 \cdot 10^{-20}\,кг = \Delta m}$$

№2.9 Дано: R, σ, $\varphi - ?$



$$\varphi(r) = \int_0^R \vec{E}\,d\vec{r} = \int_0^R \frac{\sigma \cancel{2\pi r\,dr}}{2\pi\varepsilon_0} \frac{r\,dr}{\sqrt{R^2 - r^2}} =$$

$$= \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{r\,dr}{\sqrt{R^2 - r^2}} = \frac{\sigma}{2\varepsilon_0} \left(-\sqrt{R^2 - r^2} \right) \Big|_0^R =$$

$$= \boxed{\frac{R\sigma}{2\varepsilon} = \varphi}$$

Проводники и диэлектрики в эл поле

Проводник: $\uparrow \vec{E}$

$\vec{E}_{in} \downarrow$ $E_{inz} = 0$, $\rho_{in} = 0$

σ

$\uparrow \vec{E}$

+ + + + + +

$E_{inz} = 0 \Rightarrow \varphi = const \Rightarrow$ проводник - эквипотенц. пов-сть

Поле внутри проводника:

$E_{inz} = 0$

$\vec{E}_{inz} = \vec{E} + \vec{E}_{in}$

Заряд только поверхностный

Диэлектрик

В эл. поле поляризуется. Молекулы - диполи.

$\vec{E}_0 \downarrow$ ⊕ ⊕ ⊕

σ $\uparrow \vec{E}$

Поляризация - ориентация молекул диэлектрика во внешнем электростат поле

$\vec{P} = n\vec{p}$, n - концентр диполей

$\vec{p}$ - дипольный момент

Индукция эл. поля:

В отсутствии диэл: $\vec{D} = \varepsilon_0 \vec{E}$

В диэлектрике: $\vec{D} = \varepsilon\varepsilon_0 \vec{E}$, ε - диэл. проницаемость

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\vec{P} = \varepsilon_0(\varepsilon - 1)\vec{E}$

$\chi = \varepsilon - 1$ – диэлектрическая восприимчивость

ε, χ – характеристики вещ-ва

$\vec{E}_p \uparrow\downarrow \vec{E} \Rightarrow \vec{E}_{рез} = \vec{E} + \vec{E}_p \neq 0$

σ – заряд на пов-ти, связанный заряд. (Ось z)

Теорема Гаусса

$$\oint_S \vec{D} d\vec{S} = \sum_i q_i \text{ своб.}$$

/

поток вектора $\vec{D}$ через пов-ть

$$\boxed{\operatorname{div} \vec{D} = \rho_{своб}}$$ – объёмная плотность свободного заряда

I ур-е Максвелла

Проводник:

$$\operatorname{div} \vec{E} = \frac{\rho_{своб}}{\varepsilon_0}$$

Диэлектрик

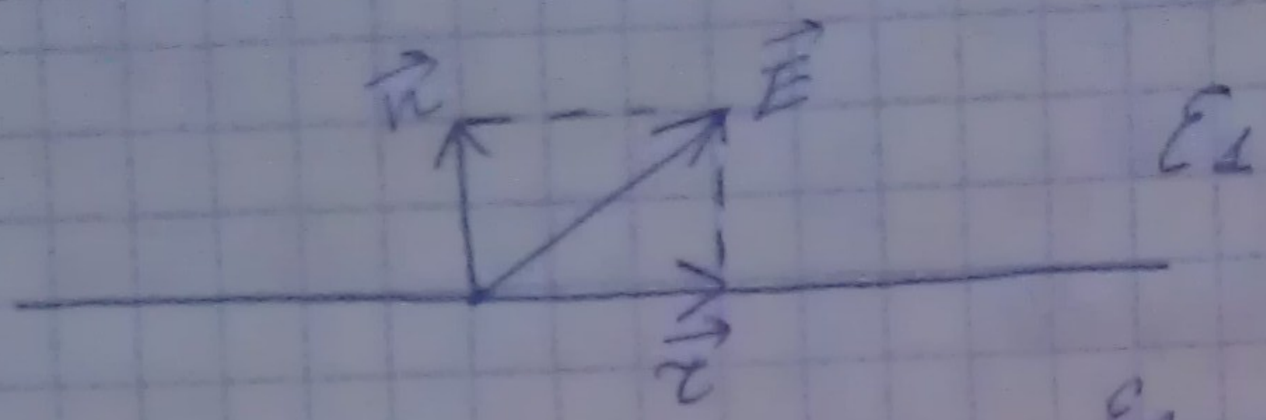
$$\operatorname{div} \vec{E} = \frac{\rho_{своб}}{\varepsilon\varepsilon_0}$$

$$\oint_S \vec{P} d\vec{S} = -\sum_i q_{i\,своб}$$

$$\operatorname{div} \vec{P} = -\rho_{своб}$$

$$\operatorname{div} \vec{E} = \frac{\rho_{своб} + \rho_{своб}}{\varepsilon_0}$$

Граничные условия:



$D_{n_2} - D_{n_1} = \sigma_{своб}$

$P_{n_2} - P_{n_1} = -\sigma_{связ}$

$E_{\tau_1} = E_{\tau_2}$ $\quad \frac{D_{\tau_1}}{\varepsilon_0 \varepsilon_1} = \frac{D_{\tau_2}}{\varepsilon_0 \varepsilon_2} \Rightarrow D_{\tau_1} \varepsilon_2 = D_{\tau_2} \varepsilon_1$

$\varepsilon_1 E_{n_1} = \varepsilon_2 E_{n_2}$ $\quad (\sigma_{своб.} = 0)$

№3.7 Дано: $R_1 < R_2 < R_3$, q $\quad \vec{E} - ?$

$E = 0$



$E = \frac{q}{4\pi\varepsilon_0 r^2}$, $r > R$

у (1) полный $-q_1$

(3) q_3

$\Rightarrow q - q_1 - q_3 = 0$

1) Внутри: $\oint \vec{E} d\vec{S} = 0 \Rightarrow \vec{E_1} = 0$

2) $\oint \vec{E} d\vec{S} = -q_1 / \varepsilon_0$

$\vec{D} = \varepsilon_0 \vec{E}$

$$\oint E(r)\,dS = \textcircled{Ф}$$

$$dS = r^2 \sin\theta\, d\varphi d\theta$$

$$dV = r^2 \sin\theta\, dr d\varphi d\theta$$

$$dV = r\, dr d\varphi dz \quad (\text{цил.})$$

$$dS = r\, d\varphi dz \quad (\text{цил.})$$

$$\textcircled{Ф} \int E(r)\, r^2 \sin\theta\, d\varphi d\theta = E(r)\, r^2 \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta =$$

$$= 4\pi r^2 \cdot E(r)$$

$$4\pi E(r) r^2 = \frac{-Q_1}{\varepsilon_0}$$

$$E(r) = \frac{-Q_1}{4\pi\varepsilon_0 r^2} \quad (\text{поле сферы совпад. с полем точечного заряда})$$

$$E(r) = \begin{cases} 0, & r < R_1,\ r > R_3 \\ \dfrac{-Q_1}{4\pi\varepsilon r^2}, & R_1 < r < R_2 \\ \dfrac{Q - Q_1}{4\pi\varepsilon_0 r^2}, & R_2 < r < R_3 \end{cases}$$

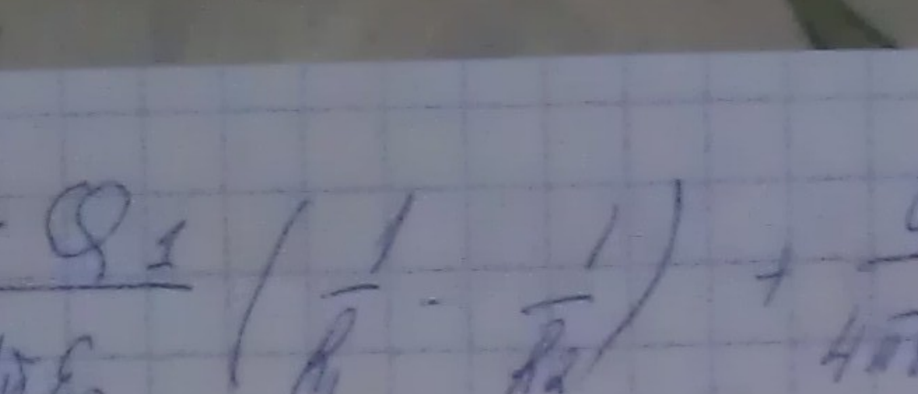
$$\Delta\varphi_{13} = 0 \Rightarrow \int_{R_1}^{R_3} E\,dr = \int_{R_1}^{R_2} E\,dr + \int_{R_2}^{R_3} E\,dr =$$

$$= -\int_{R_1}^{R_2} \frac{-Q_1\,dr}{4\pi\varepsilon_0 r^2} + \int_{R_2}^{R_3} \frac{Q - Q_1}{4\pi\varepsilon_0 r^2} =$$

$$= \frac{-q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{q-q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)=0$$

$$\Rightarrow q_1 = q\cdot\frac{R_2-R_3}{R_1-R_3}\cdot\frac{R_1}{R_2}$$

№ 3.12



l - длина нити

$$\oint \vec{E}\,d\vec{S} = \int E(r)\,r\,dz\,d\theta = \oint r\,E(r)\int_0^{2\pi} d\varphi\cdot l =$$

$$= E(r)\cdot 2\pi r l = k\cdot l/\varepsilon_0$$

$$\Rightarrow E(r) = \frac{k}{2\pi\varepsilon_0 r} \quad \text{— поле нити}$$

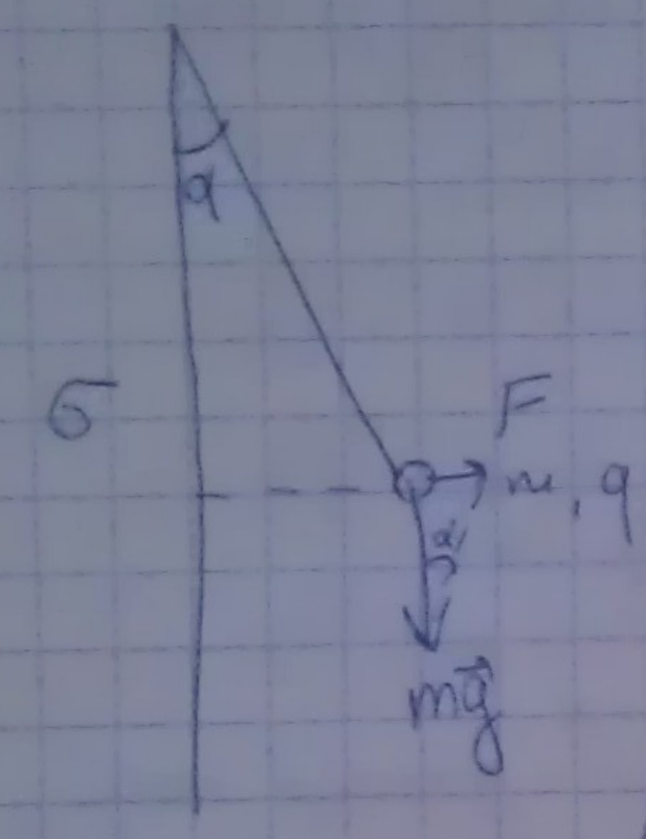
$$r = \sqrt{\frac{d^2}{4}+h^2}$$

$$\Rightarrow E_{k_+} = \frac{k\,dq_+}{\pi\varepsilon_0\sqrt{d^2+4h^2}} = \frac{k\,dq}{4\pi\varepsilon_0\left(\frac{d^2}{4}+h^2\right)}$$

$$E_+ = \frac{dk}{2\pi\varepsilon_0\left(\frac{d^2}{4}+h^2\right)}$$

Дз: 3.1, 2, 4, 6, 11, 15

№ 311 Дано: σ, m, q, α, g - ?



$$tg\,\alpha = \frac{F}{mg}$$

$$F = mg\,tg\,\alpha$$

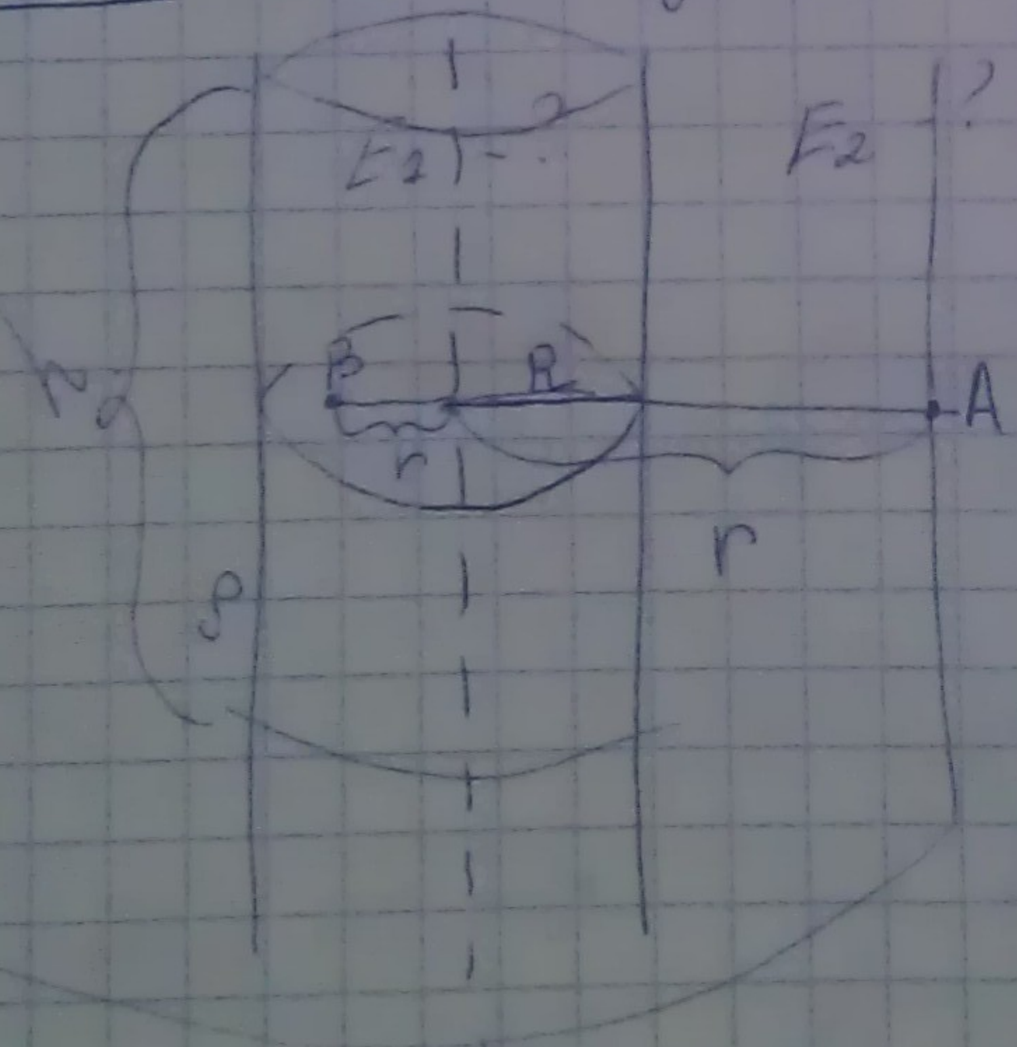
$$qE = mg\,tg\,\alpha$$

$$13.6) \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

$$\frac{q\sigma}{2\varepsilon_0} = mg\,tg\,\alpha$$

$$\boxed{q = \frac{2\varepsilon_0}{\sigma} \cdot mg\,tg\,\alpha}$$

№ 311 Дано. R, ρ E_1, E_2 - ?



$$div\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$Q = \int_V \rho dV = \rho V = \rho \pi r^2 h$$

$$\oint \vec{E} d\vec{S} = E(r) S_{бок} = E(r) 2\pi r \cdot h$$

$$\varepsilon_0$$

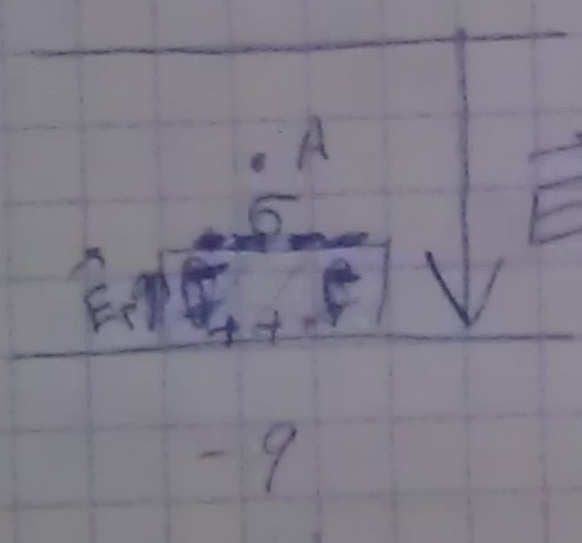
$$\Rightarrow E(r)\cdot 2\pi r l = \frac{\rho \pi r^2 l}{\varepsilon_0} \Rightarrow \boxed{E_1(r) = \frac{\rho r}{2\varepsilon_0}}$$

2) $r > R$

$$\Phi = \rho \pi R^2 l$$

$$E(r)\cdot 2\pi r l = \frac{\rho \pi R^2 l}{\varepsilon_0} \Rightarrow \boxed{E_2(r) = \frac{\rho R^2}{2r\varepsilon_0}}$$

№36



Диэл. поляризуется

$\exists \vec{E}_p \Rightarrow \exists \sigma$

№315 Дано: E_0, $\rho(\vec{r})$ - ?



$$\vec{E_0} = \frac{E_0}{r}\cdot \vec{r}$$

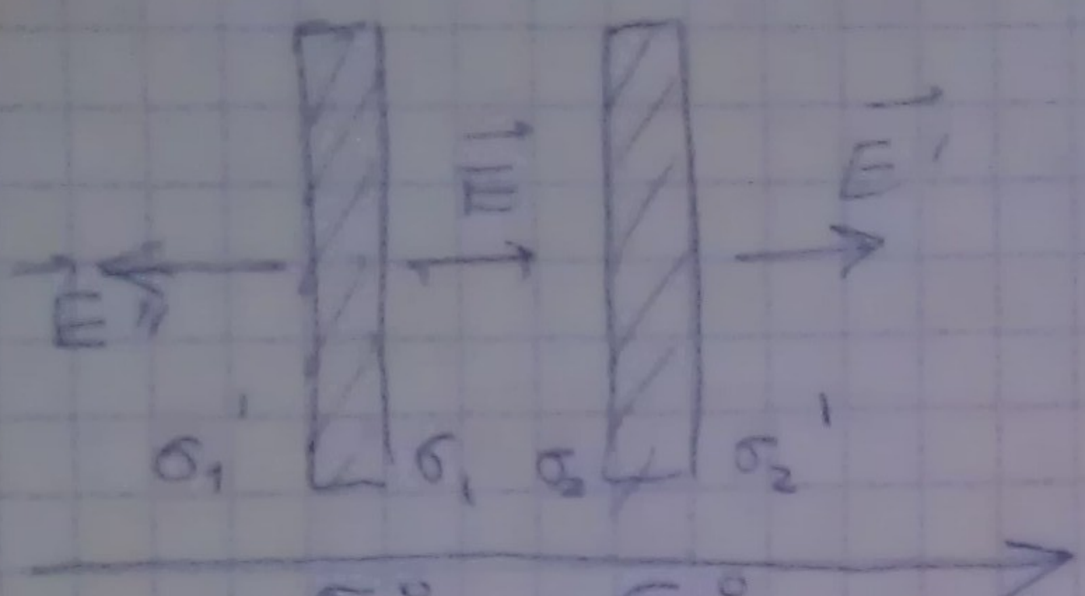
$$\operatorname{div}\vec{E_0} = \frac{\rho}{\varepsilon_0}$$

$$\operatorname{div}\varphi(r)\cdot\vec{r} = \varphi'(r)\cdot r + 3\varphi(r), \quad \varphi(r) = \frac{E_0}{r}$$

$$\operatorname{div}\left(\frac{E_0}{r}\right)\cdot\vec{r} = -\frac{E_0}{r^2}\cdot r + \frac{3E_0}{r} = \frac{2E_0}{r} = \frac{\rho(r)}{\varepsilon_0}$$

$$\Rightarrow \boxed{\rho(r) = \frac{2E_0\varepsilon_0}{r}}$$

№32 Дано: σ_1^0, σ_2^0. Найти: $E'', E, E', \sigma_1', \sigma_1, \sigma_2', \sigma_2$



$$\begin{cases} \sigma_1 + \sigma_1' = \sigma_1^0 \\ \sigma_2 + \sigma_2' = \sigma_2^0 \\ \sigma_1' - \sigma_1 - \sigma_2 - \sigma_2' = 0 \\ \sigma_1' + \sigma_1 + \sigma_2 - \sigma_2' = 0 \end{cases}$$

$$\begin{cases} \sigma_1 = \sigma_1^0 - \sigma_1' \\ \sigma_2 = \sigma_2^0 - \sigma_2' \\ 2\sigma_1' - \sigma_1^0 - \sigma_2^0 = 0 \\ \sigma_1^0 + \sigma_2^0 - 2\sigma_2' = 0 \end{cases}$$

$$\Rightarrow \boxed{\sigma_1' = \sigma_2' = \frac{\sigma_1^0 + \sigma_2^0}{2}}$$

$$\Rightarrow \boxed{\sigma_1 = \frac{\sigma_1^0 - \sigma_2^0}{2}; \quad \sigma_2 = \frac{\sigma_2^0 - \sigma_1^0}{2}}$$

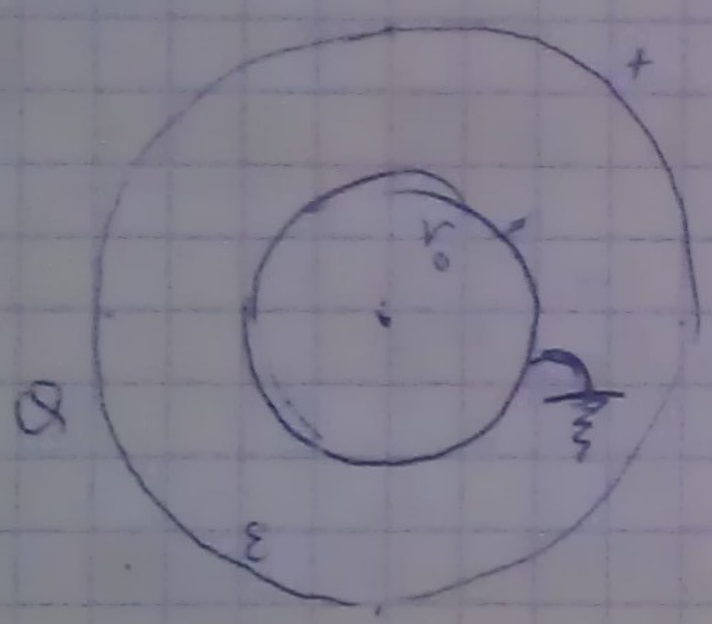
$$E'' = \frac{-\sigma_1' - \sigma_1 - \sigma_2 - \sigma_2'}{2\varepsilon_0} = \frac{-\sigma_1^0 - \sigma_2^0 - \sigma_1^0 + \sigma_2^0 - \sigma_2^0 + \sigma_1^0 - \sigma_2^0}{4\varepsilon_0}$$

$$= \boxed{\frac{-\sigma_1^0 - \sigma_2^0}{2\varepsilon_0} = E''}$$

$$E = \frac{\sigma_1' + \sigma_1 - \sigma_2 - \sigma_2'}{2\varepsilon_0} = \frac{\sigma_1^0 - \sigma_2^0 - \sigma_2^0 + \sigma_1^0}{4\varepsilon_0} = \boxed{\frac{\sigma_1^0 - \sigma_2^0}{2\varepsilon_0} = E}$$

$$E' = \frac{\sigma_1' + \sigma_1 + \sigma_2' + \sigma_2}{2\varepsilon_0} = \boxed{\frac{\sigma_1^0 + \sigma_2^0}{2\varepsilon_0} = E'}$$

№ 314 Дано: R, Q, ε, r_0. $q - ?$



$$D = \begin{cases} \dfrac{q}{\frac{4}{3}\pi r_0^3} \cdot \dfrac{4}{3}\pi r^3 \\ \dfrac{4\pi r_0^3}{4\pi r^2} \end{cases}, \quad r < r_0$$

$$\frac{q}{4\pi r^2}, \quad r_0 < r < R$$

$$\frac{Q-q}{4\pi r^2}, \quad r > R$$

$$E(r) = \begin{cases} \dfrac{qr}{4\pi r_0^3 \varepsilon_0}, & r < r_0 \\ \dfrac{q}{4\pi r^2 \varepsilon \varepsilon_0}, & r_0 < r < R \\ \dfrac{Q-q}{4\pi r^2}, & r > R \end{cases}$$

$\varphi_\infty = 0$ (т.к. заземлён)

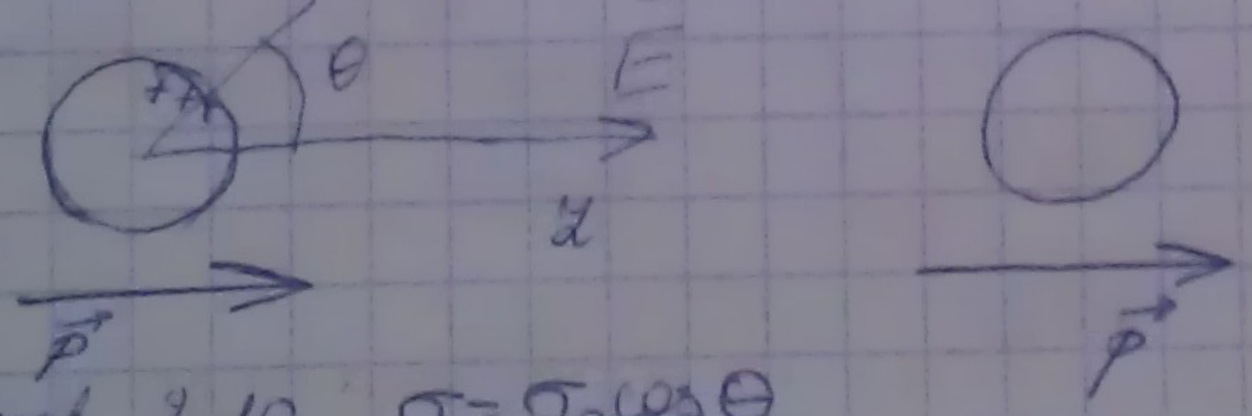
$$\varphi_{in} = 0 = \int_{r_0}^{\infty} E(r)dr = \int_{r_0}^{R} \frac{q\,dr}{4\pi\varepsilon\varepsilon_0 r^2} + \int_{R}^{\infty} \frac{\Phi - q}{4\pi\varepsilon_0 r^2} dr =$$

$$= \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{r_0} - \frac{1}{R}\right) + \frac{\Phi - q}{4\pi\varepsilon_0} \cdot \frac{1}{R} = 0$$

$$\Rightarrow q = -\frac{\varepsilon\Phi}{\left(\frac{R}{r_0} + \varepsilon - 1\right)}$$

~~№ 3.17~~ ~~Дано $q > 0$; S_1, S_2, S_3~~ Дано: a, $\vec{E}_0$ $\varepsilon \gg a$

~~$E_{1,2,3} - ?$~~ $\vec{F} - ?$



Пример 2.10. $\sigma = \sigma_0 \cos\theta$

$$E_{инд} = \frac{\sigma_0}{3\varepsilon_0}$$

$$E - E_{инд} = 0 = E_{in} \Rightarrow \sigma_0 = 3\varepsilon_0 E$$

$$\Rightarrow \sigma = 3\varepsilon_0 E \cos\theta$$

E - внешнее поле

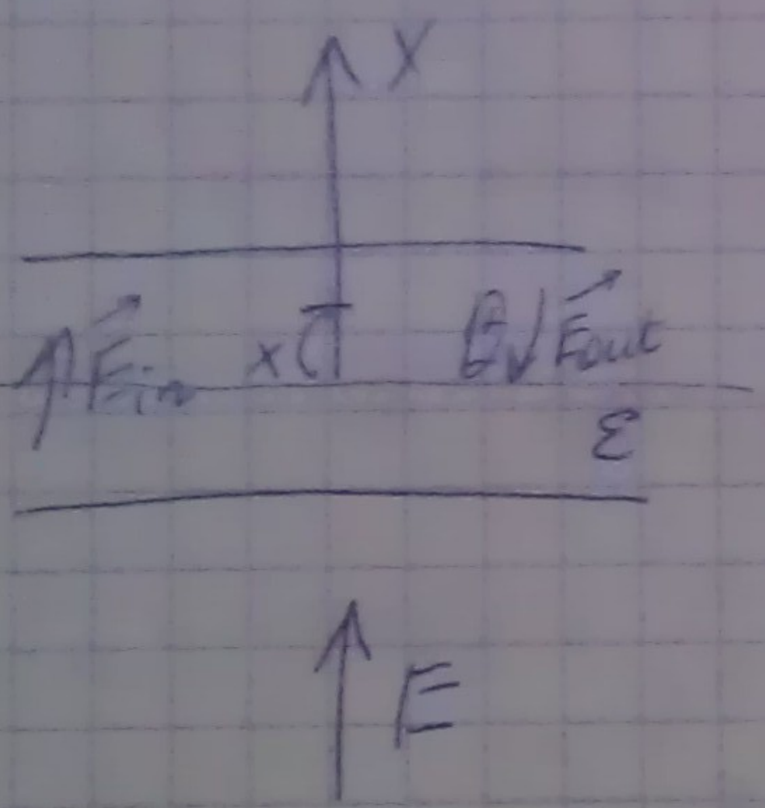
Сфера в поле ~ диполь, $\vec{p} = 4\pi\varepsilon_0 E a^3$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{z^3} = \frac{p}{2\pi\varepsilon z^3} \text{ - поле 1-го диполя}$$

$$F = (\vec{p}\cdot\vec{\nabla})E = p\frac{\partial E}{\partial z} = p\frac{p(-3)}{2\pi\varepsilon_0 z^4} =$$

$$= \frac{16\pi\varepsilon_0 E^2 a^6 \cdot -3}{2\pi\varepsilon z^4} = \frac{-24E^2a^6\pi\varepsilon_0}{z^4}$$

! №318 Дано: $2a, \varepsilon, \vec{E}$ Найти: $\varphi(x)$, $\sigma_{связ}$, ρ - ?



$$\mathcal{D}_{in} = \varepsilon\varepsilon_0 E_{in}$$

$$\mathcal{D}_{out} = \varepsilon_0 E$$

$$E_{in} = E - E_p$$

$$\mathcal{P} = \varepsilon_0(\varepsilon - 1)E_{in}$$

$$\mathcal{D}_{in} = \mathcal{D}_{out} \Rightarrow \varepsilon\varepsilon_0 E_{in} = \varepsilon_0 E$$

$$\sigma_{out} \equiv 0 \qquad \Rightarrow E_{in} = \frac{E}{\varepsilon}$$

$$\Rightarrow \mathcal{P} = \varepsilon_0 \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot E$$

$$\sigma_{связ} = \mathcal{P}_{2n} - \mathcal{P}_{1n} = -\mathcal{P}_n = -\varepsilon_0\frac{\varepsilon-1}{\varepsilon}E$$

$$in: \varphi(x) = -\int_0^x E_{in}\,dx = -\frac{E}{\varepsilon}x$$



ост $\varphi(x) = -\int_0^a E_1\,dx - \int_a^x E\,dx = -\left(\frac{E}{\varepsilon}a + E(x-a)\right) =$

$= -E\left(\frac{a}{\varepsilon} + x - a\right) = -E\left(x + \frac{a(1-\varepsilon)}{\varepsilon}\right)$

Уравнения электростатики

$\operatorname{div}\vec{D} = \rho_{своб}$

$\vec{D} = \varepsilon\varepsilon_0\vec{E}$

$\vec{E} = -\operatorname{grad}\varphi$

$$\boxed{\Delta\varphi = -\frac{\rho_{своб}}{\varepsilon\varepsilon_0}}$$ - ур-е Пуассона

№ 4.21 $\varphi(r) = \begin{cases} 0, & r \geq R \\ a\left(\frac{1}{r} + \frac{r^2}{R^3}\right), & r < R \end{cases}$

$r \to 0 \Rightarrow \exists$ точечный заряд $\boxed{q = 4\pi\varepsilon_0 a}$

$\Rightarrow \varphi = \varphi_1 + \frac{a}{r}$

$\varphi_1 = \begin{cases} -\frac{a}{r}, & r \geq R \\ -\frac{ar^2}{R^3}, & r < R \end{cases}$

$$\Delta_r = \frac{1}{r^2}\left(\frac{d}{dr} r^2 \cdot \frac{\partial}{\partial r}\right)$$

$$\Delta_r \varphi_1 = \begin{cases} \frac{1}{r^2}\left(\frac{d}{dr} r^2 \left(+\frac{a}{r^2}\right)\right) = 0, & r \geq R \\ \frac{1}{r^2}\left(\frac{d}{dr} r^2 \left(-\frac{2ar}{R^3}\right)\right) = -\frac{6ar^2}{r^2 R^3} = -\frac{6a}{R^3}, & r < R \end{cases}$$

$$\rho = \begin{cases} 0, & r > R \\ -\frac{6a\varepsilon\varepsilon_0}{R^3}, & r < R \end{cases}$$

$\Rightarrow$ сфера с пов. зарядом $q = 4\pi\varepsilon a$, $\rho = \frac{-6a\varepsilon\varepsilon_0}{R^3}$

$\sigma_{своб} = D_{2n} - D_{1n}$

$$E_R = -\operatorname{grad}\varphi = \frac{a}{r^2} \quad (r = R)$$

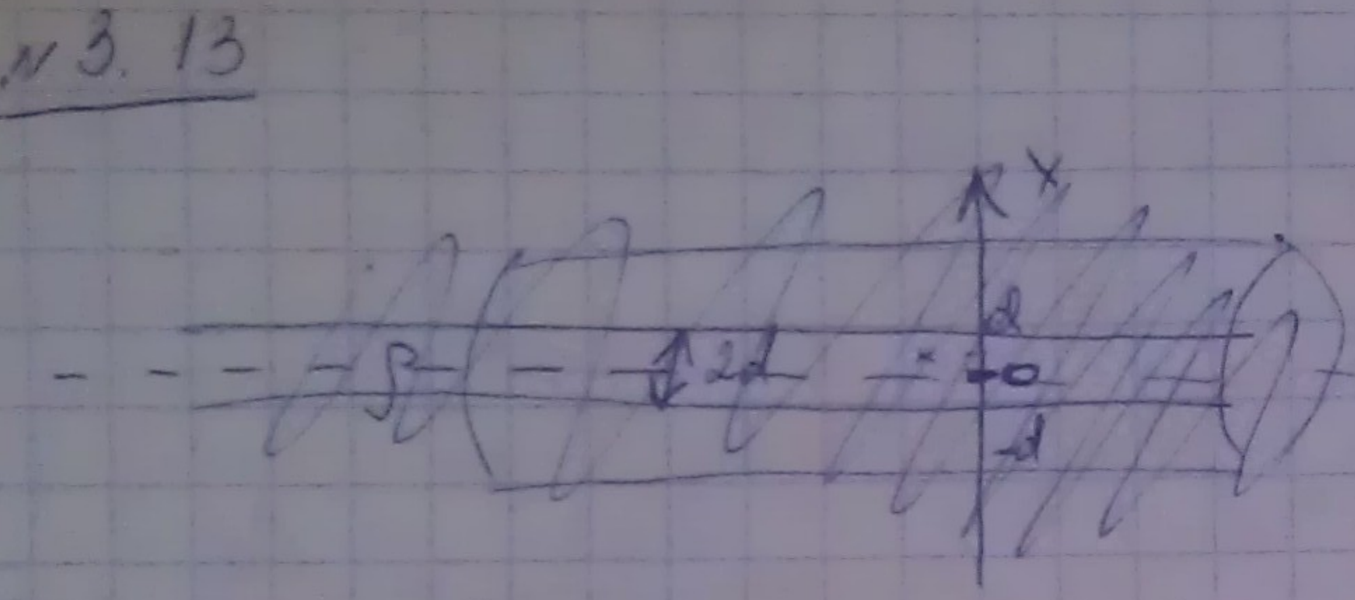
$$E_{out} = + \frac{2a}{R^3} \quad (r = R)$$

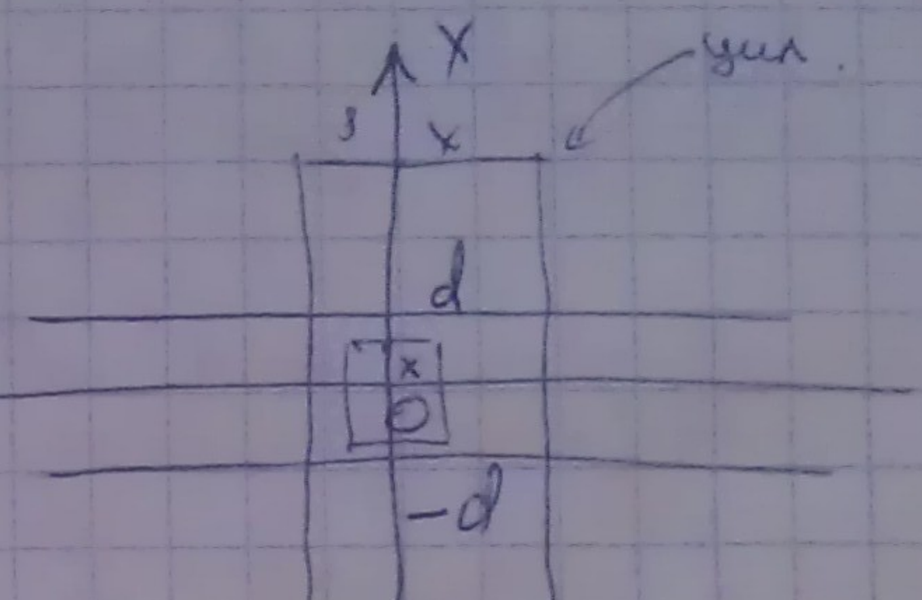
$$\sigma_{своб} = +\varepsilon_0 \frac{2a}{R^2} + \frac{\varepsilon a}{R^2} = \frac{3a\varepsilon_0}{R^2} - \text{свободный заряд}$$

Д/З: 3. ~~15~~, 16, (19), 20, 21, ~~22~~

4 ~~18~~, ~~19~~

№ 3.13





1) $\vec{E}_{in} \parallel Ox$, $|x| \le d$

$$\int \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0}$$

$$2 E_{in} S = \frac{\rho_0 x S \cdot 2}{\varepsilon_0}$$

$$E_{in} 2Sx = \frac{\rho}{\varepsilon_0} \cdot \Rightarrow E_{in} = \frac{\rho x}{\varepsilon_0}$$

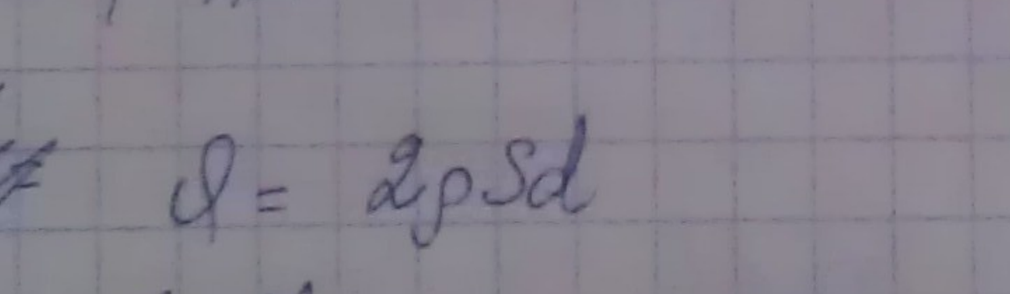
2) $\vec{E}_{out}$, $|x| \ge d$

$$Q = 2\rho S d$$

$$\frac{2\rho S d}{\varepsilon_0} = 2S \cdot E_{out}$$

$$E_{out} = \frac{\rho d}{\varepsilon_0} \cdot \operatorname{sgn} x$$

№ 3.19



$\vec{E}_{out} = 0$

№ 4.18

$$\varphi(r) = \begin{cases} \frac{a}{r}, & r \geq R \\ -\frac{ar^2}{2R^3} + \frac{3a}{2R}, & r < R \end{cases}$$

$r \neq 0$

$\varphi = \varphi_1 + \frac{a}{r} \Rightarrow \varphi_1 = \begin{cases} 0, & r \geq R \\ \frac{ar^2}{2R^3} + \frac{3a}{2R} - \frac{a}{r}, & r < R \end{cases}$

$\rho = \Delta_r \varphi_1$

$\Delta_r = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \varphi_r) = \frac{1}{r^2} \frac{d}{dr}(r^2 \frac{\partial \varphi}{\partial r})$

$\rho = \frac{1}{r^2} \frac{d}{dr}\left(r^2 \cdot \left(-\frac{ar}{2R^3} + \frac{3a}{2R} + \frac{a}{r^2}\right)\right) =$

$= \frac{1}{r^2}\left(\frac{d}{dr}\left(-\frac{ar^3}{2R^3} + \frac{3ar^2}{2R} + a\right)\right) =$

$= \frac{1}{r^2}\left(-\frac{3ar^2}{2R^3}\right) + \frac{3a}{2R} = 0, \quad r < R$

1) $\varphi = \frac{a}{r}, \quad r \geq R$

$\rho = \frac{1}{r^2}\left(\frac{d}{dr}\left(r^2\left(-\frac{a}{r^2}\right)\right)\right) = 0, \quad r \geq R$

2) $\varphi = -\frac{ar^2}{2R^3} + \frac{3a}{2R}$, $r \le R$

$r^2 \frac{\partial \varphi}{\partial r} = -\frac{ar}{R^3} \cdot r^2 = -\frac{ar^3}{R^3}$

$\frac{d}{dr}\left(-\frac{ar^3}{R^3}\right) = -\frac{3ar^2}{R^3}$

$\frac{1}{r^2}\left(\frac{d}{dr}\left(\frac{ar^3}{R^3}\right)\right) = -\frac{3a}{R^3}$, $r \le R$

$\rho = -\frac{3a\varepsilon\varepsilon_0}{R^3}$

№ 4.19

$\varphi(r) = \begin{cases} 0, & r \ge R \\ q\left(\frac{1}{r} + \frac{r^2}{2R^3} - \frac{3}{2R}\right), & r < R \end{cases}$

$r \neq 0 \Rightarrow$ точ заряд $q = 4\pi\varepsilon_0 a$

$\varphi_{**} = \frac{a}{r} + \varphi_1$

$\varphi_1 = \begin{cases} -\frac{a}{r}, & r \ge R \\ \frac{ar^2}{2R^3} - \frac{3a}{2R}, & r \ge R \end{cases}$

$\Delta_r = \frac{1}{r^2}\left(\frac{d}{dr} r^2 \cdot \frac{\partial}{\partial r}\right)$

$\rho = 0, \quad r \ge R$

$r \le R$

$$\Delta_r \varphi_1 = \frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 \cdot \left(\frac{ar}{R^3}\right)\right) = \frac{1}{r^2}\frac{d}{dr}\left(\frac{ar^3}{R^3}\right) =$$

$$= \frac{1}{r^2} \cdot \frac{3ar^2}{R^3} = \frac{3a}{R^3}$$

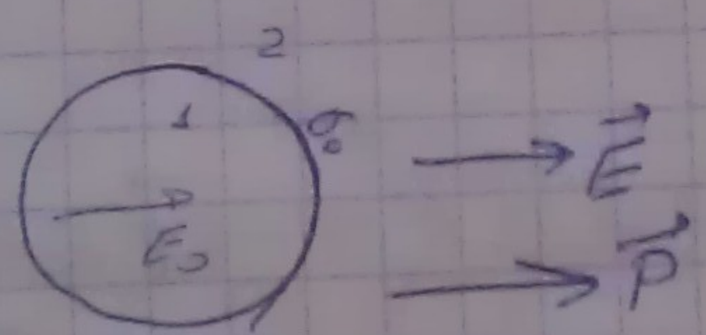
$\rho = \dfrac{3a\varepsilon\varepsilon_0}{R^3}$, $r \le R \Rightarrow$ сфера в ман. поляр.

$\varepsilon = 1$

$c\,q = 4\pi a \varepsilon_0$, $\rho = \dfrac{3a\varepsilon_0}{R^3}$

№321

ε

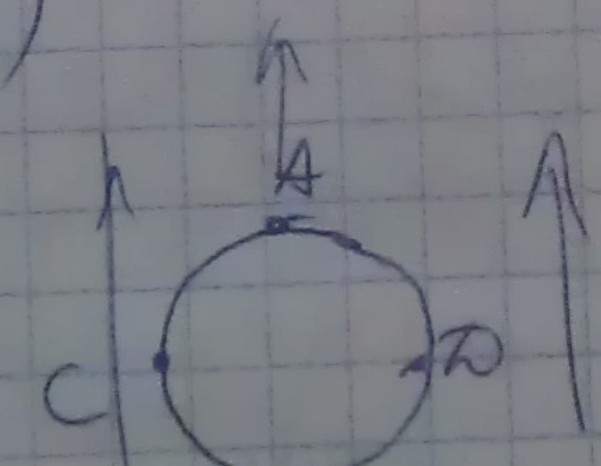


$\sigma = \sigma_0 \cos\theta$

$E_{in} = \dfrac{\sigma_0}{3\varepsilon_0} = \dfrac{P}{3\varepsilon_0}$

$(\sigma_0 = P)$

№322



$p = 4\pi\varepsilon_0 E_0 R^3$

$\vec{E} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{3(\vec{p},\vec{n})\vec{n} - \vec{p}}{r^3}$

A, B: $\dfrac{1}{4\pi\varepsilon_0} \dfrac{3\vec{p} - \vec{p}}{r^3} = \dfrac{2\vec{p}}{4\pi\varepsilon_0 R^3} = \dfrac{2 \cdot 4\pi\varepsilon_0 E_0 R^3}{4\pi\varepsilon_0 R^3} = 2E_0$

$$E_\Sigma = E_0 + 2E_0 = 3E_0$$

С.Д.: $\vec{E} = \frac{-4\pi\varepsilon_0 \vec{E_0} a^3}{4\pi\varepsilon_0 a^3} = -E_0$

$$E_\Sigma = E_0 - E_0 = 0$$

---

1) $\sigma_{связ} = P_{2n} - P_{1n}$

2) $\rho_{связ} - ?$

$$\rho_{связ} = -\operatorname{div}\vec{P} = -\vec{\nabla}\vec{P} = -\vec{\nabla}(\varepsilon_0(\varepsilon-1)\vec{E}) =$$

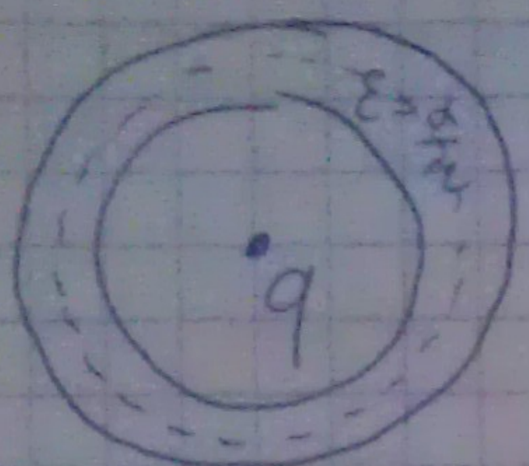
$$= -\varepsilon_0\vec{E}\vec{\nabla}(\varepsilon-1) - \varepsilon_0(\varepsilon-1)\vec{\nabla}\vec{E} = -\operatorname{div}\vec{E}$$

$$= -\varepsilon_0\vec{E}\vec{\nabla}\varepsilon - \varepsilon_0(\varepsilon-1)(\rho_{связ} + \rho_{связ})$$

$$\Rightarrow \rho_{связ} = -\frac{(\varepsilon-1)}{\varepsilon}\rho_{связ} - \varepsilon_0\vec{E}\frac{\vec{\nabla}\varepsilon}{\varepsilon}$$

$$\left(\operatorname{div}\vec{E} = \frac{\rho_{связ}}{\varepsilon\varepsilon_0} = \frac{\rho_{связ}}{\varepsilon_0} + \frac{\rho_{связ}}{\varepsilon_0}\right)$$

№1 ~~Найти вектор поляризации~~



$\rho_{связ}$ — ?

Нет свободного заряда

$$\rho_{связ} = -\varepsilon_0 E \frac{\nabla \varepsilon}{\varepsilon} = -\varepsilon_0 \frac{q}{4\pi\varepsilon_0 r^2}\left(-\frac{\sigma}{\varepsilon r^2}\right)\Big/\frac{\sigma}{\varepsilon} =$$

$$= \frac{q}{4\pi r^3}$$

$$\rho_{связ} = -\operatorname{div}\vec{P}$$

$$4\pi r^2 P = -q_{связ}(r) \quad \Big/ d$$

$$\frac{dP}{dV}, \text{ т.к. } \varepsilon = \varepsilon(r)$$

$$4\pi\, d(r^2 P) = -\rho_{связ} 4\pi r^2 dr$$

$$4\pi r^2 dP + 4\pi P \cdot 2r \cdot dr = -\rho_{связ} \cdot 4\pi r^2 dr$$

$$r^2 dP + P \cdot 2r\, dr = -\rho_{связ} r^2 dr$$

$$\Rightarrow \rho_{связ} = -\left(\frac{dP}{dr} + \frac{2P}{r}\right)$$

$$\vec{P} = \varepsilon_0(\varepsilon - 1)\vec{E} \quad \left\{\chi = \frac{\varepsilon}{\varepsilon\varepsilon_0}\right\} = \frac{(\varepsilon-1)}{\varepsilon}\vec{D} =$$

$$= \frac{\varepsilon - 1}{\varepsilon}$$

$$\vec{F} = \frac{dq}{4\pi\varepsilon\varepsilon_0 r^2} \cdot \frac{q}{4\pi\varepsilon_0 r^2}$$

$$\frac{dP}{dr} = -\frac{2q}{4\pi r^3} + \frac{q}{4\pi dr^2}$$

$$f_{выз} = -\frac{q}{2\pi r^3} + \frac{q}{4\pi dr^2} + \frac{2dq}{4\pi r^3} - \frac{2q}{4\pi dr^2} =$$

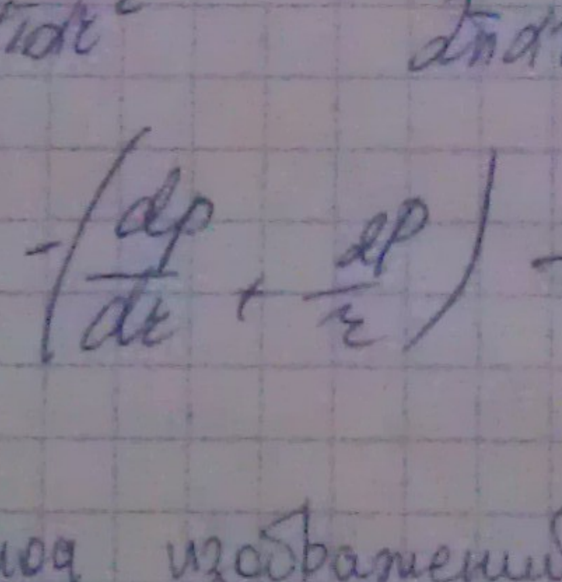
$$= \frac{q}{4\pi dr^2} - \frac{q}{2\pi dr^2}$$

$$f_{выз} = -\left(\frac{dp}{dt} + \frac{2P}{r}\right) - \text{для сферы}$$

Метод изображений

(позволяет найти заряды на пов-ти заменить на конфигурацию точечных зарядов)

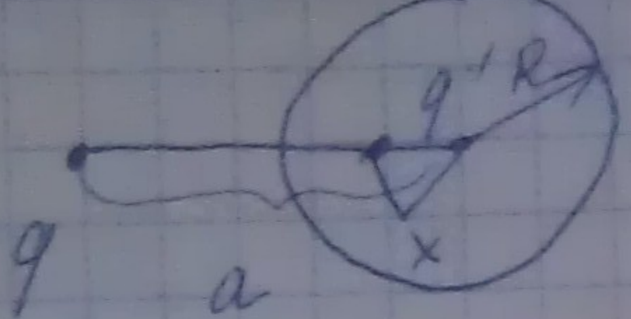
1)



$q' = -q$ пов-ти

потенциал при замене на поверхности меняться

2) проводящая сфера



а) если сфера заземлена $\varphi = 0$

$$\Rightarrow q' = -q\frac{R}{a}$$

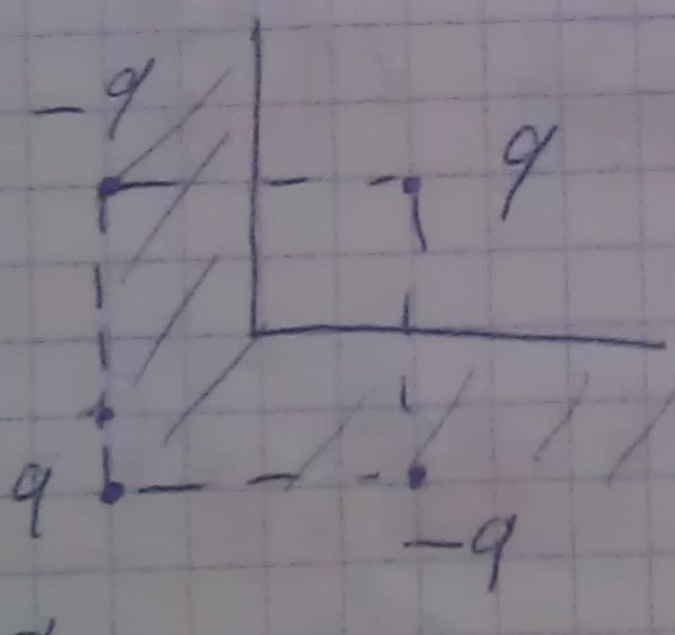
$$x = \frac{R^2}{a}, \quad q_{ин} = -q$$

б) $\varphi \neq 0$, Q – сторонний заряд на сфере

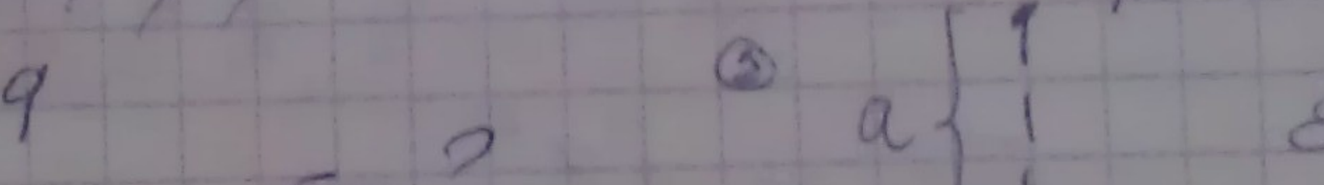
$$q' = -q\frac{R}{a}, \quad x = \frac{R^2}{a}, \quad q'' = Q + \frac{qR}{a}$$

в центре сферы

3)



№4.11 Дано: $q, \varepsilon_1, \varepsilon_2$ $F = ?$



$$F = \frac{q^2}{4\pi\varepsilon_0(2a)^2} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot q$$

↓

Сила действ. на заряд q – только для полупр-ва ①

Если нужно узнать поле и др. величины в полупр-ве

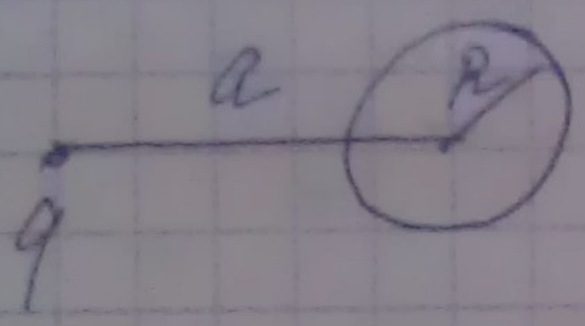
(2) ⇒ вместо заряда $q = \frac{2E_2}{E_1+E_2} q$ вместо заряда q и q'

~~q_1~~ ~~q'~~

№4.5 Дано: q, a, R

A - ? если 1) шар заземлен $\varphi = 0$

2) $\varphi \neq 0$, $Q = 0$



$$A = \int_a^\infty \vec{F} d\vec{x}$$

1) $\varphi = 0$ $\quad F = \frac{-q^2 R}{a\, 4\pi\varepsilon_0 (a-x)^2}$, $x = \frac{R^2}{a}$

2) $\varphi \neq 0$, $Q = 0$, $\quad q'' = q\frac{R}{a}$, $\quad q' = -q\frac{R}{a}$

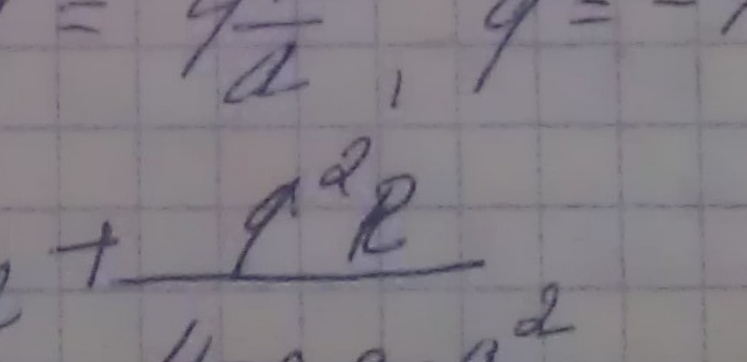
$$F = \frac{-Rq^2}{4\pi\varepsilon_0 a\left(a - \frac{R}{a}\right)^2} + \frac{q^2 R}{4\pi\varepsilon_0 a \cdot a^2}$$

a - переменная интегрирования

№4.16 Дано: ε, L, q

Найти - ?

F_S - ?



$$q' = \frac{1-\varepsilon}{1+\varepsilon} \cdot q \Rightarrow F = \frac{-1}{4\pi\varepsilon_0} \frac{q^2 \left(\frac{1-\varepsilon}{1+\varepsilon}\right)}{4L^2}$$

$\sigma = P_n$

$\vec{P} = \varepsilon_0 (\varepsilon - 1) \vec{E}$

$$|E_n| = \frac{q}{4\pi\varepsilon_0} \cdot \frac{L}{(L^2+x^2)^{3/2}} + \frac{(1-\varepsilon)}{(1+\varepsilon)} \frac{q}{4\pi\varepsilon_0} \cdot \frac{L}{(L^2+x^2)^{3/2}}$$

$$|E_n| = \frac{Lq}{(1+\varepsilon) 4\pi\varepsilon_0 (L^2+x^2)^{3/2}} \Rightarrow \vec{P}_n = \text{---}\!\!\rightarrow .$$

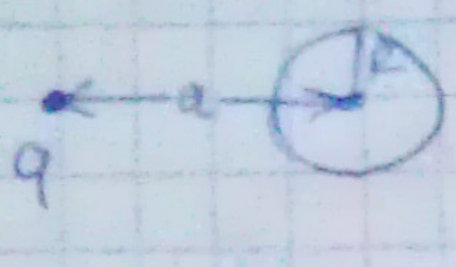
Рассм. плоскость конс. беск. круг

$$\Phi = \int_0^{\infty} dx \int_0^{2\pi} \sigma \, d\varphi$$

Д/З: ~~4.16, 4.5~~

4, (12), (23), (4), ~~6, 7, 8~~, 17, ~~15~~

~~№4.4~~ Дано: $R, q, a, \sigma - ?$



$$\Phi = S\sigma = 4\pi R^2 \sigma$$

$$q' = -\frac{qR}{a}$$

$$q'' = 4\pi R^2 \sigma + \frac{qR}{a}$$

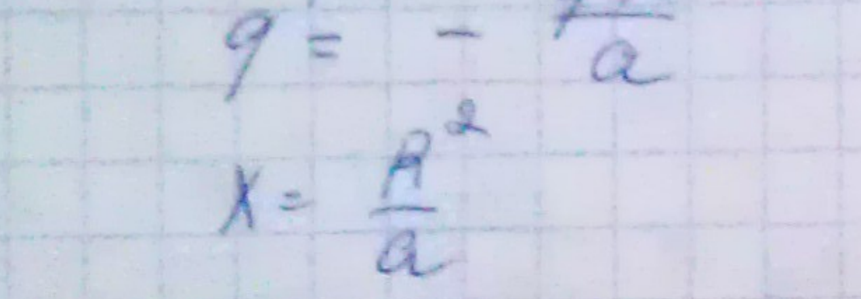
$$x = \frac{R}{a}$$

в центре

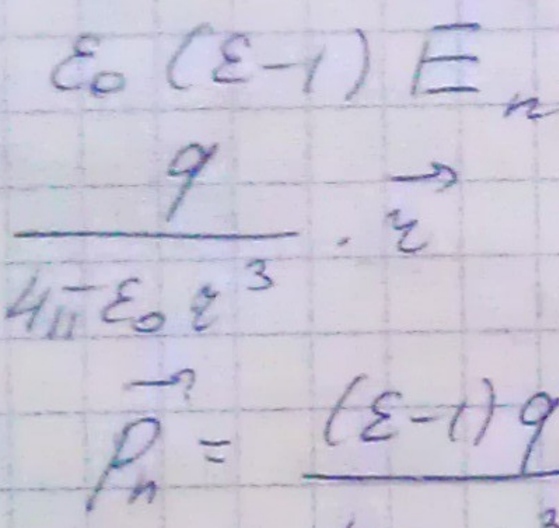
$$\sigma = P_n = \varepsilon_0 (\varepsilon - 1) E_n$$

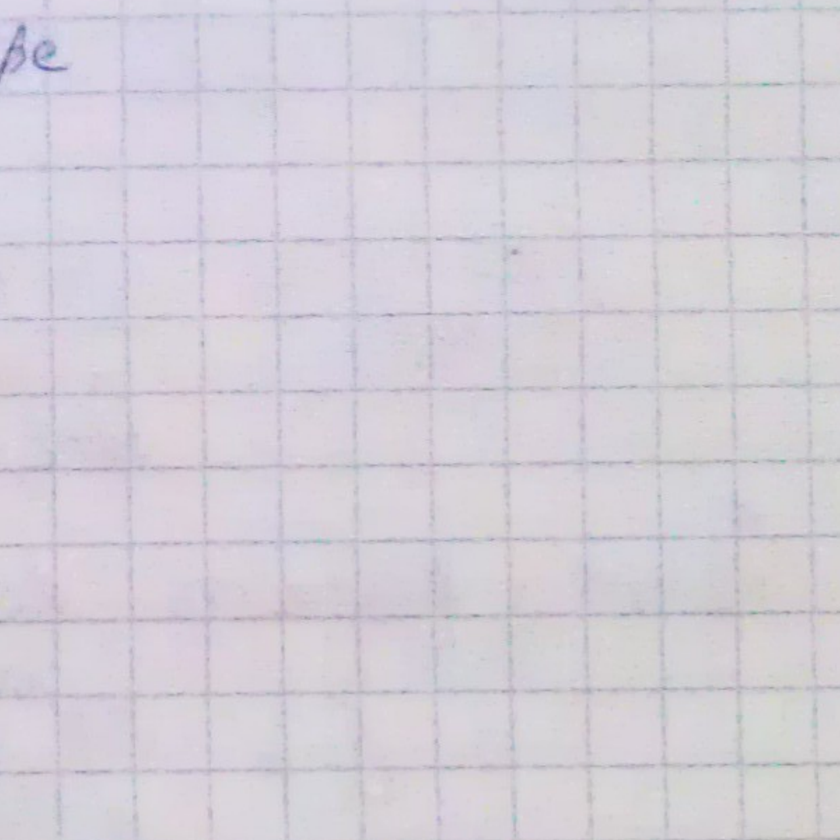
$$\vec{E} = \frac{q}{4\pi\varepsilon_0 \varepsilon^3} \cdot \vec{\varepsilon}$$

$$\vec{P_n} = \frac{(\varepsilon - 1) q}{4\pi\varepsilon_0 \varepsilon^3} \cdot \vec{\varepsilon}$$



№4.6 Дано: $q, d, F - ?$





$$Ox: \quad F_x = F_{1x} + F_{3x} + \cancel{F_{2x}}^{\,0} = -|F_1| + |F_3| \cdot \frac{\sqrt{2}}{2}$$

$$Oy: \quad F_y = \cancel{F_{1y}}^{\,0} + F_{3y} + F_{2y} = -|F_2| + |F_3| \cdot \frac{\sqrt{2}}{2}$$

$$|F_3| = \frac{q^2}{4\pi\varepsilon_0 (d\sqrt{2})^2} = \frac{q^2}{8\pi\varepsilon_0 d^2}$$

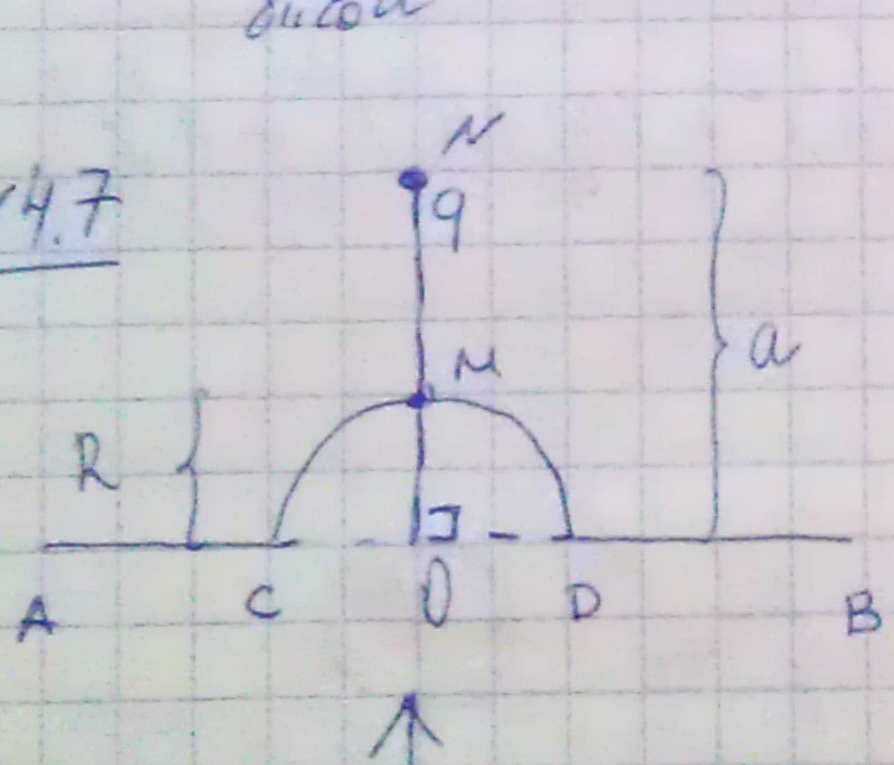
$$|F_2| = \frac{q^2}{8\pi\varepsilon_0 d^2} = |F_1|$$

$$|F_3| = \frac{q^2}{4\pi\varepsilon_0 (2d)^2} = \frac{q^2}{16\pi\varepsilon_0 d^2}$$

$$F_x = \frac{q^2}{16\pi\varepsilon_0 d^2} \cdot \frac{\sqrt{2}}{2} - \frac{q^2}{8\pi\varepsilon_0 d^2} = \frac{q^2}{8\pi\varepsilon_0 d^2}\left(\frac{\sqrt{2}}{4} - 1\right) = F_y$$

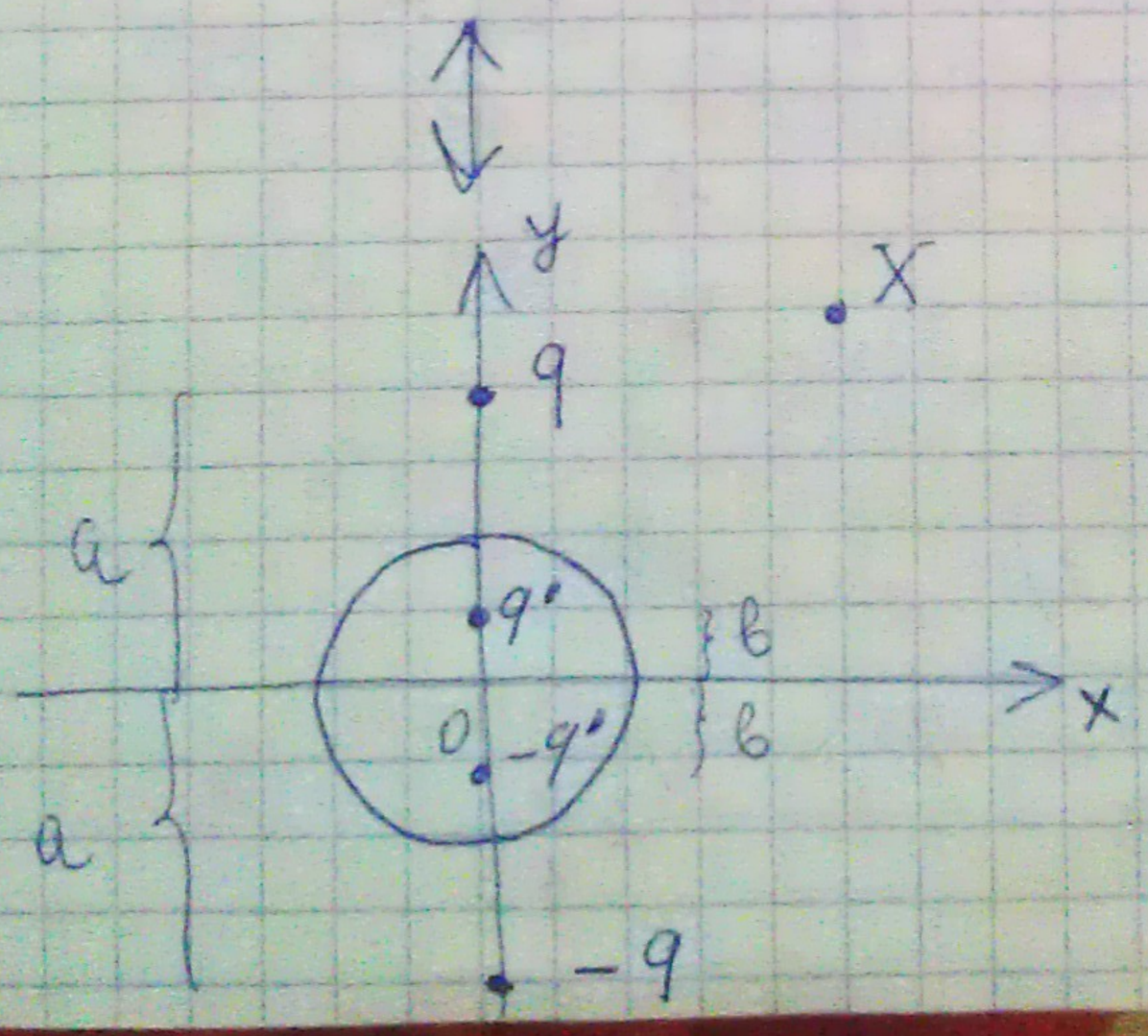
$$|F| = \frac{q^2}{8\pi\varepsilon_0 d^2}\left|\sqrt{2}\left(\frac{\sqrt{2}}{4} - 1\right)\right| = \frac{q^2}{8\pi\varepsilon_0 d^2}\left|\frac{1}{2} - \sqrt{2}\right| = \boxed{\frac{q^2}{8\pi\varepsilon_0 d^2}\left(\sqrt{2} - \frac{1}{2}\right)}$$

№4.7



Дано: q, a, R

$\varphi(x, y, z)$ – ?



$$q' = qR/a$$

$$b = R^2/a$$

$X(x, y, z)$ – произв.

$r_p - Ba$

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x^2+(y-a)^2+z^2} - \frac{1}{x^2+(y+a)^2+z^2}\right) +$$

$$+ \frac{qR}{4\pi\varepsilon_0 a}\left(\frac{1}{x^2+\left(y-\frac{R^2}{a}\right)^2+z^2} - \frac{1}{x^2+\left(y+\frac{R^2}{a}\right)^2+z^2}\right) =$$

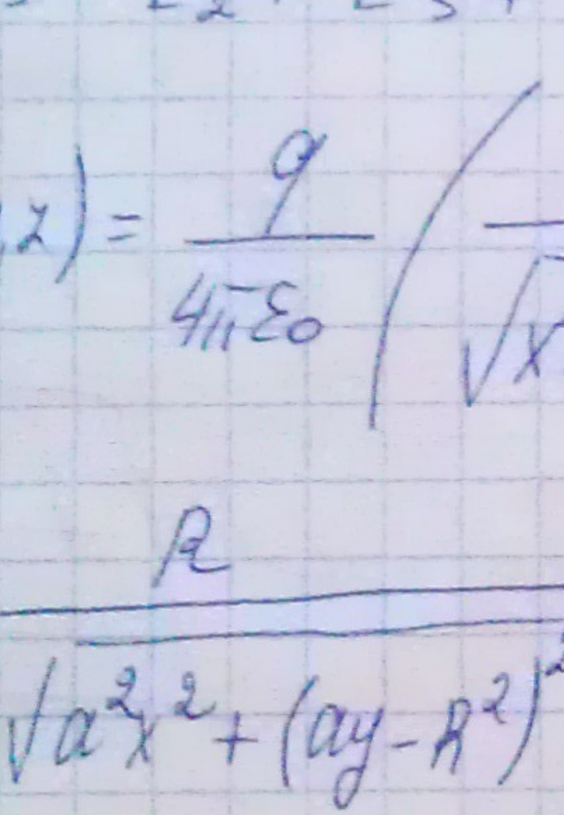
$$= E_1 + E_2 + E_3 + E_4$$

$$\Rightarrow \varphi(x,y,z) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+(y-a)^2+z^2}} - \frac{1}{\sqrt{x^2+(y+a)^2+z^2}}\right.$$

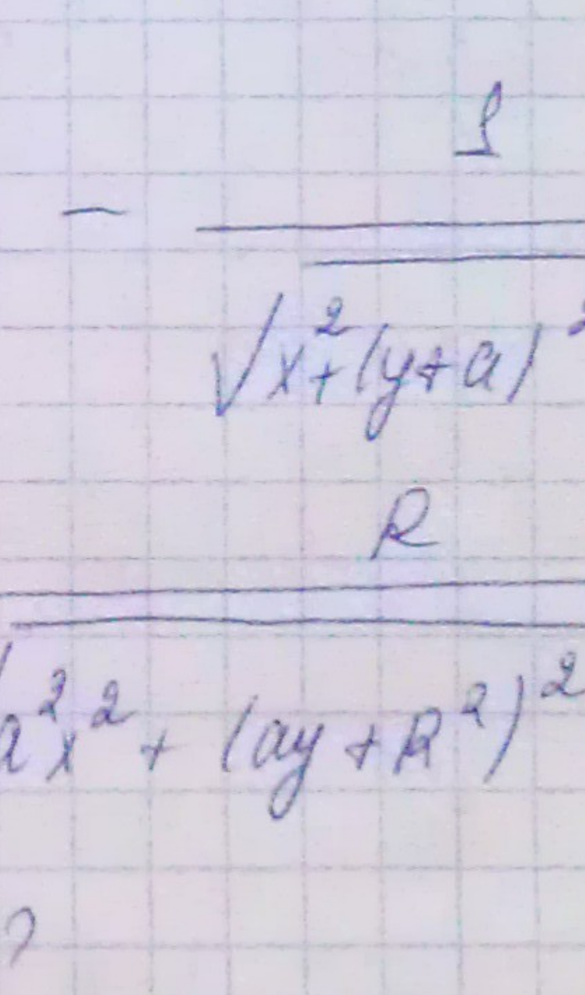
$$\left. + \frac{R}{\sqrt{a^2x^2+(ay-R^2)^2+a^2z^2}} - \frac{R}{\sqrt{a^2x^2+(ay+R^2)^2+a^2z^2}}\right)$$

№4.8 Дано: q, d, R. q' - ?

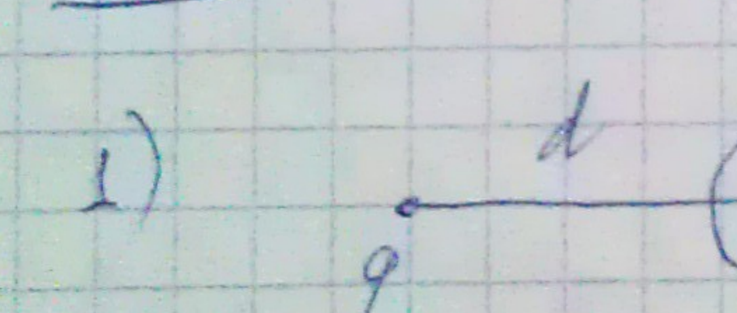
1)



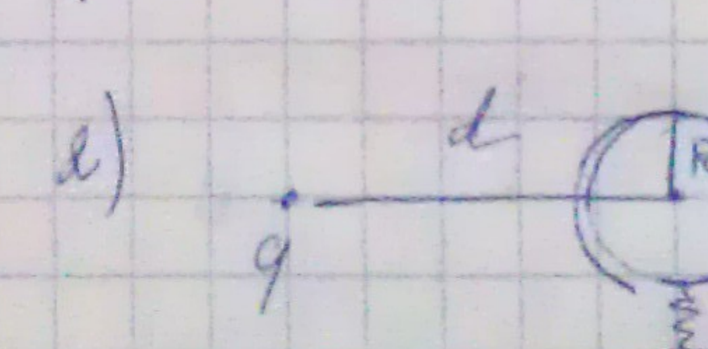
2)



1)



2)



$\varphi \neq 0$

$q_1' = -\frac{qR}{a}$

$q_1'' = Q + \frac{qR}{a}$

Q - суммарный заряд на шаре

$\Rightarrow Q = 0$

$\Rightarrow q_1'' = \frac{qR}{a}$

$x = \frac{R}{a}^2$

$\varphi = 0$

$q_2' = -\frac{qR}{a}$

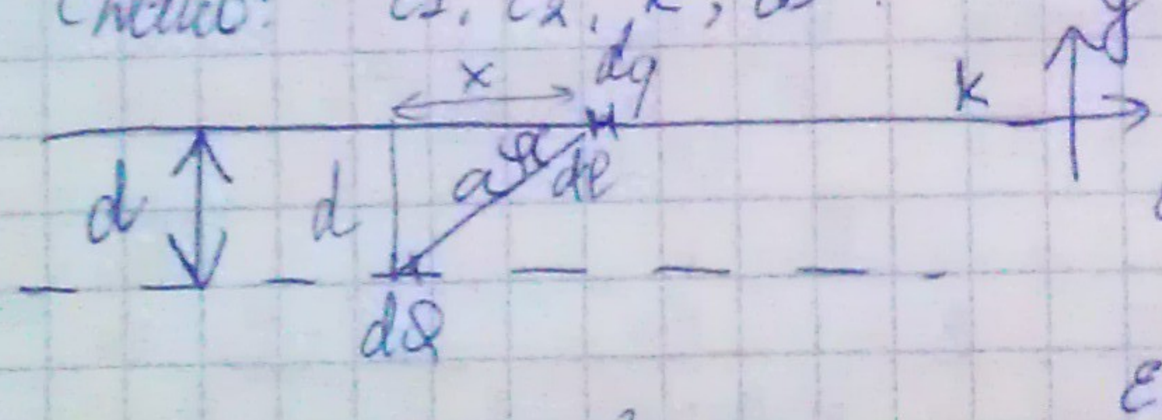
$x = \frac{R}{a}^2$

$q_2'' = 0$

$\Rightarrow \Delta q'' = q_2'' - q_1'' = -\frac{qR}{a}$

$$\boxed{\Delta q'' = -\frac{qR}{a}}$$

№ 4.15 Дано: $\varepsilon_1, \varepsilon_2, k, d$ Найти F - ?

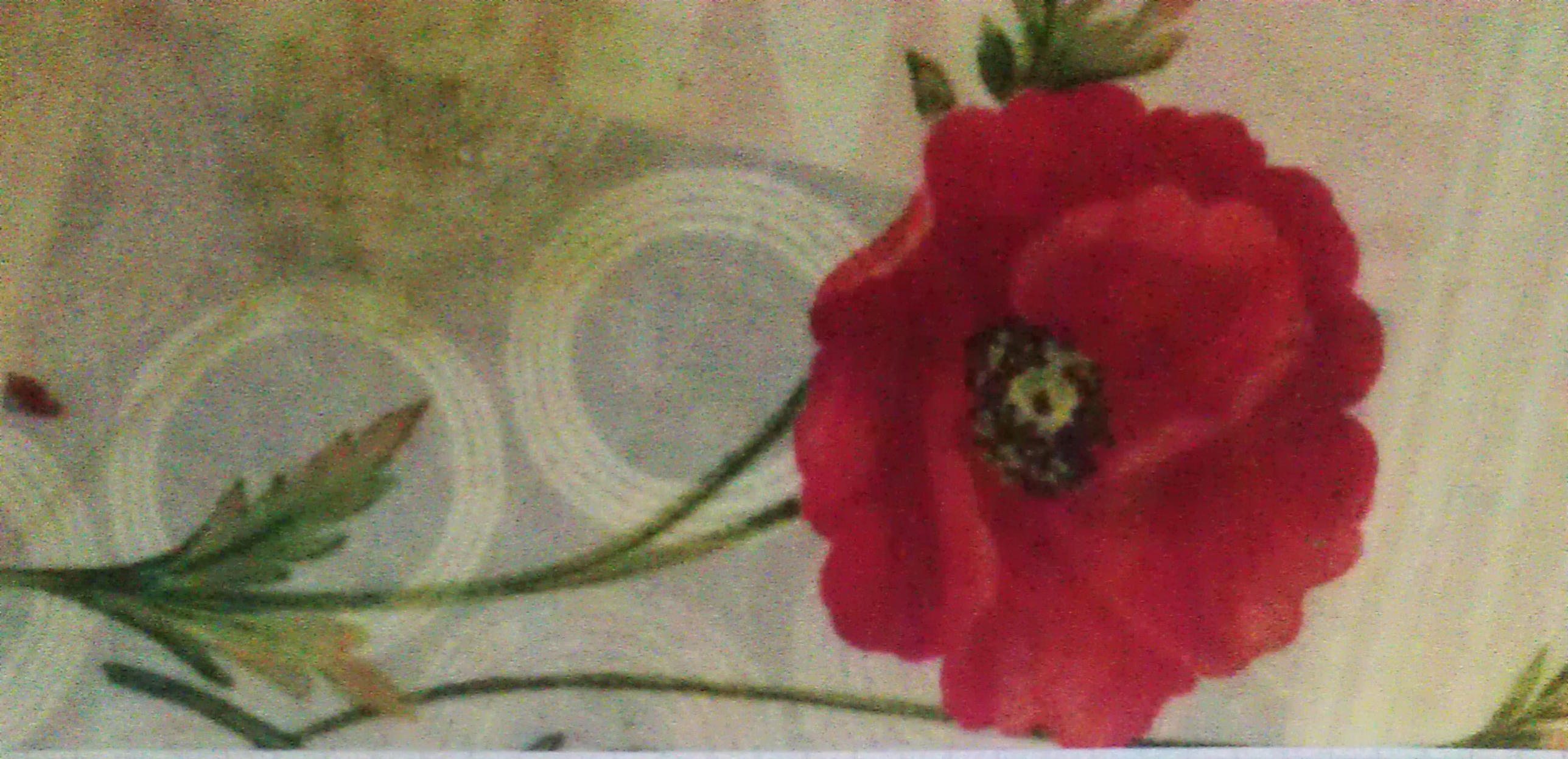


~~№ 4.14~~ $\Rightarrow F = \frac{q^2}{4\pi\varepsilon_0(2d)^2} \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$

$dq = k\,dl$

$F = \int \frac{k^2 l^2}{16\pi\varepsilon_0 d^2} \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} dl$

$dQ = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} dq = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} k\,dl$ (№ 4.16)



$$dE = \frac{dQ}{4\pi\varepsilon_0 a^2 \varepsilon_1} = \frac{k\,dl \cdot \varepsilon_1 - \varepsilon_2}{4\pi\varepsilon_0 \varepsilon_1 a^2 \varepsilon_1 + \varepsilon_2}$$

$$a = \sqrt{d^2 + x^2}$$

$$dE_x = 0$$

$$dE_y = \frac{k\,dl \sin\varphi}{4\pi\varepsilon_1 (d^2+x^2)\varepsilon_0} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}; \quad \sin\varphi = \frac{d}{\sqrt{d^2+x^2}}$$

$$dl = dx$$

$$dE_y = \frac{kd}{4\pi\varepsilon_1\varepsilon_0} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot \frac{dx}{(d^2+x^2)^{3/2}}$$

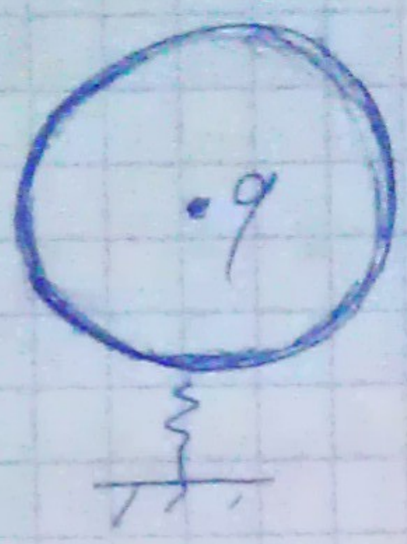
$$E = \frac{kd}{4\pi\varepsilon_1\varepsilon_0} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \int_{\mathbb{R}} \frac{dx}{(d^2+x^2)^{3/2}} =$$

$$= \frac{k}{4\pi\varepsilon_1\varepsilon_0 d} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot 2 = \frac{k}{2\pi\varepsilon_1\varepsilon_0 d} \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$F = Eq = E\,k\,dl$$

$$\Rightarrow \boxed{\frac{F}{dl} = \frac{k^2}{2\pi\varepsilon_1\varepsilon_0 d} \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}}$$

№4.17



№4.23 – внутри №5.23

Электроёмкость. Энергия эл. поля

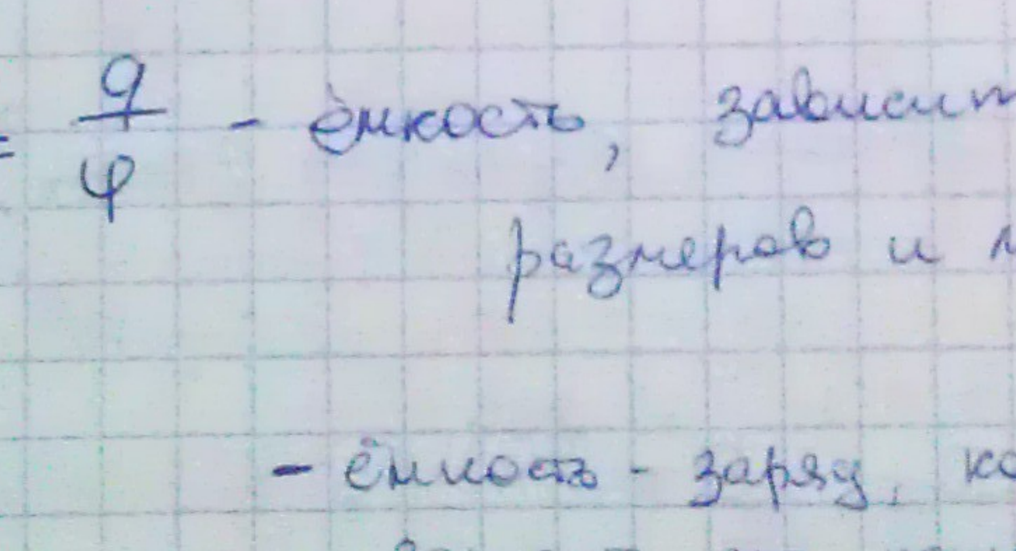
$\omega = \frac{1}{2} \varepsilon \varepsilon_0 E^2$ – объёмная плотность энергии поля

$W = \int_V \omega dV$, $W = \frac{CU^2}{2}$ – ёмкость конденсатора

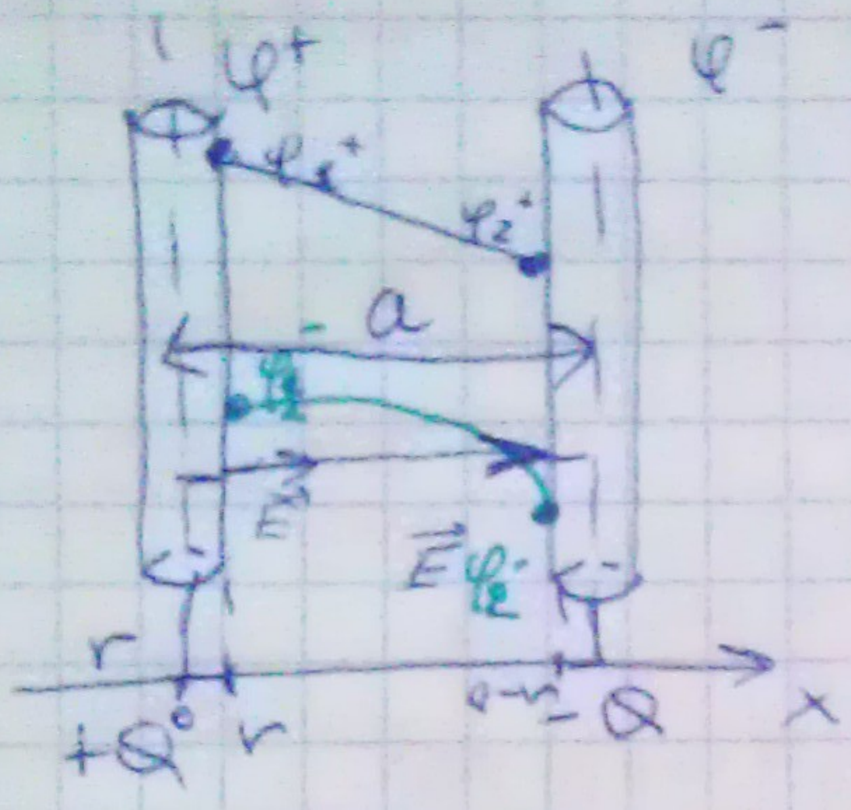
$C = \frac{q}{\varphi}$ – ёмкость, зависит только от формы, размеров и материала тела.

– ёмкость – заряд, кот. нужно сообщ. телу, чтоб увеличить его потенциал на 1 В

№5.7



Дано: $a \gg r$, $C_0 - ?$



$$U = \int_a^b E\,dr = Q \cdot (\dots)$$

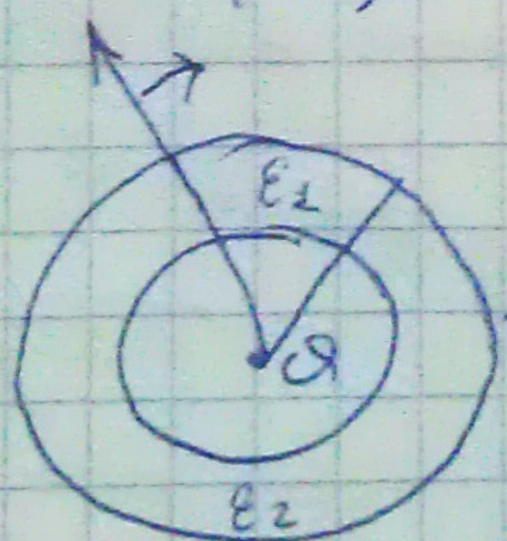
$$C = \frac{Q}{U}$$

$\vec{E} = \dfrac{\varphi}{2\pi r l \varepsilon_0}$ – поле нити (один)

$$U = \int_r^{a-r} \frac{Q\,dr}{4\pi\varepsilon_0 r l} = \frac{Q}{\pi\varepsilon_0 l} \ln\frac{a-\varepsilon}{\varepsilon} \sim \frac{\varphi}{\pi\varepsilon_0 l} \ln\frac{a}{\varepsilon}$$

$$\Rightarrow C_0 = \frac{\pi\varepsilon_0}{\ln a/\varepsilon} = \frac{C}{l} \text{ (ёмкость единицы длины)}$$

№5.23 Дано: $\varepsilon_1, \varepsilon_2, \Omega_1, \Omega_2$, $E = ?$



$$Q = \int \mathcal{D}\,dS$$

$$\mathcal{D} = \varepsilon\varepsilon_0 E$$

$E_{\varepsilon_1} = E_{\varepsilon_2} = E$ – одинаковое поле

$$\int \varepsilon_1 \varepsilon_0 E\, dS_1 + \int \varepsilon_2 \varepsilon_0 E\, dS_2 = Q$$

$$d\Omega = \sin\theta\, d\theta\, d\varphi$$

$$dS = r^2 d\Omega$$

$$\varepsilon_0 E \left[\int_0^{\Omega_1} r^2 \varepsilon_1 d\Omega + \int_{\Omega_1}^{4\pi} r^2 \varepsilon_2 d\Omega \right] =$$

$$= \varepsilon_0 E r^2 (\varepsilon_1 \Omega_1 + \varepsilon_2 \Omega_2) = Q \qquad (\Omega_1 + \Omega_2 = 4\pi)$$

$$\Rightarrow E = \frac{Q}{\varepsilon_0 r^2 (\varepsilon_1 \Omega_1 + \varepsilon_2 \Omega_2)}$$

$$U = \int_{R_1}^{R_2} \frac{Q dr}{\varepsilon_0 r^2 (\ldots)} = \frac{Q}{\varepsilon_0 (\ldots)} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow C = \frac{\varepsilon_0 (\varepsilon_1 \Omega_1 + \varepsilon_2 \Omega_2) R_1 R_2}{R_2 - R_1}$$

↓

опред-ся геометрия системы и св-ва вещ-ва.

№ 5.15 Дано: $S, d, \xi, \varepsilon_1 = \varepsilon, \varepsilon_2 = 1$

$\Delta W = ?$ $A = ?$

1) $\xi = const$, 2) $Q = const$

2) $Q = const$

$+ \vdash$ $C_{изм} \downarrow$

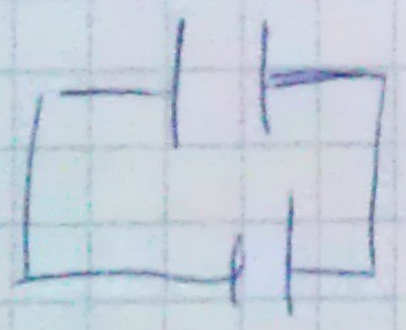
$C = \frac{\varepsilon\varepsilon_0 S}{d}$, $C = \frac{q}{u} \Rightarrow u_{изм} \uparrow$

$W = \frac{q^2}{2C}$

$W_2 - W_1 = \frac{q^2 d}{2\varepsilon_0 S}(1-\varepsilon) = \frac{q^2 d}{2\varepsilon_0 S}\underbrace{\left(\frac{\varepsilon-1}{\varepsilon}\right)}_{>0} > 0$

$\Delta W = A$, $q = \xi C = \xi \frac{\varepsilon\varepsilon_0 S}{d}$

1) $\xi = const$ $(\Rightarrow \Delta C)$



$C_{изм} \downarrow$, $\xi = const \Rightarrow q \downarrow$

$W = \frac{Cu^2}{2}$

$W_2 = W_1 = \frac{\xi^2 \varepsilon_0 S}{2d}(1-\varepsilon) < 0$

$\Delta W = A + A_{ист}$

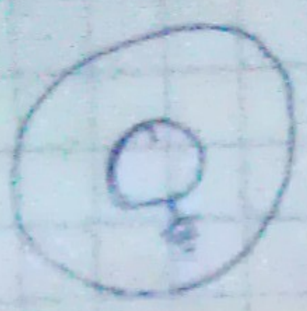
↙ ↘

внешняя работа источника $= \xi \Delta q =$

$= \xi(q_2 - q_1) = \xi(C_2\xi - C_1\xi) = \frac{\xi^2 \varepsilon_0 S(1-\varepsilon)}{d} = 2\Delta W$

$\Rightarrow A = \Delta W - A_{ист} = -\frac{\xi^2 d}{2\varepsilon_0 S}(1-\varepsilon) = -\Delta W$

№5.2 R_1, R_2 ($R_2 > R_1$), U, ΔW - ?

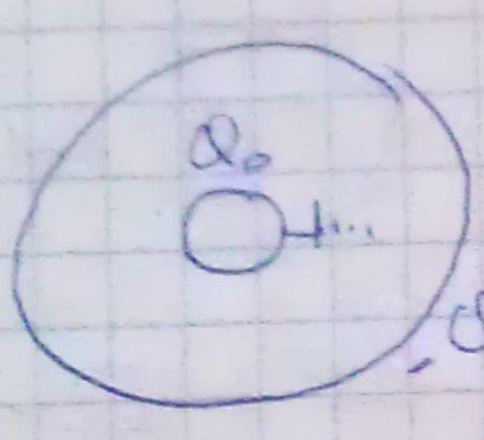


1) $W = \frac{CU^2}{2}$

$C = 4\pi\varepsilon_0 \frac{R_2 R_1}{R_2 - R_1}$ - емкость сферич конд.

$W_1 = \frac{U^2}{2} \cdot 4\pi\varepsilon_0 \frac{R_2 R_1}{R_2 - R_1}$

2)



$\varphi = 0$

$W = \int \varepsilon_0 \cdot \frac{1}{2} \cdot E^2 dV =$

$= \int_0^{R_2} \int_0^{2\pi} \int_0^{\pi} \varepsilon_0 \frac{1}{2} \frac{Q_0^2}{16\pi^2\varepsilon_0^2 r^4} r^2 \sin\theta \, dr d\varphi d\theta =$

$= \frac{Q_0^2}{32\pi^2\varepsilon_0} \cdot 4\pi \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{Q_0^2}{8\pi\varepsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$\varphi = \int_{R_1}^{R_2} \frac{Q_0 \, dr}{4\pi\varepsilon_0 r^2} + \int_{R_2}^{\infty} \frac{Q_0 - Q}{4\pi\varepsilon_0 r^2} dr = 0$

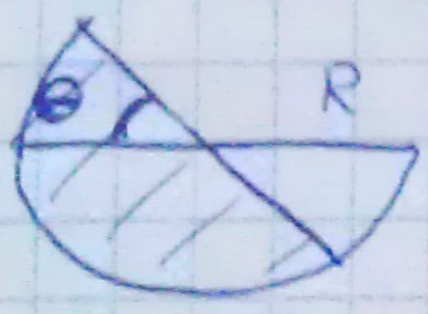
$Q_0 \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{Q_0 - Q}{R_2} = 0$

$\frac{Q_0}{R_1} - \frac{Q_0}{R_2} + \frac{Q_0}{R_2} - \frac{Q}{R_2} = 0 \Rightarrow \frac{Q_0}{R_1} = \frac{Q}{R_2}$

$$\Rightarrow W = \frac{q^2 R_1 (R_2 - R_1)}{8\pi\varepsilon_0 R_2^2 R_1 R_2} = \left\{ q = CU \right\} =$$

$$= \frac{U^2 R_2^2 R_1^2 (4\pi\varepsilon_0)^2 \cdot R_1}{(R_2 - R_1) \cdot 8\pi\varepsilon_0 R_2^3} = \frac{U^2 R_1^3 \cdot 2\pi\varepsilon_0}{(R_2 - R_1) R_2^3}$$

№ 6.24 Дано: $R, \alpha, \varepsilon, \xi$ M - ?



$$K = \frac{I\omega^2}{2}, \quad K_t' = \frac{I}{2} \cdot 2\omega \cdot \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} =$$

$\omega = \omega(\theta(t))$

$$\frac{dK}{dt} = I\omega \frac{d\omega}{dt} = I\varepsilon \cdot \omega = M\omega$$

$$dK = M\omega dt = M d\theta$$

$$\Rightarrow A = M d\theta$$

$\xi = const$

$$\Delta W = \frac{\xi^2 \Delta C}{2}$$

$$C = C_1 + C_2$$

$$C_1 = \frac{\varepsilon\varepsilon_0 S_1}{d}, \quad C_2 = \frac{\varepsilon_0 S_2}{d}$$

$$C = \frac{\varepsilon_0}{d}(\varepsilon S_1 + S_2)$$

[ ] [ ] - параллельное соед-е

$$S_1 = \frac{(\pi - \Theta) R^2}{2}$$

$$S_2 = \frac{\Theta R^2}{2}$$

$$\Rightarrow C = \frac{\varepsilon_0}{d}\left(\varepsilon(\pi - \Theta) R^2 + \Theta R^2 \right) =$$

$$= \frac{\varepsilon_0 R^2}{2d}\left(\varepsilon(\pi - \Theta) + \Theta \right) = \frac{\varepsilon_0 R^2}{2d}\left(\Theta(1 - \varepsilon) + \pi\varepsilon \right)$$

$$\Delta C = \frac{\varepsilon_0 R^2}{2d}(1 - \varepsilon)\Delta\Theta$$

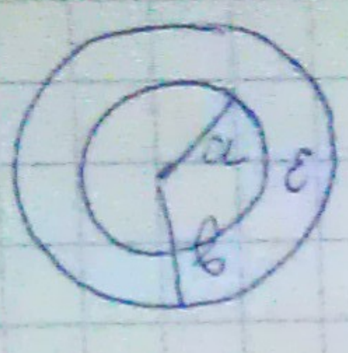
$$A_{исп} = \xi^2 \Delta C$$

$$\Delta W = \frac{\xi^2 \Delta C}{2} \Rightarrow A = -\frac{\xi^2 \Delta C}{2} = \frac{\varepsilon_0 R^2 (\varepsilon - 1)\Delta\Theta\, \xi^2}{4d}$$

$$\Rightarrow M = (\varepsilon - 1)\frac{\varepsilon_0 \xi^2 R^2}{4d}$$

Дз: §5 ~~20~~, ~~18~~, 22, ~~16~~, ~~10~~, ~~14~~, ~~8~~, ~~8~~, ~~6~~, 3

№5.6



Дано: $a, b, \varepsilon = \frac{\varepsilon_a Q}{r}$, u

$C - ?$

$W - ?$

$$\omega = \frac{1}{2}\varepsilon\varepsilon_0 E^2 = \frac{1}{2}\frac{\varepsilon_0 \varepsilon_a Q}{r} E^2$$

$$W = \int_V \omega dV = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_a^b \frac{1}{2}\varepsilon_a\varepsilon_0 \frac{a}{r} E^2 dr \cdot r^2 \sin\theta \, dr =$$

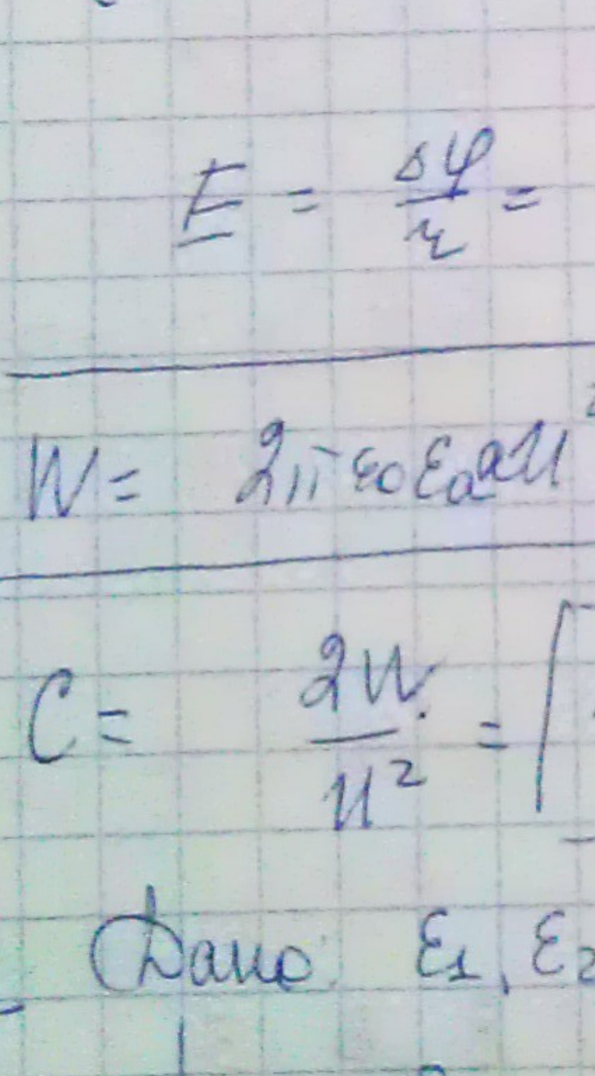
$$= 2\pi\varepsilon_0\varepsilon_a a \int_a^b rE^2 dr$$

$$\underline{E} = \frac{\Delta\varphi}{r} = \frac{U}{r} \Rightarrow E^2 = \frac{U^2}{r^2}$$

$$\boxed{W = 2\pi\varepsilon_0\varepsilon_a a U^2 \ln(b/a)} = \frac{CU^2}{2}$$

$$C = \frac{2W}{U^2} = \boxed{4\pi\varepsilon_0\varepsilon_a a \ln(b/a) = C}$$

№5.8 Дано: $\varepsilon_1, \varepsilon_2, \psi_1, \psi_2, \ell, Q, R_1, R_2$. $E - ?$; $C - ?$



$$Q = \int_S \mathcal{D} dS \qquad dS = \ell \cdot dr$$

$$\mathcal{D} = \varepsilon\varepsilon_0 E$$

$$\varphi = \int \varepsilon_1\varepsilon_0 E \, dS_1 + \int \varepsilon_2\varepsilon_0 E \, dS_2$$

$$q = E\varepsilon_0\left(\varepsilon_1\int_{S_1} dS_1 + \varepsilon_2\int_{S_2} dS_2\right) =$$

$$= E\varepsilon_0\left(\varepsilon_1 l_2\int_0^{\psi_1} d\psi_1 + \varepsilon_2 l_2\int_{\psi_1}^{2\pi} d\psi\right) = E\varepsilon_0 l_2\left(\varepsilon_1\psi_1 + \varepsilon_2(2\pi-\psi_2)\right)$$

$$= E\varepsilon_0 l_2(\varepsilon_1\psi_1 + \varepsilon_2\psi_2)$$

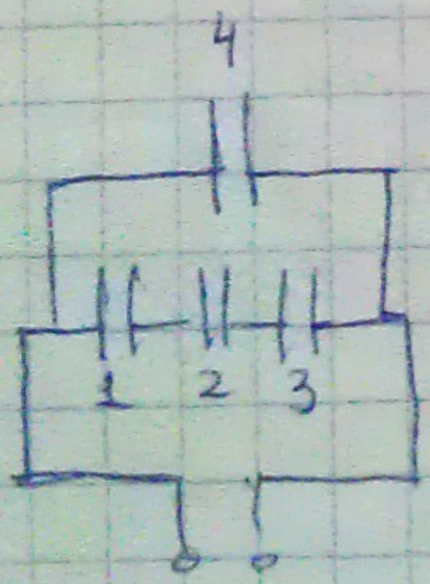
$$\Rightarrow \boxed{E = \frac{q}{\varepsilon_0 l_2(\varepsilon_1\psi_1+\varepsilon_2\psi_2)}}$$

$$U = \int_{R_1}^{R_2} E\,dr = \frac{q}{\varepsilon_0 l(\varepsilon_1\psi_1+\varepsilon_2\psi_2)}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{q\ln R_2/R_1}{\varepsilon_0 l(\varepsilon_1\psi_1+\varepsilon_2\psi_2)}$$

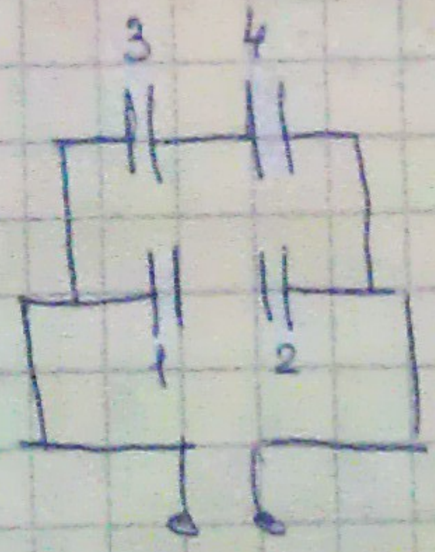
$$C = \frac{q}{U}$$

$$\Rightarrow \boxed{C = \frac{\varepsilon_0 l(\varepsilon_1\psi_1+\varepsilon_2\psi_2)}{\ln R_2/R_1}}$$

№ 5.12



а)



б)

1) $C_1 = C_2 = C_3 = C_4 = C$

а) $C_a = \frac{1}{\frac{3}{C}} + C = \frac{4C}{3}$

$\Rightarrow \boxed{C_a > C_б}$

б) $C_б = \frac{1}{\frac{2}{C}} + \frac{1}{\frac{2}{C}} = C$

2) а) $C_a = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} + C_4 = \frac{C_1 C_2 C_3}{C_1C_2 + C_1C_3 + C_2C_3} + C_4$

б) $C_б = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} + \frac{1}{\frac{1}{C_3} + \frac{1}{C_4}} =$

$= \frac{C_1C_2}{C_1+C_2} + \frac{C_3C_4}{C_3+C_4}$

$C_a = C_б \Rightarrow \frac{C_1C_2}{C_1+C_2} + \frac{C_3C_4}{C_3+C_4} = C_4 + \frac{C_1C_2C_3}{C_1C_2 + C_1C_3 + C_2C_3}$

$\boxed{C_4 = \frac{C_1C_2}{C_1+C_2}}$

$\frac{\cancel{C_3}C_4}{C_3+C_4} = \frac{C_1C_2\cancel{C_3}}{C_1C_2 + C_1C_3 + C_2C_3}$

$$\frac{C_1C_2}{(C_1+C_2)\left(C_3+\frac{C_1C_2}{C_1+C_2}\right)} = \frac{C_1C_2}{C_1C_2+C_1C_3+C_2C_3}$$

$$\frac{C_1C_2}{C_1C_3+C_2C_3+C_1C_2} = \frac{C_1C_2}{C_1C_2+C_1C_3+C_2C_3} \quad \boxed{\forall\, C_3}$$

№5.16 +q, -q, S, d, d_0

Дано: q, S, d, d_0, A - ?

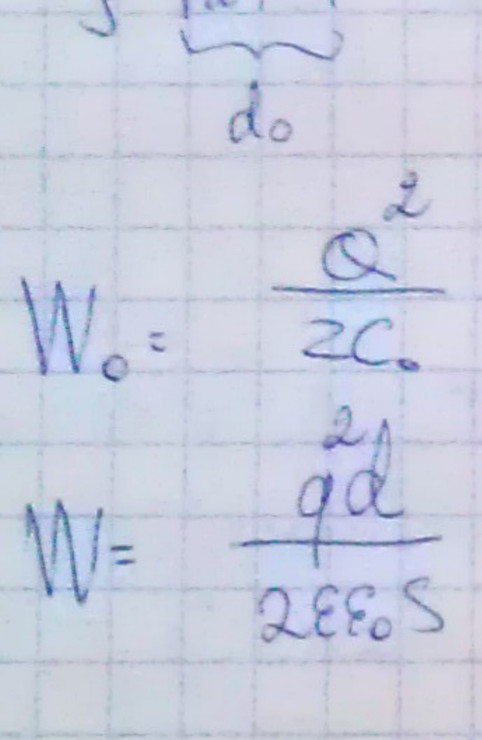
$A = \Delta W$ $\quad C = \frac{\varepsilon\varepsilon_0 S}{d}$

$$W_0 = \frac{Q^2}{2C_0} = \frac{q^2 d_0}{2\varepsilon\varepsilon_0 S}$$

$$W = \frac{q^2 d}{2\varepsilon\varepsilon_0 S}$$

$$\Rightarrow A = W_0 - W = \boxed{\frac{q^2}{2\varepsilon\varepsilon_0 S}(d_0 - d) = A}$$

№5.18



Дано: q, ε, b, a. W - ?

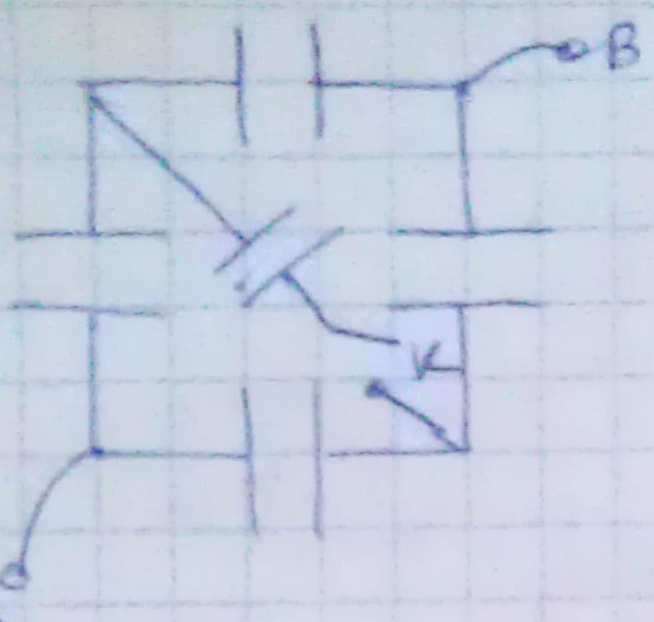
$$W = \frac{q^2}{2C} \; ; \; C = \frac{q}{u}$$

$$W = \frac{qu}{2}$$

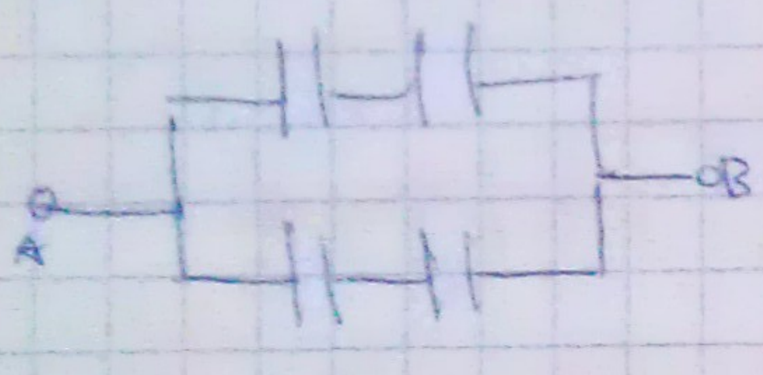
$$u = \int_a^b E\,dr = \int_a^b \frac{q\,dr}{4\pi\varepsilon\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Rightarrow \boxed{W = \frac{q^2}{8\pi\varepsilon\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

№5.10



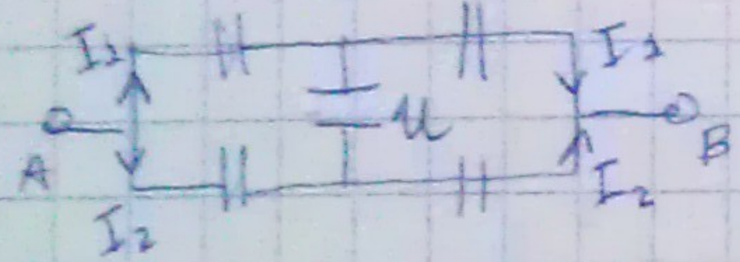
1)



$$C_1 = \frac{C}{2} + \frac{C}{2} = C$$

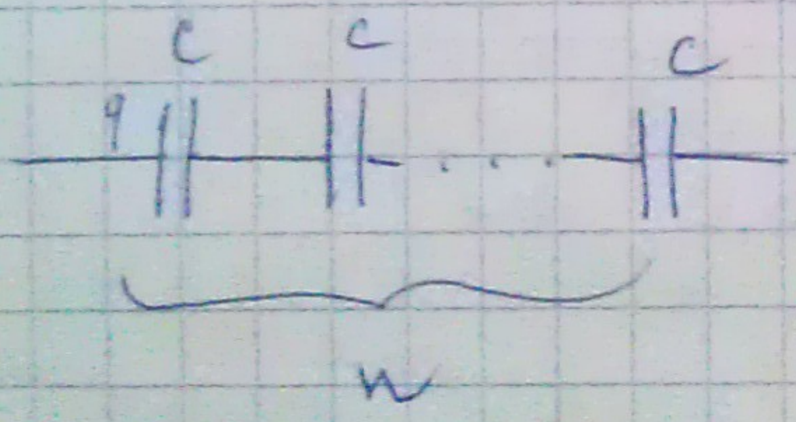
2) K-замкнут

C - одинак $\Rightarrow I_1 = I_2 \Rightarrow u = 0$



$\Rightarrow 1 \sim 2$, $C_2 = C_1 = C$

№5.9



$$C_0 = \frac{C}{n}$$

а) C опред. св-вами системы, не завис. от зарядов

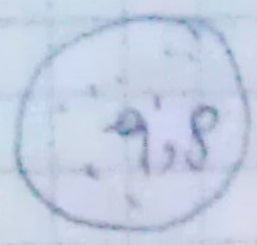
$$\Rightarrow C_1 = const = \frac{C}{n} = C_0$$

б) $u = \frac{Q}{C}$

$$u_0 = \frac{nq}{C_0}, \quad u_1 = \frac{q(n-1)}{C_1} = \frac{q(n-1)}{C_0}$$

$\Rightarrow \frac{u_1}{u_0} = \frac{n-1}{n} = 1 - \frac{1}{n} < 1 \Rightarrow u \downarrow$

№5.80 Дано: $q = 10^{-10}$ Кл, $r = 1$ см. $A - ?$

1) 

2) 

$A = \Delta W$

$E_1 = \frac{q}{4\pi\varepsilon_0} \cdot \frac{x}{r^3}$ – поле внутри шара на расст. x от центра

$E_2 = 0 \Rightarrow W_2 = 0$

$A = W_1 = \int_0^r \frac{\varepsilon\varepsilon_0}{2} E^2 dV = \frac{\varepsilon\varepsilon_0}{2} \cdot \left(\frac{q}{4\pi\varepsilon_0}\right)^2 =$

$w = \frac{\varepsilon\varepsilon_0}{2} E^2 = \frac{\varepsilon\varepsilon_0}{2} \cdot \frac{q^2}{16\pi^2\varepsilon_0^2} \cdot \frac{x^2}{r^6} = \frac{\varepsilon q^2 x^2}{32\pi^2\varepsilon_0 r^6}$

$W_1 = \int_V w dV = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_0^r \frac{\varepsilon q^2 x^2}{32\pi^2\varepsilon_0 r^6} \sin\theta \cdot x^2 dx =$

$= \frac{\varepsilon q^2 \cdot 4\pi}{32\pi^2\varepsilon_0 r^6} \int_0^r x^4 dx = \frac{\varepsilon q^2}{8\pi\varepsilon_0 r^6} \frac{x^5}{5}\Big|_0^r = \boxed{\frac{\varepsilon q^2}{40\pi\varepsilon_0 r} = A}$

8

Стационарное поле

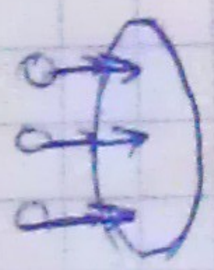
Ток - упорядоченное движение заряж. частиц

Квазистационарные токи - в быстрых пр-ва

Проводимость $\lambda = \frac{1}{\rho}$, ρ - удельная проводимость

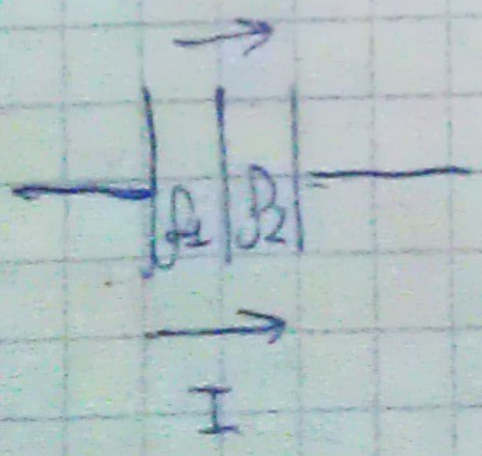
Плотность тока $\vec{j} = \frac{dI}{d\vec{s}}$, $I = \oint_S \vec{j} d\vec{s}$

$I = \frac{dQ}{dt}$



3 и Ома в диф форме: $\vec{j} = \lambda \vec{E}$

№6.7 Дано: $\rho_1, \rho_2, S, d, \xi_0$. σ - ?
$\varepsilon_1, \varepsilon_2$



$\sigma = D_{2n} - D_{1n}$

$\vec{j} = \lambda \vec{E}$, $\rho = \frac{1}{\lambda}$

$j = \frac{I}{S}$

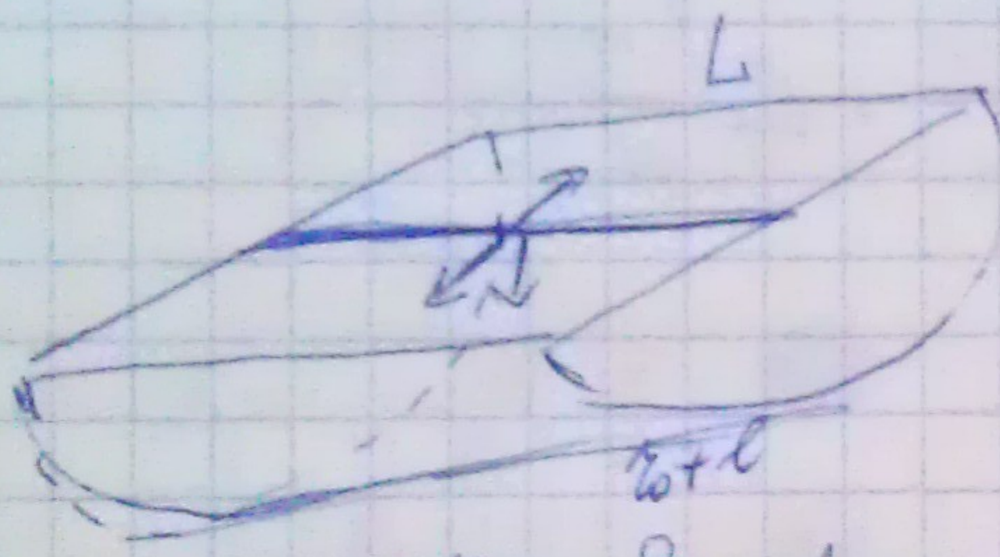
$E_1 = \frac{I\rho_1}{S}$, $E_2 = \frac{I\rho_2}{S}$

$D_1 = \varepsilon_1 \varepsilon_0 \frac{I\rho_1}{S}$, $D_2 = \frac{\varepsilon_1 \varepsilon_0 I \rho_2}{S}$ $\Rightarrow \sigma = \frac{I\varepsilon_0}{S}(\varepsilon_2\rho_2 - \varepsilon_1\rho_1)$

$$I = \frac{\mathcal{E}_0}{R} = \frac{\mathcal{E}_0}{\rho_1 \frac{d}{S} + \rho_2 \frac{d}{S}} \qquad R = \frac{\rho d}{S}$$

$$\sigma: \frac{\mathcal{E}_0 \varepsilon_0}{2d} \left(\frac{\varepsilon_2 \rho_2 - \varepsilon_1 \rho_1}{\rho_1 + \rho_2} \right)$$

№ 6.12 Дано: I, L, λ, r_0, l . $U - ?$



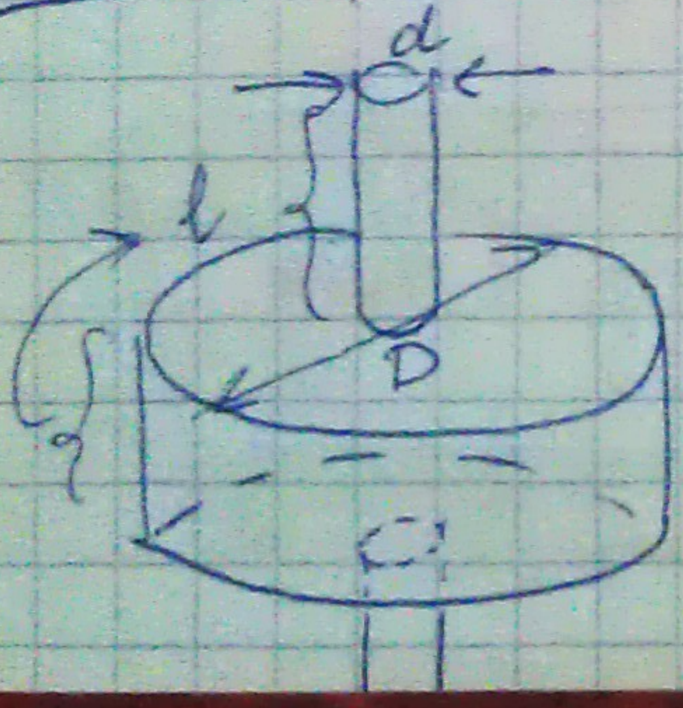
Нужно поле $\Rightarrow U = \int_{r_0}^{} E\,dr$ $\qquad E = \frac{\lambda}{\varepsilon}$ $\qquad \lambda = \frac{I}{\pi r L}$

$$\Rightarrow \quad E = \frac{I}{\varepsilon \pi r L}$$

$$U = \int_{r_0}^{r_0+l} \frac{I}{\varepsilon \pi r L}\,dr = \frac{I}{\varepsilon \pi L} \ln\left(1 + \frac{l}{r_0}\right)$$

№ 6.13 Дано: d, n, δ, l, λ . $R = ?$

$\delta << d$



$U = \int E\,dr = \Phi(\ldots)^{1/c}$ $\qquad E \sim \Phi$

$U = \int E\,dr = I(\ldots)$ $\qquad E \sim I$

$\Downarrow R$

1) 

$$j = \frac{I}{2\pi r \delta}$$

Крышка

$$E = \frac{I}{2\pi r \lambda \delta}$$

$$U = \int_{d/2}^{D/2} \frac{I}{2\pi\lambda\delta r}\, dr = \frac{I}{2\pi\lambda\delta} \cdot \ln\frac{D}{d}$$

$$\Rightarrow \boxed{R_к = \frac{\ln D/d}{2\pi\lambda\delta}}$$

2) Стенка:



$$j = \frac{I}{S}$$

$$S = \pi\left(\left(\frac{D}{4}\right)^2 - \left(\frac{D}{2} - \delta\right)^2\right)$$

$$j = \frac{I}{\pi\lambda\left(\left(\frac{D}{4}\right)^2 - \left(\frac{D}{2} - \delta\right)^2\right)}$$

$$E = \frac{I}{\pi\lambda(D\delta - \delta^2)}$$

$$U = \int_0^\ell \frac{I}{\pi\lambda(D\delta - \delta^2)}\, dr = \boxed{\frac{I\ell}{2\pi\lambda(D\delta - \delta^2)} = R_{бок}}$$

$$R = 2R_{тс} + R_{бок}$$

№ 6.16 Дано R_1, R_2, U_0, $\varepsilon = 1$, $\lambda = k/r^2$.

ρ, σ, $E(r)$ - ?



$$j = \frac{I}{S} = \frac{I}{2\pi r L}$$

$$E = \frac{I}{2\pi r L \varepsilon_0} = \frac{I r}{2\pi k L}$$

$$I = \frac{U_0}{R};$$

$$U = \int_{R_1}^{R_2} E\,dr = \int_{R_1}^{R_2} \frac{I r}{2\pi k L} = \frac{I}{4\pi k L}(R_2^2 - R_1^2)$$

$$R = \frac{(R_2^2 - R_1^2)}{4\pi k L}$$

$$E = \frac{U_0^2 \cdot 4\pi k L}{(R_2^2 - R_1^2) 2\pi k L} = \left| \frac{2U_0^2}{R_2^2 - R_1^2} = E \right.$$

$$\sigma_{внутр} = D_{1n} - D_{1n} \to 0 = \left| \frac{2U_0 R_1 \varepsilon_0}{R_2^2 - R_1^2} = \sigma_{внутр} \right.$$

$$\sigma_{внеш} = D_{2n} - D_{1n} \to 0 = \left| \frac{-2U_0 R_2 \varepsilon_0}{R_2^2 - R_1^2} = \sigma_{внеш} \right.$$

$\Delta\varphi = \frac{\rho}{\varepsilon_0}$ - ур-е Пуассона

$$\varphi = \int_r^{\varphi=0} E\,dr = \int_r^0 \frac{2\mu_0 r\,dr}{R_2^2 - R_1^2} = \frac{-\mu_0 r^2}{R_2^2 - R_1^2}$$

$\Delta_r = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$ – в цил. сист. коорд.

$$\Delta_r \varphi = \frac{-\mu_0 \cdot 4}{R_2^2 - R_1^2} = -\frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \boxed{\rho = \frac{4\mu_0\varepsilon_0}{R_2^2 - R_1^2}}$$

№6.22 $\lambda, \varepsilon, l, R_1, R_2, U(0) = U_0$. $I(t) - ?$

$$I(t) = -\frac{dQ}{dt}$$

$Q = CU$; $U = IR \Rightarrow Q = CIR$

$[CR] = \frac{1}{c}$

$$I = CR\frac{dI}{dt}$$

$$dt = -CR\frac{dI}{I} \Rightarrow \ln I + C = -\frac{t}{RC}$$

$$\Rightarrow I = Ce^{-t/CR}$$

$$t=0 \Rightarrow I = \frac{U_0}{R} = C$$

$$\Rightarrow I(t) = \frac{U_0}{R} e^{-t/CR}$$

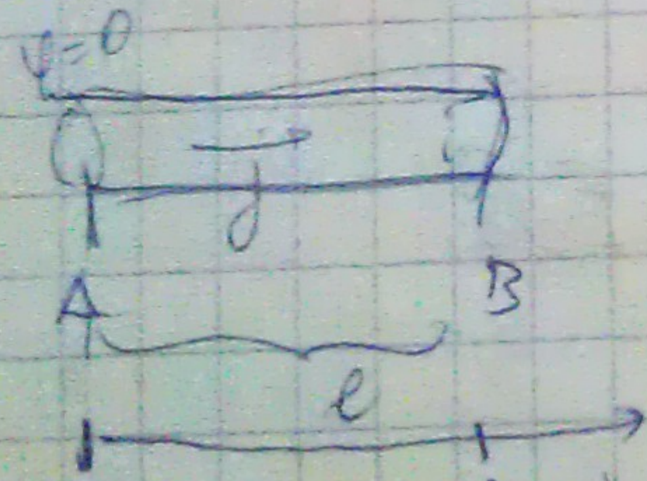
(6.13): $R = \frac{\ln R_2/R_1}{2\pi\lambda L}$, $C = \frac{2\pi\varepsilon L \varepsilon_0}{\ln R_2/R_1}$

$$\Rightarrow CR = \frac{\varepsilon\varepsilon_0}{\lambda}$$

$$\Rightarrow \boxed{I = \frac{U_0 \cdot \ln R_2/R_1}{2\pi\lambda L} \cdot e^{\frac{-t\lambda}{\varepsilon\varepsilon_0}}}$$

ДЗ: теория 11-20 / §6 ~~1~~, ~~6~~, ~~8~~, 10, (14), (15), ~~19~~, ~~24~~, ~~25~~

№6.19 Дано: j, l, λ_1, λ_2. $\varphi - ?$



$$\lambda(x) = x\frac{\lambda_2-\lambda_1}{l} + \lambda_1 = \frac{x(\lambda_2-\lambda_1)+l\lambda_1}{l}$$

$$j = \lambda E \Rightarrow E = \frac{j}{\lambda}$$

$$E = \frac{jl}{x(\lambda_2-\lambda_1)+l\lambda_1}$$

~~$\Delta\varphi = \int \frac{jl\,dx}{x(\lambda_2-\lambda_1)+l\lambda_1}$~~

$$\Delta\varphi = \int_l^0 \frac{j\,dx}{x(\lambda_2-\lambda_1)+l\lambda_1} = \frac{j}{\lambda_2-\lambda_1}\int_l^0 \frac{d(x(\lambda_2-\lambda_1)+l\lambda_1)}{x(\lambda_2-\lambda_1)+l\lambda_1} =$$

$$= \frac{j}{\lambda_2-\lambda_1}\ln\left(x(\lambda_2-\lambda_1)+l\lambda_1\right)\Big|_l^0 =$$

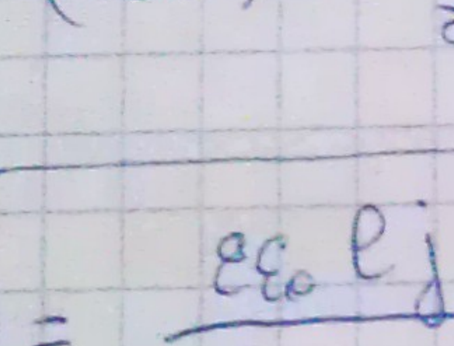
$$= \frac{j}{\lambda_2-\lambda_1}\left(\ln l\lambda_1 - \ln l\lambda_2\right) = \frac{j}{\lambda_2-\lambda_1}\ln\frac{\lambda_1}{\lambda_2} \longrightarrow$$

$$\operatorname{div}\vec{D} = \rho, \quad \vec{D} = \varepsilon\varepsilon_0\vec{E}$$

$$\operatorname{div}(\varepsilon\varepsilon_0\vec{E}) = \frac{\partial}{\partial x}\left(\frac{\varepsilon\varepsilon_0 l j}{x(\lambda_2-\lambda_1)+l\lambda_1}\right) = \frac{-\varepsilon\varepsilon_0 l j}{(x(\lambda_2-\lambda_1)+l\lambda_1)^2}\,(\lambda_2-\lambda_1)$$

$$\boxed{\rho = \frac{\varepsilon\varepsilon_0 l j\,(\lambda_1-\lambda_2)}{(x(\lambda_2-\lambda_1)+l\lambda_1)^2}}$$

№6.21 Дано: λ, ν_0, R_1, R_2. $\frac{Q}{t}$ - ?



$$\frac{Q}{t} = \nu_0 I = N$$

$$I = \oint_S \vec{j}\,d\vec{s} = \lambda \oint_S \vec{E}\,d\vec{s}$$

$$N = \nu_0 I, \quad I = \frac{\nu_0}{R}$$

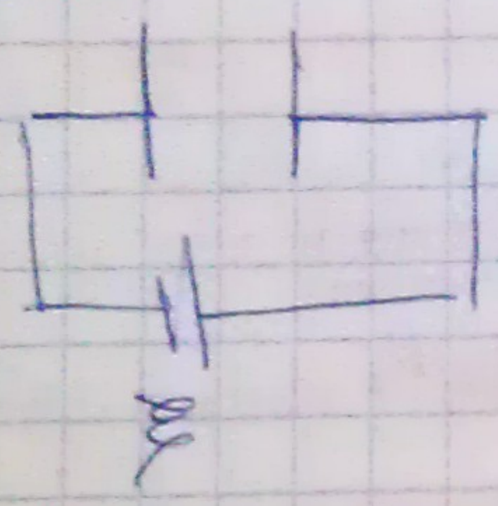
$$N = \frac{\nu_0^2}{R}$$

$$\frac{I}{S} = \lambda E \Rightarrow E = \frac{I}{\lambda S} = \frac{I}{4\pi\varepsilon\varepsilon_0 \lambda r^2}$$

$$\int E dr = \frac{I}{4\pi\varepsilon\varepsilon_0\lambda} \int_{R_1}^{R_2} \frac{dr}{r^2} = I \cdot \left(\frac{1}{4\pi\varepsilon\varepsilon_0\lambda}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right) = R$$

$$\Rightarrow \boxed{N \frac{4\pi\varepsilon\varepsilon_0 \lambda_0^2}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$

№6.23 Дано: $\mathcal{E}$, $\vartheta = const$, S, d_0. $I(t) - ?$



$$d(t) = d_0 + \vartheta t$$

$$I(t) = -\frac{dq}{dt}$$

$$q = C\mathcal{E} = \frac{\varepsilon\varepsilon_0 S \mathcal{E}}{d(t)} = \{\varepsilon = 1\} = \frac{\varepsilon_0 S \mathcal{E}}{d_0 + \vartheta t}$$

$$\Rightarrow I(t) = -\varepsilon_0 S \mathcal{E} \frac{d}{dt}\left(\frac{1}{d_0 + \vartheta t}\right) = \boxed{\frac{\varepsilon_0 S \mathcal{E} \vartheta}{(d_0 + \vartheta t)^2} = I(t)}$$

№6.8 Дано: N, λ. $R - ?$

$$(6.14) \Rightarrow R = \frac{\varepsilon\varepsilon_0}{\lambda C}, \quad (5.2) \Rightarrow C_{сф} = N\pi\varepsilon\varepsilon_0 N$$

$$C = \frac{C_{сф}}{2} = 2\pi\varepsilon\varepsilon_0 N \Rightarrow \boxed{R = \frac{1}{2\pi\lambda N}}$$

№6.1 Дано: R_1, R_2, U. U_1, U_2 - ?

$U = U_1 - U_2$, (6.13) $\Rightarrow \varepsilon\varepsilon_0 I = \lambda q_1 = \lambda q_2$

$q_1 = q$

$I = \frac{\lambda}{\varepsilon\varepsilon_0} q_1 = \frac{\lambda}{\varepsilon\varepsilon_0} q_2$. $q_2 = -q_1 = -q$,

$U_1 = \frac{q}{C_1} = \frac{\varepsilon\varepsilon_0 I}{\lambda \cdot 4\pi\varepsilon\varepsilon_0 R_1} = \frac{I}{4\pi\lambda R_1}$

$U_2 = \frac{-I}{4\pi\lambda R_2}$

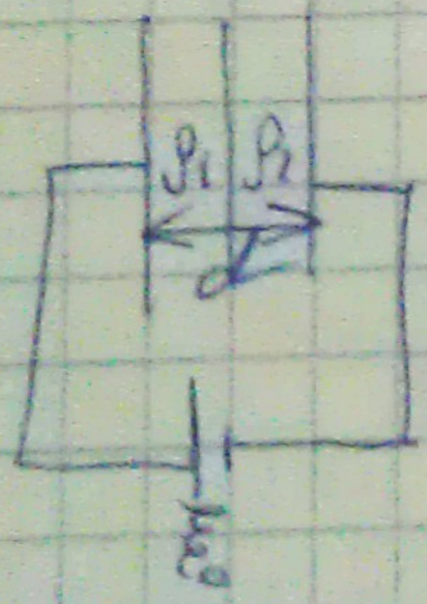
$U = U_1 - U_2 = \frac{I}{4\pi\lambda}\left(\frac{1}{R_2} + \frac{1}{R_1}\right) = \frac{I(R_1+R_2)}{4\pi\lambda R_2 R_1}$

$\Rightarrow I = \frac{U \cdot 4\pi\lambda R_2 R_1}{R_2 + R_1}$

$\Rightarrow \boxed{U_1 = \frac{U R_2}{R_1+R_2}}$

$\boxed{U_2 = \frac{-U R_1}{R_1+R_2}}$

№6.6 Дано: $\varepsilon_0, \rho_1, \rho_2, S, d$. N_1, N_2 - ?



$N_1 = U_1 I = \frac{U_1^2}{R_1}$ $E_1 = \frac{I\rho_1}{S}$

$N_2 = \frac{U_2^2}{R_2}$ $E_2 = \frac{I\rho_2}{S}$

$$U_1 = \int_0^{d/2} E\,dr = \frac{I\rho_1 d}{2S} \qquad U_2 = \int_{d/2}^{d} E\,dr = \frac{I\rho_2 d}{2S}$$

$$I = \frac{\mathcal{E}_0}{\frac{d}{2S}(\rho_1+\rho_2)} \Rightarrow U_1 = \frac{\rho_1 \mathcal{E}_0}{\rho_1+\rho_2}, \quad U_2 = \frac{\rho_2 \mathcal{E}_0}{\rho_1+\rho_2}$$

$$N_1 = \frac{\rho_1^2 \cdot \mathcal{E}_0^2}{(\rho_1+\rho_2)^2 \cdot \rho_1 \cdot \frac{d}{2S}} = \left| \frac{\rho_1 \mathcal{E}_0^2 \cdot 2S}{d(\rho_1+\rho_2)^2} = N_1; \; N_2 = \frac{\rho_2 \mathcal{E}_0^2 \cdot 2S}{d(\rho_1+\rho_2)^2} \right|$$

Магнитное поле квазистационарных токов

Магнитное поле создаётся движ. зарядами и действует на движ. заряды.

$\vec{B}$ - индукция магнитного поля

$\operatorname{div}\vec{B} = 0$ - ур. Максвелла

$\Downarrow$ поле соленоидально (линии замкнуты)

Поток: $\int_S \vec{a}\,dS$

Циркуляция: $\int_L \vec{a}\,dl$

Вихрь: $\operatorname{rot}\vec{a}$

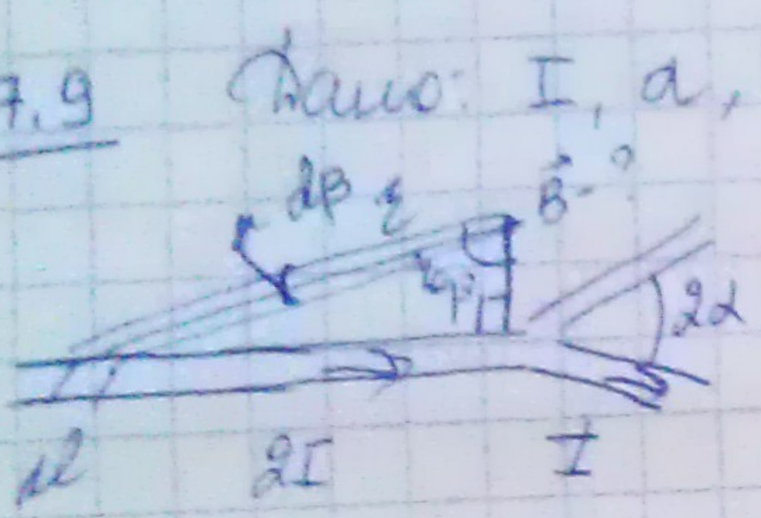
Точечный источник вектора: $\operatorname{div}\vec{a}$

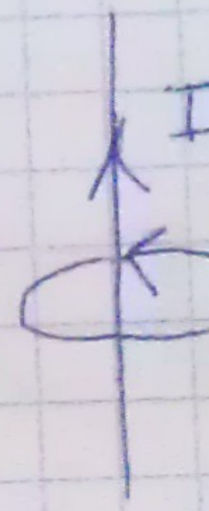
$\operatorname{div}\vec{B} = 0 \Rightarrow$ нет точки, из кот. исходят магнитные поле

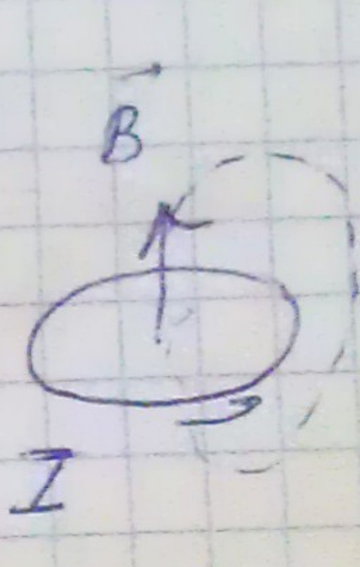
Зак-н Био-Савара-Лапласа $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{[Id\vec{l}\,\vec{r}]}{r^3}$

$$\Rightarrow \vec{B} = \int_L d\vec{B}$$

№ 7.9 Дано: I, a, r_0







$dl = r_0 d\beta$

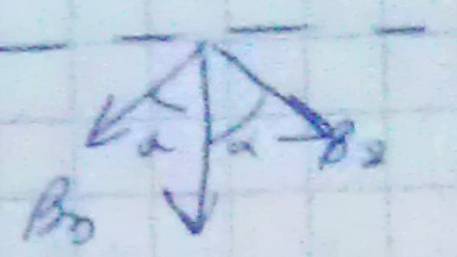
$r = \frac{r_0}{\cos\beta} \Rightarrow dl = r\cos\beta\, d\beta$

$$B_1 = \frac{\mu_0 I}{2\pi} \int \frac{dl \cdot r \cdot \cos\beta}{r^3} = \frac{\mu_0 I}{2\pi} \int \frac{\cos^2\beta\, d\beta}{r} \cdot \frac{\mu_0 I}{2\pi r_0} \int_0^{\pi/2} \cos^3\beta\, d\beta =$$

$$= \frac{\mu_0 I}{4\pi r_0} \int_0^{\pi/2} (1 - \sin^2\beta)\, d\sin\beta = \frac{\mu_0 I}{2\pi r_0} \left(\sin\beta - \frac{\sin^3\beta}{3} \right) \Big|_0^{\pi/2}$$

$$= \frac{\mu_0 I}{3\pi r_0}$$

$$B_2 = \frac{\mu_0 I}{6\pi r_0}$$



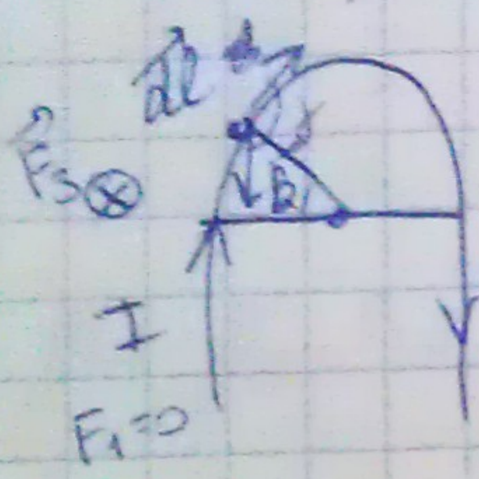
$$B_2 = B_1 + 2B_2\cos\alpha =$$

$$= \boxed{\frac{\mu_0 I}{3\pi r_0}(1+\cos\alpha) = B}$$

№ 7.10

Сила Ампера $F_A = I\int [dl, B]$

Дано I, R, B



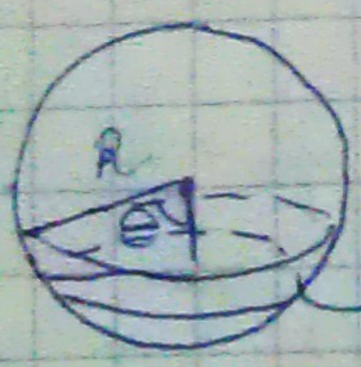
$\gamma = \pi - \alpha$

$dl = R d\alpha$

$$F_3 = I\int_0^{\pi} R\,d\alpha\, B\sin\alpha = IRB\left(-\cos\alpha\Big|_0^{\pi}\right) = \boxed{2IRB = F}$$

$F = F_3$

№ 7.6 Дано: R, σ, ω. B - ?

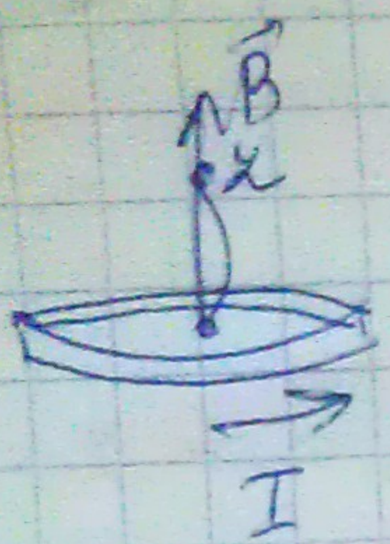


$x = R\sin\theta$

$$dI = \frac{dq}{T} = \frac{dq}{2\pi}\omega$$

$dq = \sigma dS$

$dS = \underbrace{R d\theta}_{l\text{ - высота "цилиндра"}} \cdot 2\pi \cdot R\sin\theta = 2\pi R^2 \sin\theta\, d\theta$



(7.3):

$$B(z) = \frac{\mu_0 I r^2}{2(r^2+z^2)^{3/2}}$$

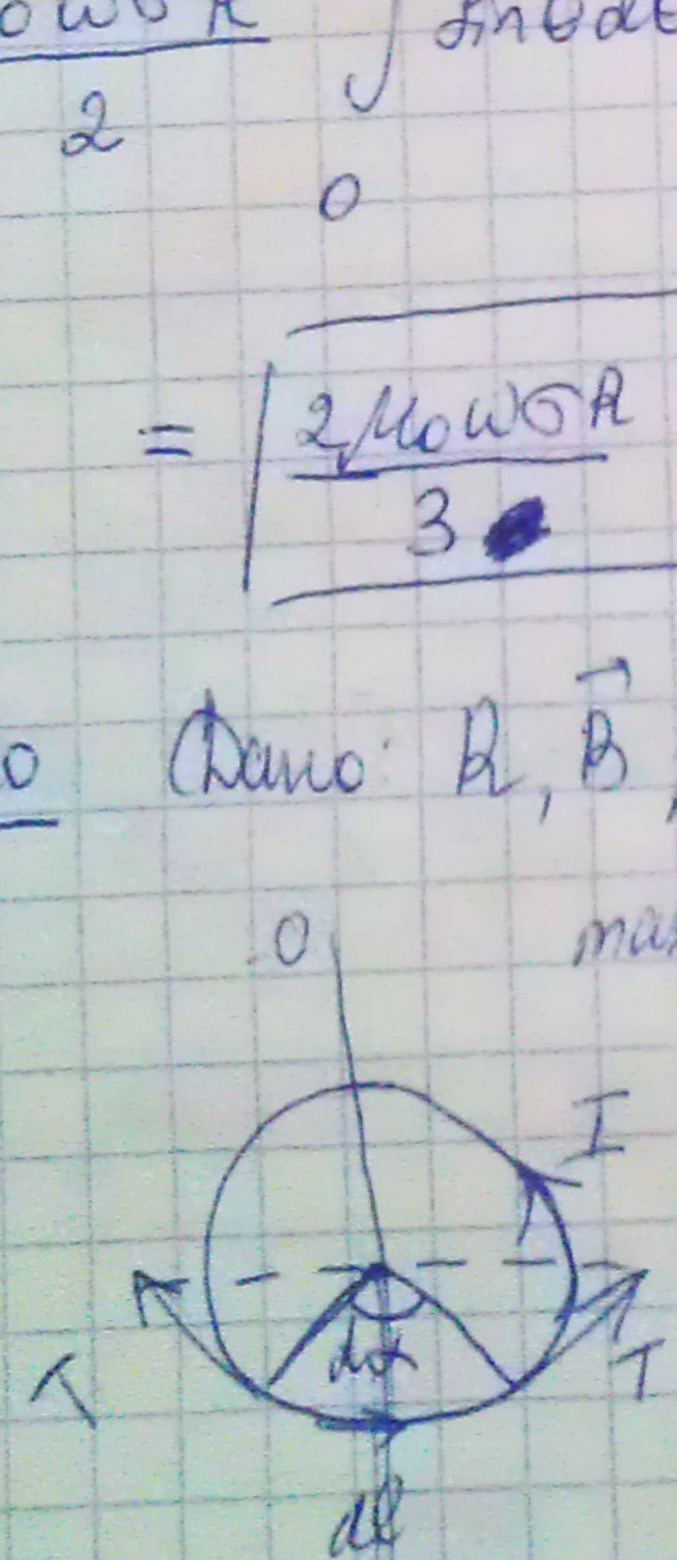
$z =$

$$B = \int \frac{\mu_0 r^2 dI}{2(r^2+z^2)^{3/2}} = \int \frac{\mu_0 R^2 \sin^2\theta \omega \sigma \cancel{2\pi} R \sin\theta d\theta}{\cancel{2\pi} \cdot 2 (R^2 \sin^2\theta + R^2 \cos^2\theta)^{3/2}} =$$

$$\frac{\mu_0 \omega \sigma R}{2} \int_0^{\pi} \sin^3\theta d\theta = -\frac{\mu_0 \omega \sigma R}{2} \left(\underbrace{\quad \cos\theta - \frac{\cos^3\theta}{3} \quad}_{-4/3} \right) \Big|_0^{\pi}$$

$$= \boxed{\frac{2\mu_0 \omega \sigma R}{3} = B}$$

№ 7.20 Дано: R, B, T I_{max} - ?



max напряжение

$$F_A = 2T_x$$

$$dF_A = IB\,dl$$

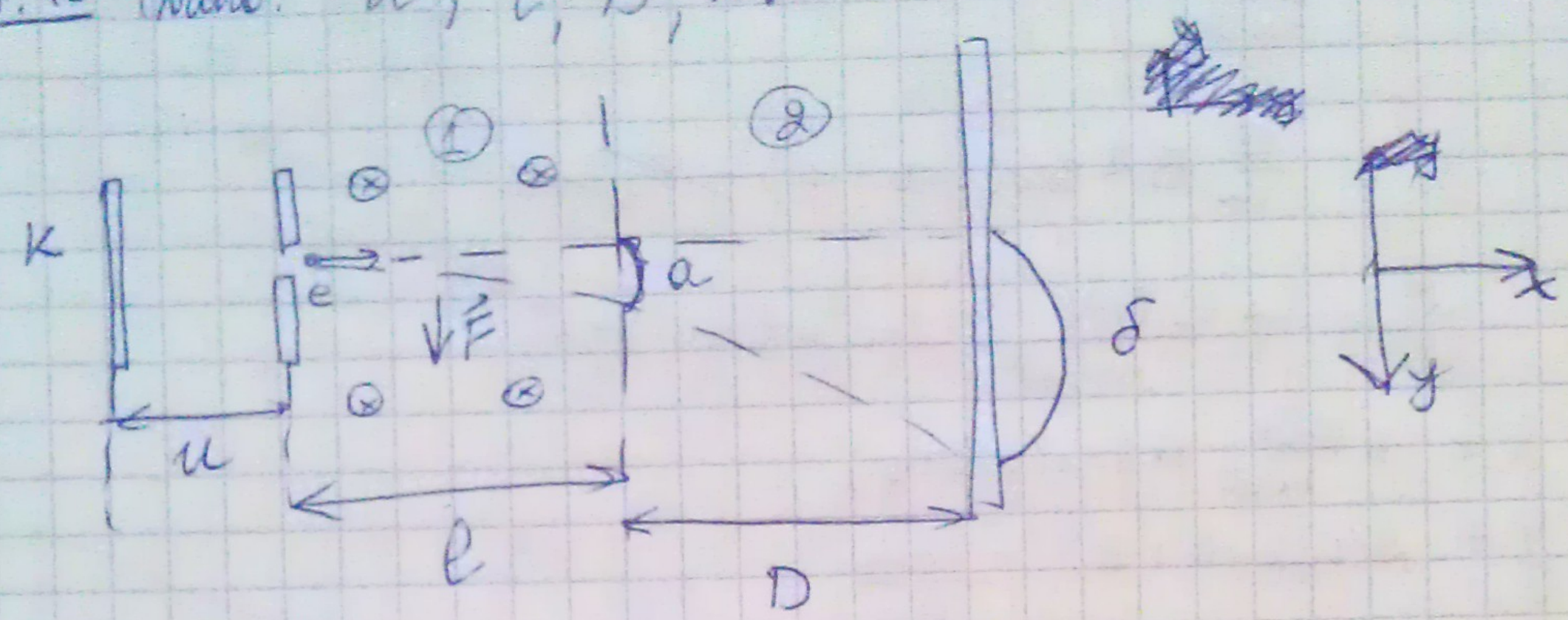
$$T_x = T \sin\frac{d\alpha}{2} \sim T\frac{d\alpha}{2}$$

$$dl = R\,d\alpha$$

$$IR\,d\alpha B = T\,d\alpha$$

$$\Rightarrow \boxed{I_{max} = \frac{T}{RB}}$$

№ 7.12 Дано: U, l, B, D. δ - ?



$$\vec{F} = q[\vec{v}, \vec{B}] \qquad \vec{F} = e[\vec{B}, \vec{v}] \qquad (q = -e)$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \\ 0 & 0 & \end{vmatrix} = \vec{i}(v_y B_z) - \vec{j}(v_x B_z) + \vec{k} \cdot 0$$

① $$\begin{cases} m\ddot{x} = +e\dot{y}B \\ m\ddot{y} = -e\dot{x}B \end{cases}$$

$$\dot{x} = \frac{e}{m} y B + \dot{x}(0)$$

$$\frac{m v_0^2}{2} = eU \Rightarrow v_0 = \sqrt{\frac{2eU}{m}} = v_{0x} = \dot{x}(0)$$

$$\ddot{y} = -\frac{e^2}{m^2} B^2 y - v_0 \frac{e}{m} B$$

$$\ddot{y} + \left(\frac{eB}{m}\right)^2 y = v_0 \frac{e}{m} B$$

Замена: $y = z - \frac{v_0 m}{Be} \Rightarrow z = A\cos(\omega t + \varphi_0)$

$\omega = \frac{eB}{m}$ $\quad y = A\cos(\omega t + \varphi) - \frac{v_0 m}{Be}$

$\dot{x} = \frac{e}{m} BA\cos(\omega t + \varphi) - \cancel{v_0 + v_0}$

$\dot{x} = + \frac{eBA}{m\omega} \sin(\omega t + \varphi) + C^{\to 0}$

$x = + \frac{eB}{m\omega} A\sin(\omega t + \varphi)$

$y = A\cos(\omega t + \varphi) - \frac{v_0 m}{Be}$

$\left.\begin{array}{l} x = 0 \\ y = 0 \\ \dot{x} = v_0 \end{array}\right\} \Rightarrow A, \varphi$

$x = \ell \Rightarrow t \Rightarrow y(t) = a$

② $\dot{y} = -A\omega \sin(\omega t + \varphi)$

$\dot{x}(t_0), \dot{y}(t_0)$ – нач. скор в обл ②

$\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = 0 \end{cases}$

$\begin{cases} \dot{x} = \text{const} \\ \dot{y} = \text{const} \end{cases}$

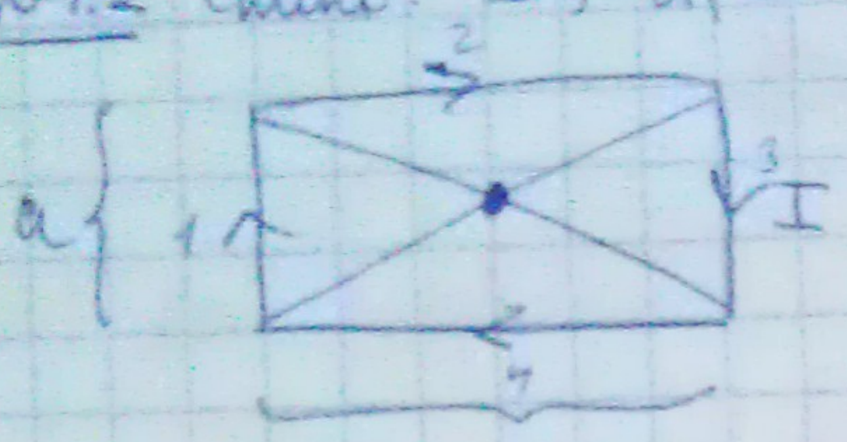
$\begin{cases} x = \dot{x}(t_0)\, t + x_0 = \ell \\ y = y(t_0)\, t + y_0 = a \end{cases}$

$x = \ell + b \Rightarrow t \Rightarrow y(t) = \delta$

ДЗ: 7.18 – доделать

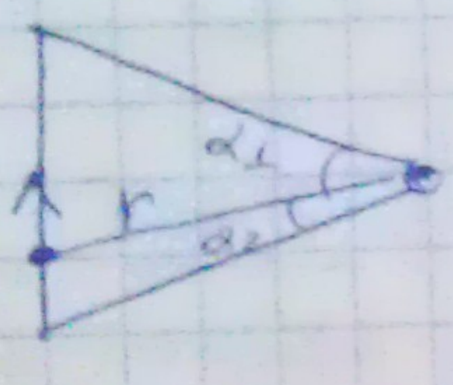
~~7.5~~, ~~7~~, 8, ~~11~~, ~~14~~, 15, ~~16~~

№7.1 Дано: I, a, b. B - ?



$$dB = \frac{\mu_0 I}{4\pi} \frac{[d\vec{l}, \vec{r}]}{r^3}$$

$$\vec{B} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \vec{B_4} = 2\vec{B_a} + 2\vec{B_b}$$



$$B = \frac{\mu_0 I}{4\pi r} (\sin\alpha_1 + \sin\alpha_2)$$

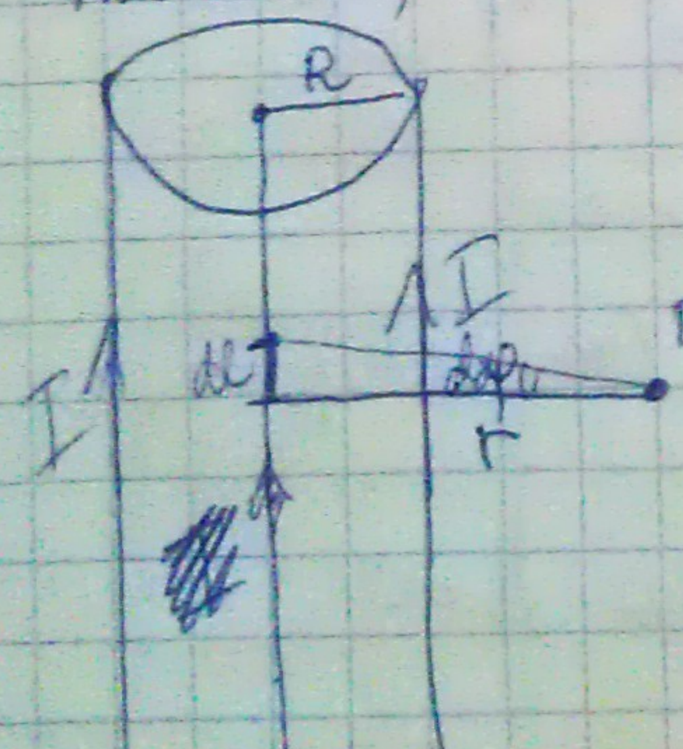
$$B_a = \frac{\mu_0 I}{2\pi b} \cdot \frac{2a}{\sqrt{a^2+b^2}} = \frac{\mu_0 I}{\pi\sqrt{a^2+b^2}} \cdot \frac{a}{b}$$

$$B_b = \frac{\mu_0 I}{\pi\sqrt{a^2+b^2}} \cdot \frac{b}{a}$$

$$B = \frac{2\mu_0 I}{\pi\sqrt{a^2+b^2}} \left(\frac{a}{b} + \frac{b}{a}\right) = \frac{2\mu_0 I}{\pi\sqrt{a^2+b^2}} \frac{a^2+b^2}{ab}$$

$$\boxed{B = \frac{2\mu_0 I \sqrt{a^2+b^2}}{\pi ab}}$$

№7.4 Дано: R, I. B(r) - ?



I равном. распред. по пов-ти $\Rightarrow B(r) = 0$, $0 \le r \le R$

$$dB = \frac{\mu_0 I}{4\pi} \frac{[d\vec{l}, \vec{r}\,]}{r^3} =$$

$= \frac{\mu_0 I}{4\pi} \frac{dl \cdot R \sin\varphi}{r^2} = \frac{\mu_0 I \sin\varphi\, dl}{r^2}$

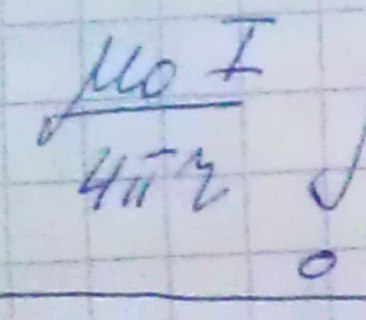
$dl = r\, tg\, d\varphi \sim r\, d\varphi$

$dB = \frac{\mu_0 I \sin\varphi\, d\varphi}{4\pi r}$

$B = \frac{\mu_0 I}{4\pi r} \int_0^{\pi} \sin\varphi\, d\varphi = \frac{\mu_0 I}{4\pi r} (-\cos\varphi|_0^{\pi}) = \frac{2\mu_0 I}{2\cdot 4\pi r} = \frac{\mu_0 I}{2\pi r}$

$$B(r) = \begin{cases} \frac{\mu_0 I}{2\pi r}, & r > R \\ 0, & 0 \le r \le R \end{cases}$$

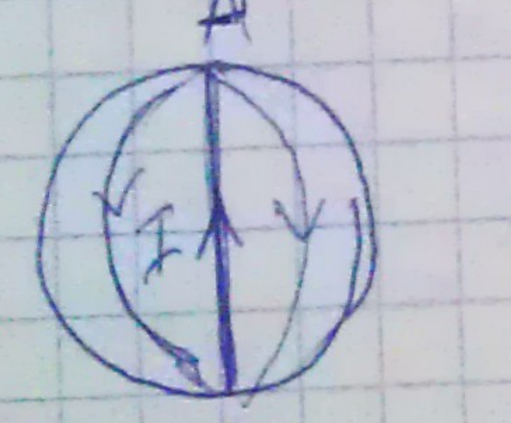
№ 7.8



$\vec{B}$ - ?

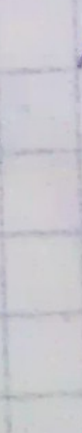
1) Внутри сферы: $\vec{B} = \vec{B}_I + \sum_{i=0}^{+\infty} B_i$

B_i - инд. маг. поля, созд. "дугой"



I_{i1} и I_{i2} сонапр.

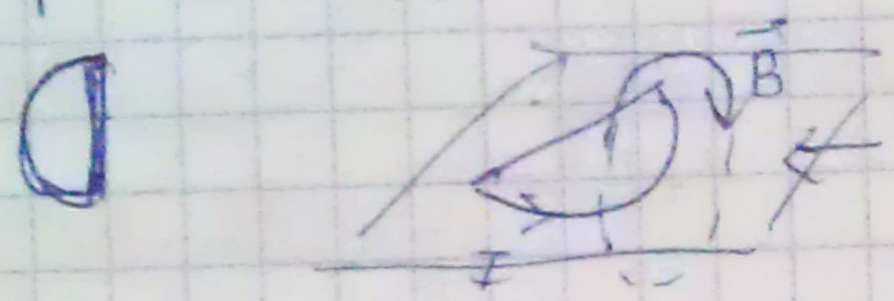
$\Rightarrow \sum_{i=0}^{+\infty} B_i = 0$ (компенсируются)

B_I:  - ≈ 7.4 при $R = 0$

$\Rightarrow B_{in} = B_I = \frac{\mu_0 I}{2\pi r}$, r - расст. от проводника

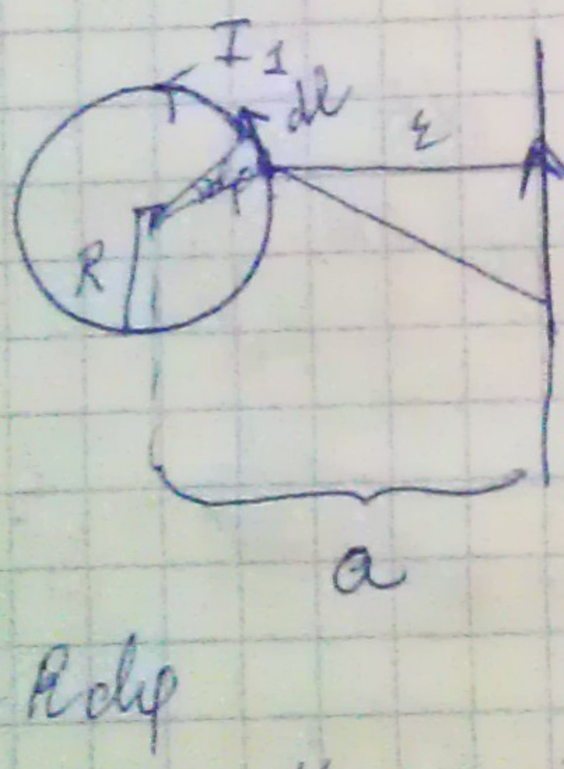
я) Вне сферы

Сфера - совокупность замкн. проводников



$\Rightarrow$ т.к "кусочка" 3 "кусок" создающий поле, конечное, данное $\Rightarrow B_{out} = 0$.

№ 7.11 Дано: R, I_1, I_2, a F - ?



$\vec{F} = I_1 \int_L [d\vec{l}, \vec{B}]$

$B = \frac{\mu_0 I}{2\pi r}$

$dl = R\,d\varphi$

$dF = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{r} dl = \frac{\mu_0 I_1 I_2}{2\pi r} dl = \frac{\mu_0 I_1 I_2 R\,d\varphi}{2\pi r}$

$r = r(\varphi)$

$r = a - x$, $x = R\sin\varphi$

$r = a - R\sin\varphi$

$$dF = \frac{\mu_0 I_1 I_2}{2\pi} \frac{R\,d\varphi}{a - R\sin\varphi}$$

$$I = \int_0^{2\pi} \frac{R\,d\varphi}{a - R\sin\varphi} = \int_0^{2\pi} \frac{d\varphi}{R\left(\frac{a}{R} - \sin\varphi\right)}$$

$$\sin\varphi = \frac{2\operatorname{tg}\varphi/2}{1+\operatorname{tg}^2\varphi/2} = \frac{2t}{1+t^2}$$

$$t = \operatorname{tg}\varphi/2, \qquad \varphi = 2\operatorname{arctg} t \Rightarrow d\varphi = \frac{2}{1+t^2}\,dt$$

$$I = \int_0^{+\infty} \frac{\frac{2}{1+t^2}}{R\left(\frac{a}{R} - \frac{2t}{1+t^2}\right)}\,dt = \frac{2}{R}\int_0^{+\infty} \frac{dt}{\frac{a}{R}(1+t^2) - 2t} =$$

$$= \frac{2}{R}\int_0^{+\infty} \frac{dt}{\frac{a}{R} + \frac{a}{R}t^2 - 2t} = \frac{2}{R}\int_0^{+\infty} \frac{dt}{\frac{a}{R}\left(t^2 - \frac{2R}{a}t + 1\right)} =$$

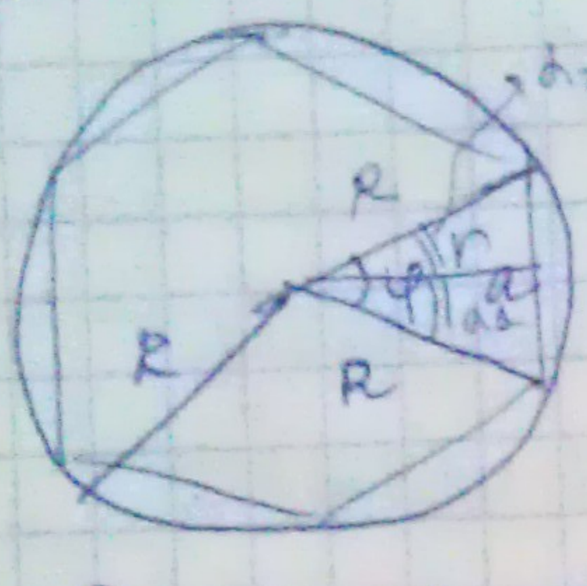
$$= 2a\int_0^{+\infty} \frac{dt}{(t - \frac{R}{a})^2 - \frac{R^2}{a^2}} = \left(\frac{1}{\sqrt{1 - R^2/a^2}} - \frac{1}{1}\right)$$

$$= \frac{2}{R}\left(\frac{1}{\sqrt{1 - R^2/a^2}} - 1\right)$$

$$\Rightarrow F = \frac{\mu_0 I_1 I_2}{2\pi} \cdot \frac{1}{R}\left(\frac{1}{\sqrt{1 - R^2/a^2}} - 1\right)\underbrace{\left(\int_0^{2\pi R} dl\right)}_{L} =$$

$$\Rightarrow \boxed{F = \mu_0 I_1 I_2 \left(\frac{1}{\sqrt{1 - R^2/a^2}} - 1\right)}$$

№ 7.14 Дано: I, n, R $\quad \vec{B_0} - ?$



$$\varphi = \frac{2\pi}{n}$$

$$a^2 = R^2 + R^2 - 2R^2\cos\frac{2\pi}{n} = 2R^2\left(1-\cos\frac{2\pi}{n}\right)$$

$$a = R\sqrt{2}\cdot\sqrt{1-\cos 2\pi/n} = 2R\sin\frac{\pi}{n}$$

$$P = n\cdot 2R\sin\frac{\pi}{n}$$

$$r = R\cos\frac{\pi}{n}$$

Для одной грани: $B_1 = \dfrac{\mu_0 I}{4\pi b}\left(\cos\alpha_1 - \cos\alpha_2\right)$

$$\cos\alpha_1 = -\cos\alpha_2 = \frac{a/2}{\sqrt{r^2+\left(\frac{a}{2}\right)^2}}$$

$$\Rightarrow B_1 = \frac{\mu_0 I}{4\pi r}\cdot\frac{a}{2}\cdot\frac{1}{\sqrt{r^2+\left(\frac{a}{2}\right)^2}}$$

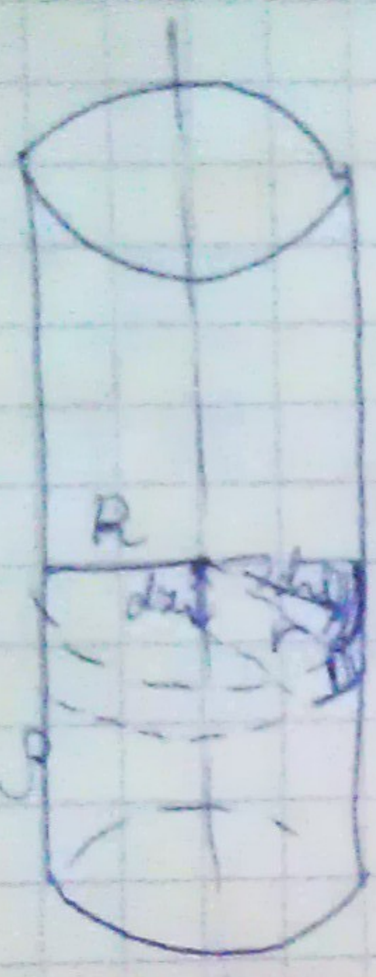
$$\frac{a}{2} = \frac{2R\sin\pi/n}{R\cos\pi/n} = 2\operatorname{tg}\pi/n$$

$$\sqrt{r^2+\left(\frac{a}{2}\right)^2} = \sqrt{R^2\cos^2\pi/n + R^2\sin^2\pi/n} = R$$

$$\Rightarrow B_1 = \frac{\mu_0 I\cdot 2\operatorname{tg}\pi/n}{2\pi}\cdot\frac{1}{R} \Rightarrow \quad B = nB_1$$

$$\Rightarrow \boxed{B = \frac{n\mu_0 I\operatorname{tg}\pi/n}{2\pi R}}$$

N7.19 Дано: R, ρ, ω. $B_{in}, B_{out} - ?$



$$dI = \frac{dq}{T} = \frac{dq}{2\pi}\omega$$

$$dq = \rho dV = \rho r\, dr\, d\varphi\, dz$$

$\varphi \in [0, 2\pi]$, $z \in (-\infty; +\infty)$

$r \in [0, +\infty)$

$r > R \Rightarrow \rho = 0 \Rightarrow dq = 0 \Rightarrow B_{out} = 0$

$r \in [0, R]$

Кольца рад. r: $B(x) = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$,

$dq = 2\pi\rho r\, dr\, dz$, $x \sim z$, $dI = \rho r \omega\, dr\, dz$

$dB = \frac{\mu_0 r^2}{2\pi(r^2+z^2)^{3/2}} dI = \frac{\mu_0 r^2}{2\pi(r^2+z^2)^{3/2}} \cdot \rho r \omega\, dr\, dz =$

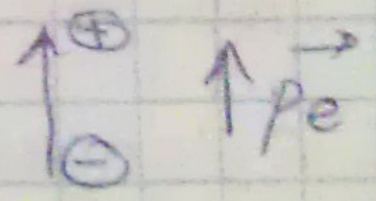
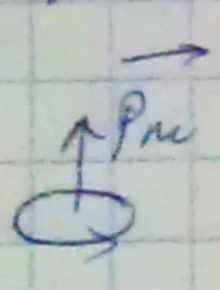
$= \frac{\mu_0 r^3 \rho\omega\, dr\, dz}{2\pi(r^2+z^2)^{3/2}}$, d – расст. от оси до точки, dz

$$B = 2\int_d^R dr \int_0^{+\infty} \frac{\mu_0 r^3 \rho\omega\, dz}{2\pi(r^2+z^2)^{3/2}} = \frac{2\mu_0\rho\omega}{2\pi} \int_d^R dr \int_0^{+\infty} \frac{r^3\, dz}{(r^2+z^2)^{3/2}} =$$

$$= \frac{\mu_0\rho\omega}{2\pi} \int_d^R r^3 \cdot \frac{1}{r^2} \cdot \frac{r}{r}\, dr = \frac{\mu_0\rho\omega}{\pi} \int_d^R r\, dr =$$

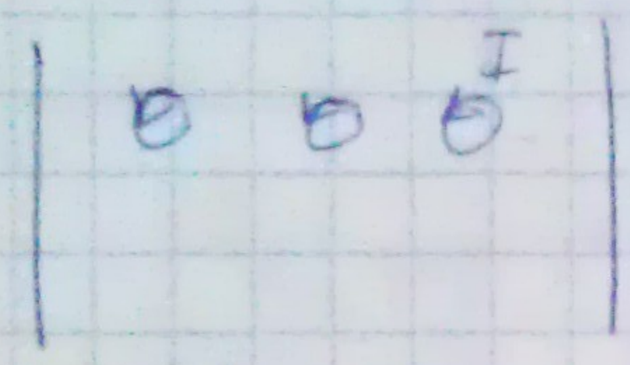
$$= \boxed{\frac{\mu_0\rho\omega}{2\pi}(R^2 - d^2) = B(d)}$$

Магнитное поле в веществе

Магнитное поле	Электр поле
$\vec{B} = \mu_0 \vec{H}$	$\vec{D} = \varepsilon\varepsilon_0 \vec{E}$
$\vec{H}$ - напр-ть ~~индукция~~ магн. поля	$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$
$\vec{B} = \mu_0 \vec{H} + \vec{J}$	
$\vec{J}$ - намагниченность среды	
$\vec{J} = \frac{\sum \vec{p_m}}{V}$	$\vec{P} = n\vec{p_e}$ 
$\vec{p_m}$ - магнитный момент  $\vec{p_m}$ создаётся эл. тока (молек.) токами	
$\vec{p_m}$ ориентируются по полю (тоже поворачиваемая)	
$\vec{J} = \chi_м \vec{H}$	$\vec{P} = \chi \vec{E}$
$\chi_м$ - магнитная восприимчивость	
μ - магнитная проницаемость	$\varepsilon = 1 + \chi$
$\mu = 1 + \chi_м$	
$I_{пров}$ - токи проводимости	Ø своб
$I'_м$ - токи намагничивания	Ø связ

$i'_м$ – линейная плотность тока

$j'_м$ – объёмная плотность тока



$I'_м(j'_м) \neq 0$

1) $\mu(x)$

2) $I_{пров} \neq 0$, $j_{пров} \neq 0$

$j = \frac{dI}{dS}$, $i = \frac{dI}{dl}$

Теор. о циркуляции напр-ти

$\oint \vec{H} d\vec{l} = I_{пров}$

$rot \vec{H} = j$

$div \vec{H} = 0$ (нет источника)

↓ Вихревое магнитное поле порождается токами

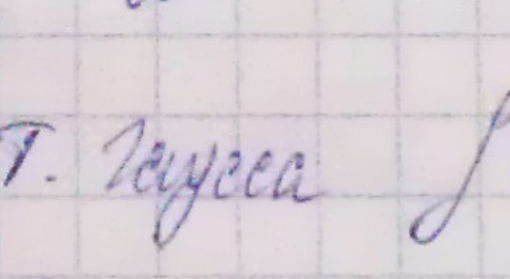
$\oint \vec{B} d\vec{l} = \mu_0 (I + I')$

$\oint \vec{J} d\vec{l} = I'$

$H_{2\tau} - H_{1\tau} = i_{пров}$

$H_{\tau 1} = H_{\tau 2}$

$\sigma_{своб}$ $\rho_{своб}$



$\rho_{своб} \neq 0$

1) $\varepsilon(x)$ – неодноф

2) $\rho_{своб} \neq 0$

$\rho = \frac{dq}{dV}$, $\sigma = \frac{dS}{dS}$

Т. Гаусса $\oint \vec{D} d\vec{S} = q_{своб}$

$div \vec{D} = \rho$

$rot \vec{E} = 0$ (невихревое поле)

$\oint \vec{E} d\vec{S} = \frac{q_{своб} + q_{связ}}{\varepsilon_0}$

$\oint \vec{P} d\vec{S} = q_{связ}$

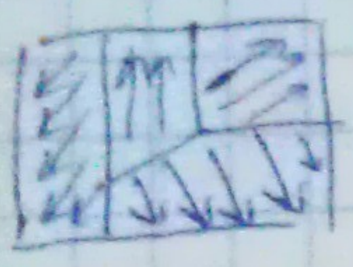
$D_{2n} - D_{1n} = \sigma_{своб}$

$E_{\tau 1} = E_{\tau 2}$

$J_{1n} - J_{2n} = i'$

$\chi > 0$ парамагнетики } ↑↑
$\chi < 0$ диамагнетики }

$\chi \gg 1$ ферромагнетики

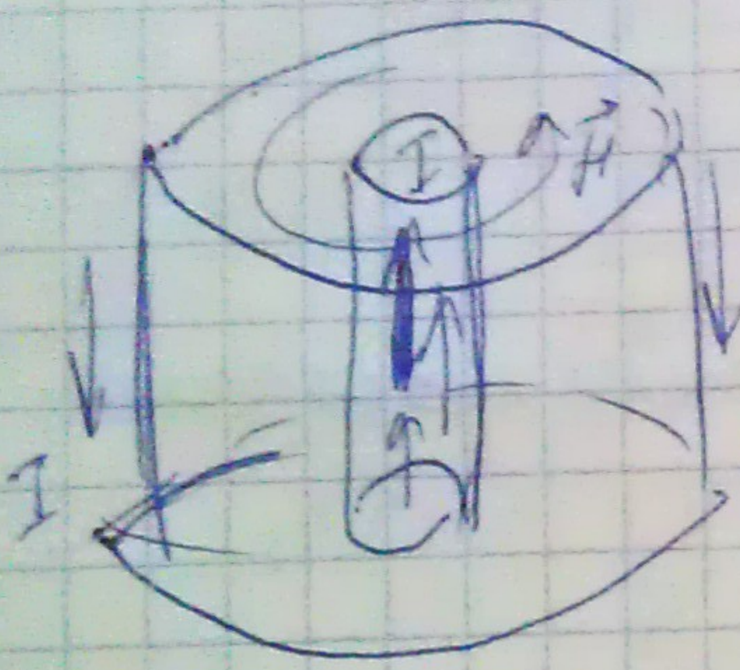


- усиливают внешнее поле

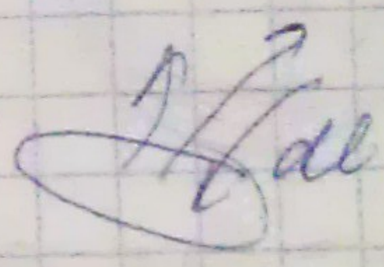
$\chi = 0 \Rightarrow$ вакуум

$B_{1n} - B_{2n} = 0$ свед

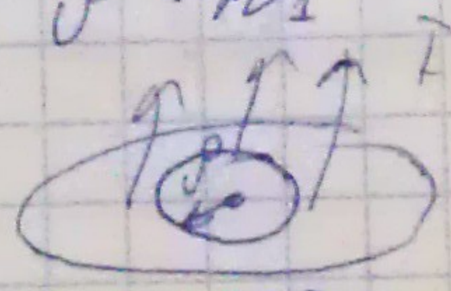
№ 8.1 R_1, R_2, I $H(\rho)$ - ?



$$\oint \vec{H}\, d\vec{l} = I$$



1) $\rho < R_1$



$dl \uparrow\uparrow H$

$$\oint H\, dl = \int_0^{2\pi} H_\rho\, d\varphi = \frac{I}{\pi R^2} =$$

$$= H 2\pi\rho = \frac{I \pi \rho^2}{\pi R^2}, \quad j = \frac{I}{\pi R^2}$$

$$\boxed{H_1 = \frac{I\rho}{2\pi R^2}}$$

$I \quad \frac{S_1}{S_2}$

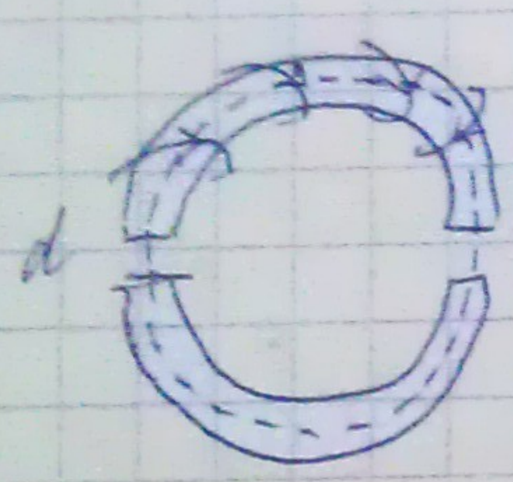
2) $R_1 < \rho < R_2$

$H_2 \cdot 2\pi\rho = I \Rightarrow \boxed{H_2 = \frac{I}{2\pi\rho}}$

3) $\rho > R_2$ $\quad H_3 \cdot 2\pi\rho = 0 \Rightarrow \boxed{H_3 = 0}$

№ 8.3 Дано: μ_1, μ_2, d, L, N, I $\quad B - ?$

$$\oint \vec{H} d\vec{l} = NI$$



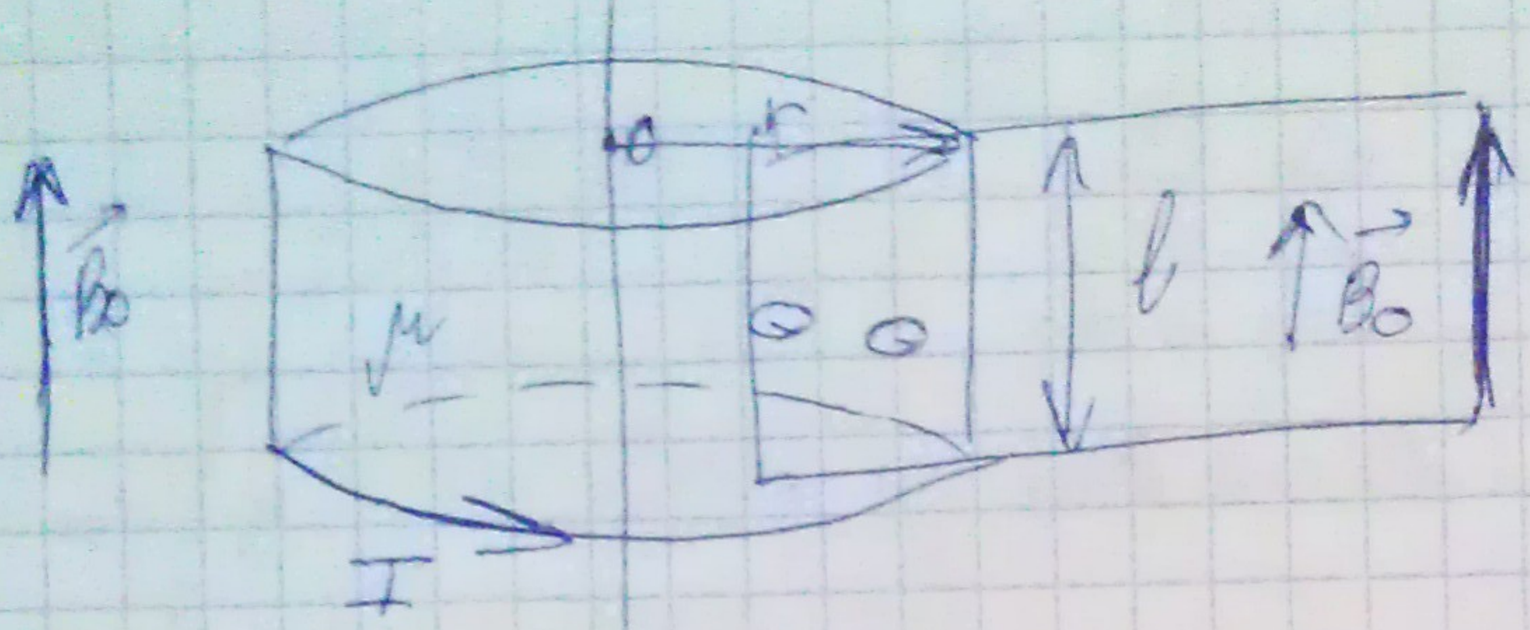
$d \quad d \quad \vec{H} = \frac{\vec{B}}{\mu\mu_0}$

$B_{n1} = B_{n2} \Rightarrow B \perp$ пов-ти, есть только постоянная норм. сост-ю.

$$NI = \int_{\frac{L}{2}-d} \frac{B}{\mu_1\mu_0} dl + \int_{2d} \frac{B}{\mu_0} dl + \int_{\frac{L}{2}-d} \frac{B dl}{\mu_2\mu_0}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 NI}{\left(\frac{L}{2}-d\right)\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)+2d}}$$

№ 8.10 Дано: $N, l, \mu, \vec{B_0}$. $\quad \vec{B} - ?$ (соленоид)



1) $\mu \neq f(x)$

2) $I_{\text{ов}} = 0$ $\Rightarrow$ $I'_{\text{ов}} = 0$, $j_{\mu}' = 0$

I' - поверхностный ток $\Rightarrow$ ЭВ

$$dB(z) = \frac{z^2 \mu \mu_0 dI'}{(z^2 + r^2)^{3/2}}$$

$$dI' = \underbrace{\frac{I'}{l}}_{i'} dz$$

$$B_{\mu} = \int_0^l dB(z) = \frac{\mu \mu_0 I' l}{2l\sqrt{l^2 + r^2}} = \{l \ll r\} = \frac{\mu \mu_0 I'}{2r\sqrt{1 + \frac{r}{l^2}}} =$$

$$= \frac{\mu \mu_0 I'}{2r}$$

$B = B_{\mu} + B_0$

$I' - ?$

$\oint B\, dl = I'$

$B_{out} l - B_{in} l = I' \mu_0$, $B_{out} = B_0$

$H_{2\tau} = H_{1\tau}$

$\frac{B_{2\tau}}{\mu\mu_0} = \frac{B_{1\tau}}{\mu_0}$

$B_{\tau 2} \approx B_0$

$B_{\tau 1} \approx B_{in}$

$\Rightarrow \frac{B_0}{\mu} = B_{in}$

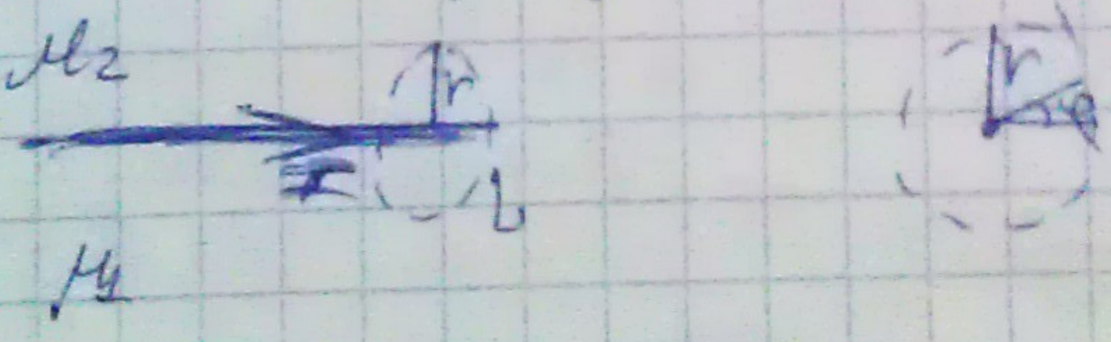
$B_0 l - \frac{B_0 l}{\mu} = I' \mu_0 \Rightarrow I' = \frac{B_0 l}{\mu_0}\left(1 - \frac{1}{\mu}\right) = \frac{B_0 l}{\mu_0 \mu}(\mu - 1)$

$$B = \frac{\mu\mu_0}{2\pi} \cdot \frac{B_0 l}{\mu\mu_0}(\mu - 1) + B_0 = \boxed{B_0\left(\frac{l(\mu-1)}{2\pi} + 1\right) \approx B}$$

Д/З Теория 31-40

~~3~~, ~~4~~, ~~5~~, ~~6~~, (12), ~~13~~, ~~14~~, ~~15~~

№8.4 Дано: I, μ_1, μ_2 $\vec{B}$ - ?



$I = \oint_L \vec{H} d\vec{l}$, $\vec{r}$ - ферм., $d\vec{l} = \vec{r} d\varphi$

Циркуляция вектора напряженности

$$\vec{H}(\varphi) = \begin{cases} \frac{\vec{B}}{\mu_2 \mu_0}, & \varphi \in [0, \pi] \\ \frac{\vec{B}}{\mu_1 \mu_0}, & \varphi \in [\pi, 2\pi] \end{cases}$$

$$I = \int_0^{\delta} \frac{B_2}{\mu_2 \mu_0} d\varphi + \int_{\pi}^{2\pi} \frac{B_2}{\mu_4 \mu_0} d\varphi = \frac{B \pi 2}{\mu_0} \left(\frac{1}{\mu_2} + \frac{1}{\mu_1} \right)$$

$$\Rightarrow \boxed{B = \frac{I \mu_0 \mu_1 \mu_2}{\pi 2 (\mu_1 + \mu_2)}}$$

№8.5 Дано: $\mu_1, \mu_2, \vec{H_0}(\vec{r}), H_1, H_2, B$ - ?

Граничн. усл $\Rightarrow$ $H_1 \mu_1 = H_2 \mu_2 \Rightarrow H_2 = \frac{\mu_1}{\mu_2} H_1$

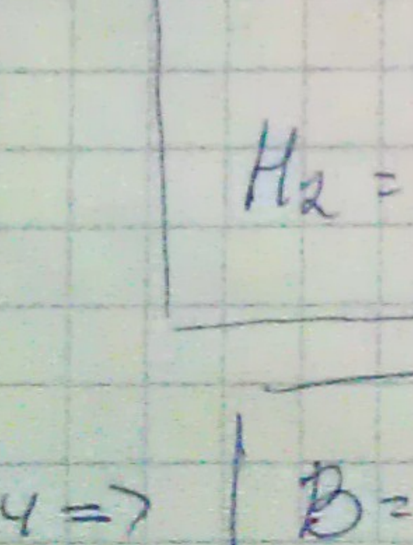
$2H_0 = H_1 + H_2$

$$2H_0 = H_1 \left(1 + \frac{\mu_1}{\mu_2}\right) = H_1 \left(\frac{\mu_2 + \mu_1}{\mu_2} \right)$$

$$\Rightarrow \boxed{\begin{array}{l} H_1 = \frac{2 \mu_2 H_0}{\mu_2 + \mu_1} \\ H_2 = \frac{2 \mu_1 H_0}{\mu_2 + \mu_1} \end{array}}$$

№8.4 $\Rightarrow$ $\boxed{B = \frac{2 \mu_0 \mu_1 \mu_2}{\mu_1 + \mu_2} H_0}$

№8.8



Дано: $I, R, \mu, \vec{B}$ - ?

1) $0 \le r \le R$

$$I = \oint_L \vec{H} d\vec{l} = \frac{I \pi r^2}{\pi R^2} = \frac{I r^2}{R^2}$$

$\vec{H}\,d\vec{\ell} = H\,dl = H r\,d\varphi$

$H_1 \cdot 2\pi = \frac{I}{R^2} r^2 \Rightarrow H_1 = \frac{Ir}{2\pi R^2} \Rightarrow B_1 = \frac{\mu\mu_0 Ir}{2\pi R^2}$

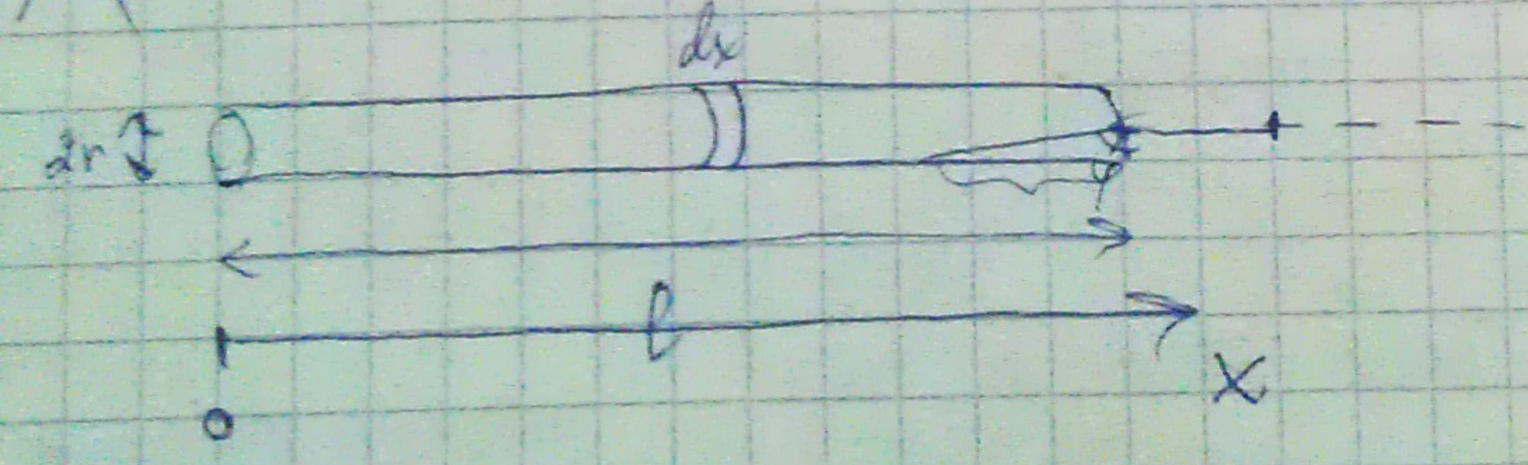
2) $r > R$

$I = \oint_L \vec{H}\,d\vec{l}$

$H_2 \cdot 2\pi r = I \Rightarrow H_2 = \frac{I}{2\pi r} \Rightarrow B_2 = \frac{\mu\mu_0 I}{2\pi r}$

$$B = \begin{cases} \frac{\mu\mu_0 Ir}{2\pi R^2}, & 0 \le r \le R \\ \frac{\mu\mu_0 I}{2\pi r}, & r > R \end{cases}$$

~~№2~~ Дано: l, r, J_m. $B - ?$



$J = kx + b$

$J(0) = 0 \Rightarrow b = 0$

$J(l) = J_m = kl \Rightarrow k = \frac{J_m}{l}$

$\Rightarrow J(x) = \frac{J_m x}{l}$

$x = l - R\operatorname{tg}\varphi$

$\operatorname{tg}\varphi = \frac{l-x}{R}$

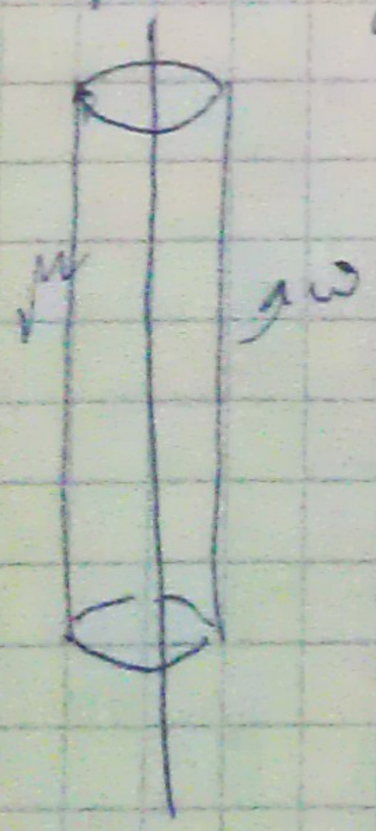
$\varphi = \operatorname{arctg}\frac{l-x}{R}, \quad d\varphi = \frac{-dx}{1+\frac{(l-x)^2}{R^2}}$

$B = \frac{1}{2}\mu_0 J(x)\left(\frac{\frac{l(x)}{2}-l}{\sqrt{r^2+(l(x)/2-l)^2}} + \frac{\frac{l(x)}{2}+l}{\sqrt{r^2+(\frac{l(x)}{2}+l)^2}}\right)$

$l(x) = \oint_L \vec{J}\,d\vec{l} = \oint_L J(x)\,dl = \int_0^l \frac{J_m x}{l}\,dx =$

$= \frac{J_m}{l2} x^2\Big|_0^l = \frac{J_m l}{2} = I_m'$

№ 8.13 Дано: μ, ω, b, R, Q. B – ?



$dI = \frac{dQ}{2\pi}\omega$

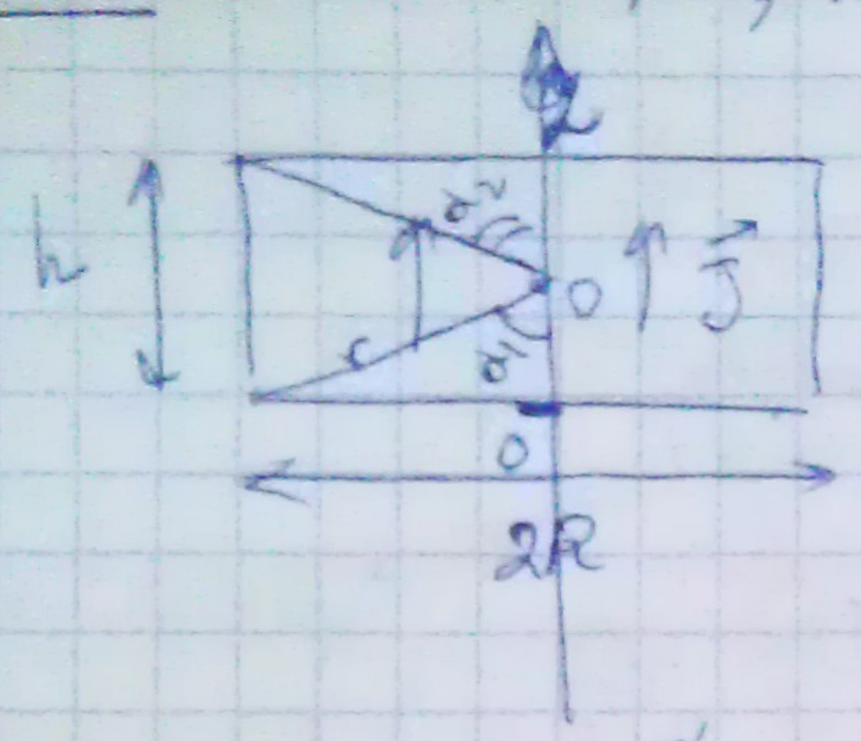
$\oint_L \vec{B}\,d\vec{l} = \mu\mu_0 I$

$\int_0^B B\,dl = \mu\mu_0 \int \frac{dQ}{2\pi}\omega\,dN$

$Bb = \mu\mu_0 \frac{\omega}{2\pi}\varphi$

$\left[B = \frac{\mu\mu_0 Q\omega}{2\pi b}, \quad H = \frac{Q\omega}{2\pi b}\right]$

№ 8.14 Дано: $R, \vec{J}, h$ $\vec{B}$ - ?



$$dB = \frac{\mu_0}{2} J\, dz\, \frac{R\sin\alpha}{z^3}$$

$$z = \frac{R}{\sin\alpha} \Rightarrow dz = -\frac{R\, d\alpha}{\sin^2\alpha}$$

$$I = -J/\ell$$

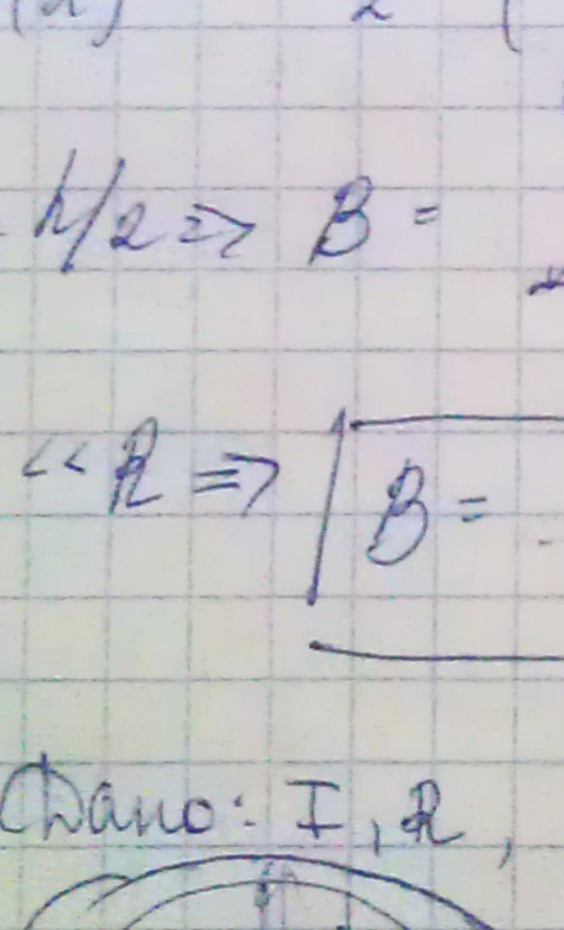
$$B = \frac{\mu_0 J}{2} \int_{\pi-\alpha_1}^{\alpha_2} (-\sin\alpha)\, d\alpha = \frac{\mu_0 J}{2}(\cos\alpha_1 - \cos\alpha_2)$$

$$B(z) = \frac{\mu_0 J}{2}\left(\frac{h/2 - z}{\sqrt{R^2 + (h/2 - z)^2}} + \frac{h/2 + z}{\sqrt{R^2 + (h/2 + z)^2}}\right)$$

$$z = h/2 \Rightarrow B = \frac{\mu_0 J}{2}\left(\frac{h}{\sqrt{R^2 + h^2}}\right) = \frac{\mu_0 J}{2}\left(\frac{h}{R\sqrt{1 + \frac{h^2}{R^2}}}\right)$$

$$h \ll R \Rightarrow \boxed{B = \frac{\mu_0 J h}{2R}}$$

№ 8.15 Дано: I, R, χ_M, r $B(r)$ - ?



1) $0 \le r \le R$

$$I_\Sigma = I + I_M$$

$$I_\Sigma = \frac{I r^2}{R^2}$$

$$\oint_L H\, dl = \frac{I r^2}{R^2}$$

$$\int_0^{2\pi} \chi H \, d\varphi = \frac{I r^2}{R^2}$$

$$H = \frac{I r}{2\pi R^2}$$

$$B = \mu_0 \mu H = \mu_0 (1+\chi) H$$

$$B = \frac{\mu_0 (1+\chi) I r}{2\pi R^2}$$

2) $r > R$

$$I_2 = I$$

$$\oint_L H \, dl = I$$

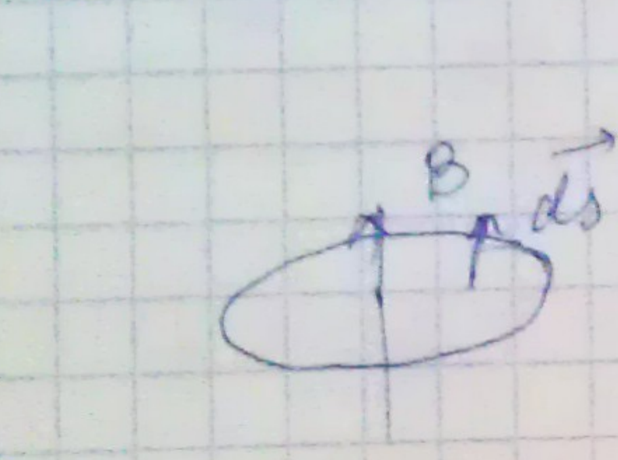
$$2\pi r H = I$$

$$H = \frac{I}{2\pi r}$$

$$B = \mu_0 \mu H = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \left| B = \begin{cases} \dfrac{\mu_0 (1+\chi) I r}{2\pi R^2}, & 0 \le r \le R \\ \dfrac{\mu_0 I}{2\pi r}, & r > R \end{cases} \right|$$

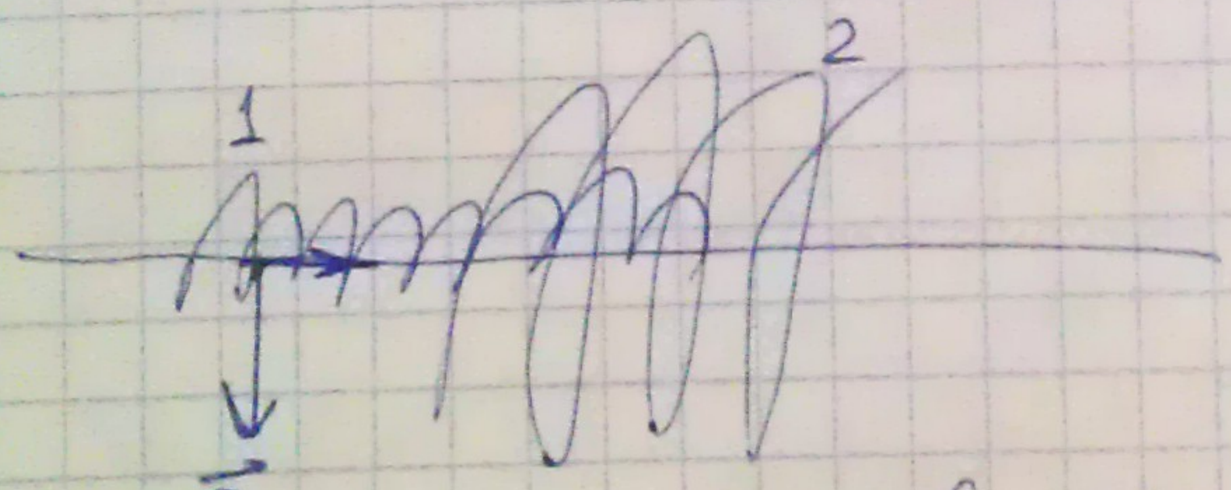
Магнитный поток. Индуктивность. Энергия маг. поля



$\Phi = \int \vec{B}\, d\vec{S}$ - маг. поток

$\Phi \sim I$

$L = \frac{\Phi}{I}$ - индуктивность контура

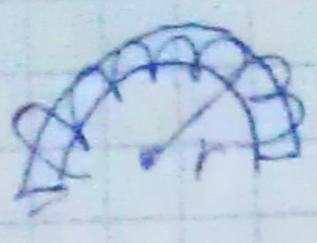


$\Phi_{1 \text{ через } 2} = \int B_1\, dS_2$

$L_{12} = \frac{\Phi_{12}}{I_1}$ - взаимная индукция контуров

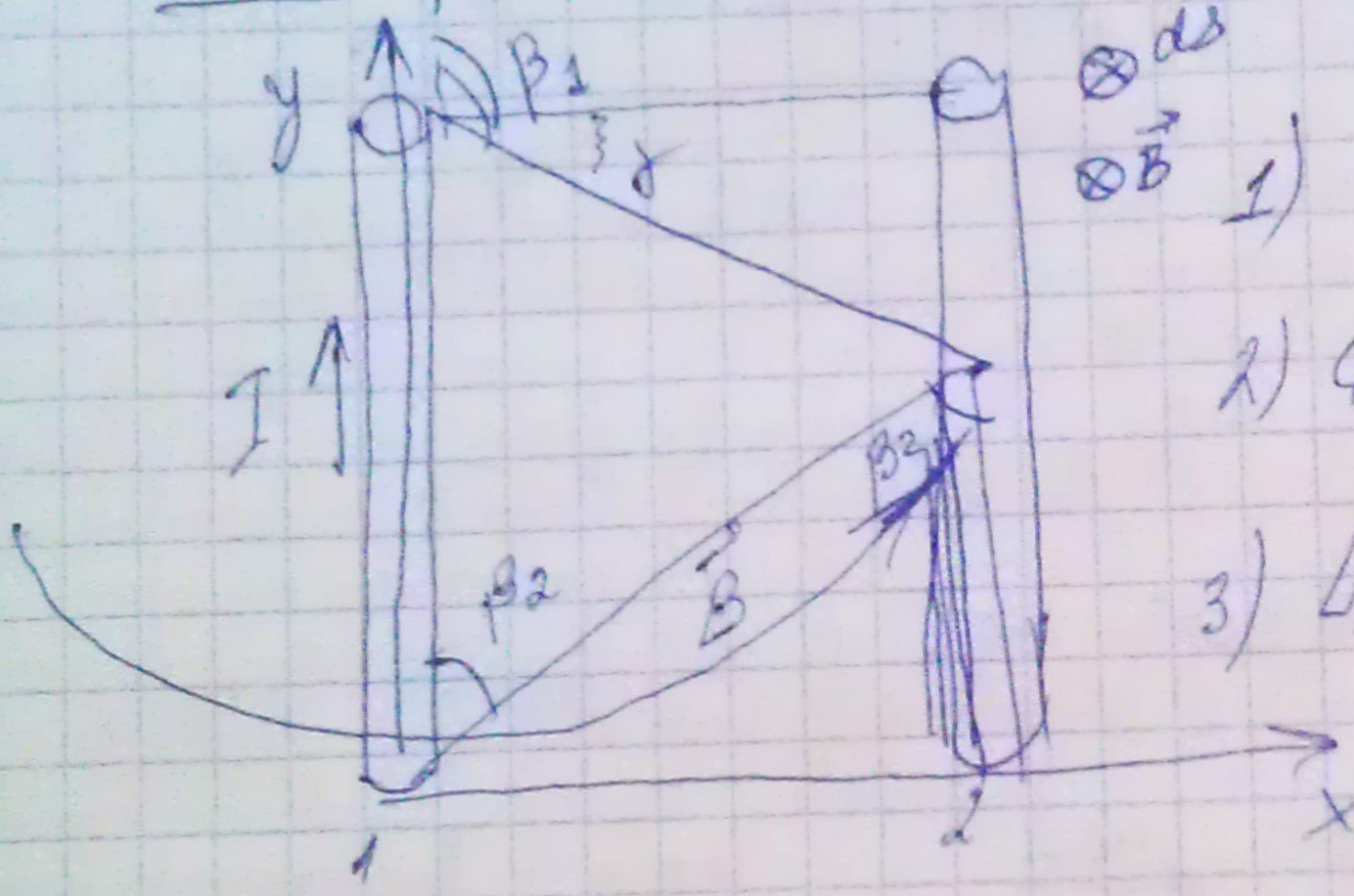
Соленоид: ~~⌀⌀⌀

Теория:



$$B(\text{в центре}) = \frac{\mu \mu_0 N I}{2R}$$

№ 9.2 Дано: a, d, l L_{12} - ?



1) B_2

2) Φ_1 через 2

3) $L_{12} = \frac{\Phi_{12}}{I}$

1) $B_1 = \frac{\mu_0 I}{4\pi x}(\cos\beta_2 - \cos\beta_1)$

$$\cos\beta_2 = \frac{\gamma}{\sqrt{y^2+d^2}}$$

$$\cos\beta_1 = \cos(\pi/2+\gamma_1) = -\sin\gamma_1 = + \frac{-l+y}{\sqrt{(l-y)^2+d^2}}$$

2) $\Phi = \int_0^l \int_{d-a}^{d+a} \frac{\mu_0 I}{4\pi x}\left(\frac{y}{\sqrt{y^2+d^2}} + \frac{l-y}{\sqrt{(l-y)^2+d^2}}\right) dx\, dy =$

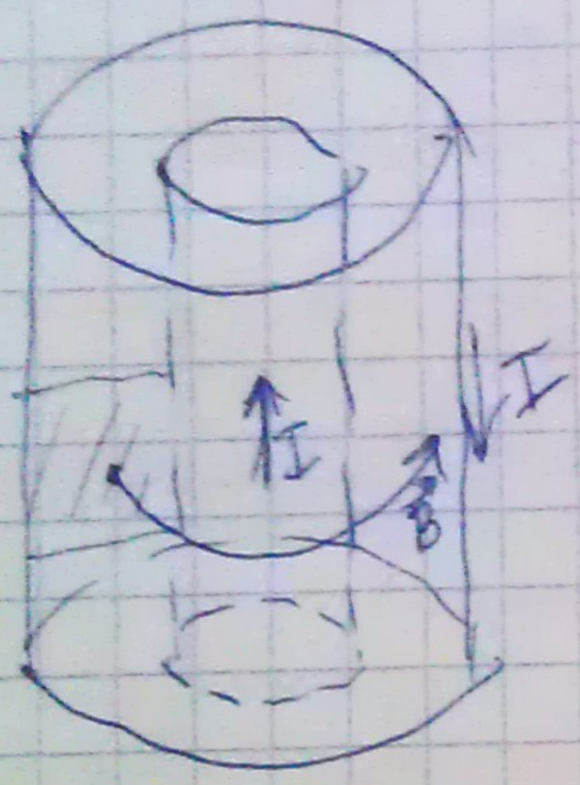
$$= \frac{\mu_0 I}{4\pi} \ln\frac{d+a}{d-a}\left((y^2+d^2)^{1/2} - ((l-y)^2+d^2)^{1/2}\right)\Big|_0^l =$$

$$= \frac{\mu_0 I}{4\pi} \ln \frac{d+a}{d-a} \left(\sqrt{l^2+d^2} - d\right) \cdot 2$$

$$\Phi \Rightarrow L_{12} = \frac{\mu_0}{2\pi} \ln \frac{d+a}{d-a} \left(\sqrt{l^2+d^2} - d\right)$$

$a \ll d$, $l \ll d$

№ 9.3 Дано: l, R_1, R_2 L - ?



$$2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

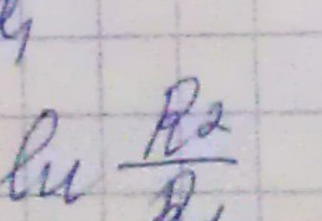
$$\Phi = \int_0^l \int_{R_1}^{R_2} \frac{\mu_0 I}{2\pi r} dr\, dz = \frac{\mu_0 l I}{2\pi} \ln \frac{R_2}{R_1}$$

$$\Psi \Rightarrow L_{12} = \frac{\mu_0 l}{2\pi} \ln \frac{R_2}{R_1}$$

Закон Электромагн. индукции

$$\mathcal{E}_и = -\frac{\partial \Phi}{\partial t} = \oint E dl$$

$$I_{инд} = \frac{\mathcal{E}_и}{R}$$



Диф. форма $rot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Переменное магн. поле порождает вихревое электрич.

$\Phi = \Phi(t)$, если

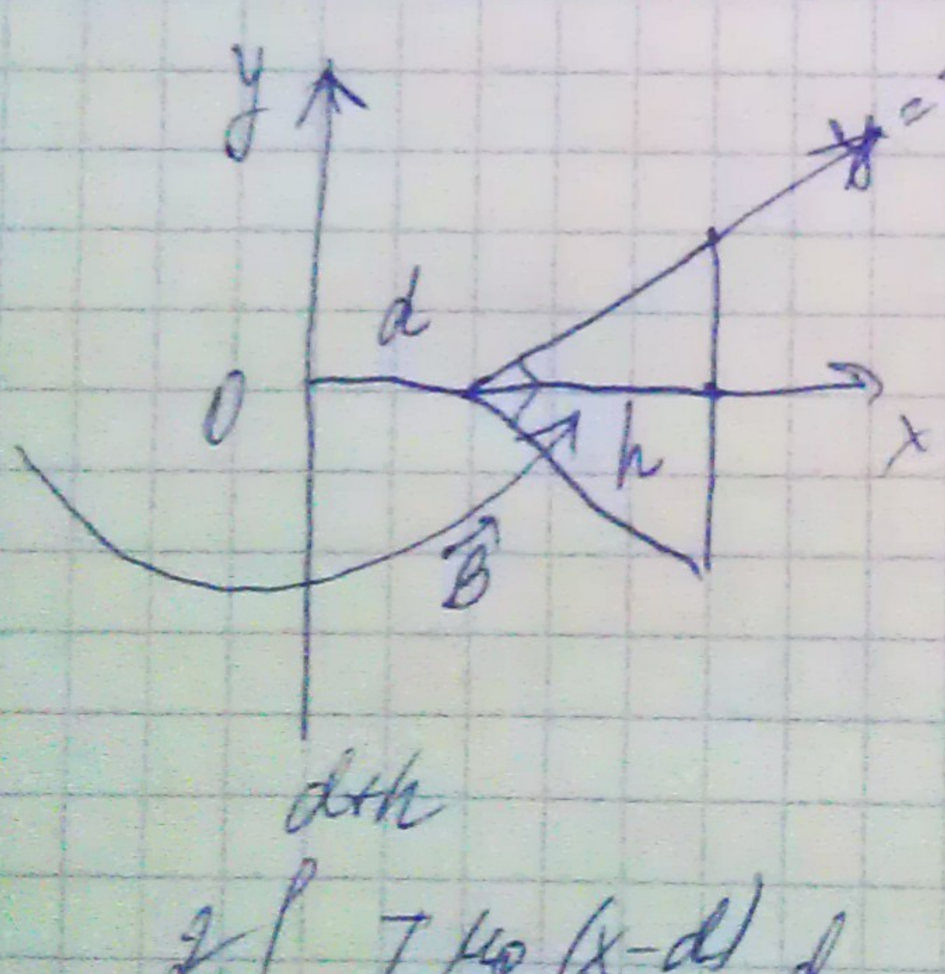
- $B = B(t)$ $(I = I(t))$
- $S = S(t)$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = IR = \frac{dq}{dt} R$$

$$\Rightarrow \Delta q = -\frac{1}{R} \Delta \Phi$$

№10.4 Дано: h, R, d, I_0, τ $\quad I_m - ?$

$$I = I_0 e^{-(t/\tau)^2}$$

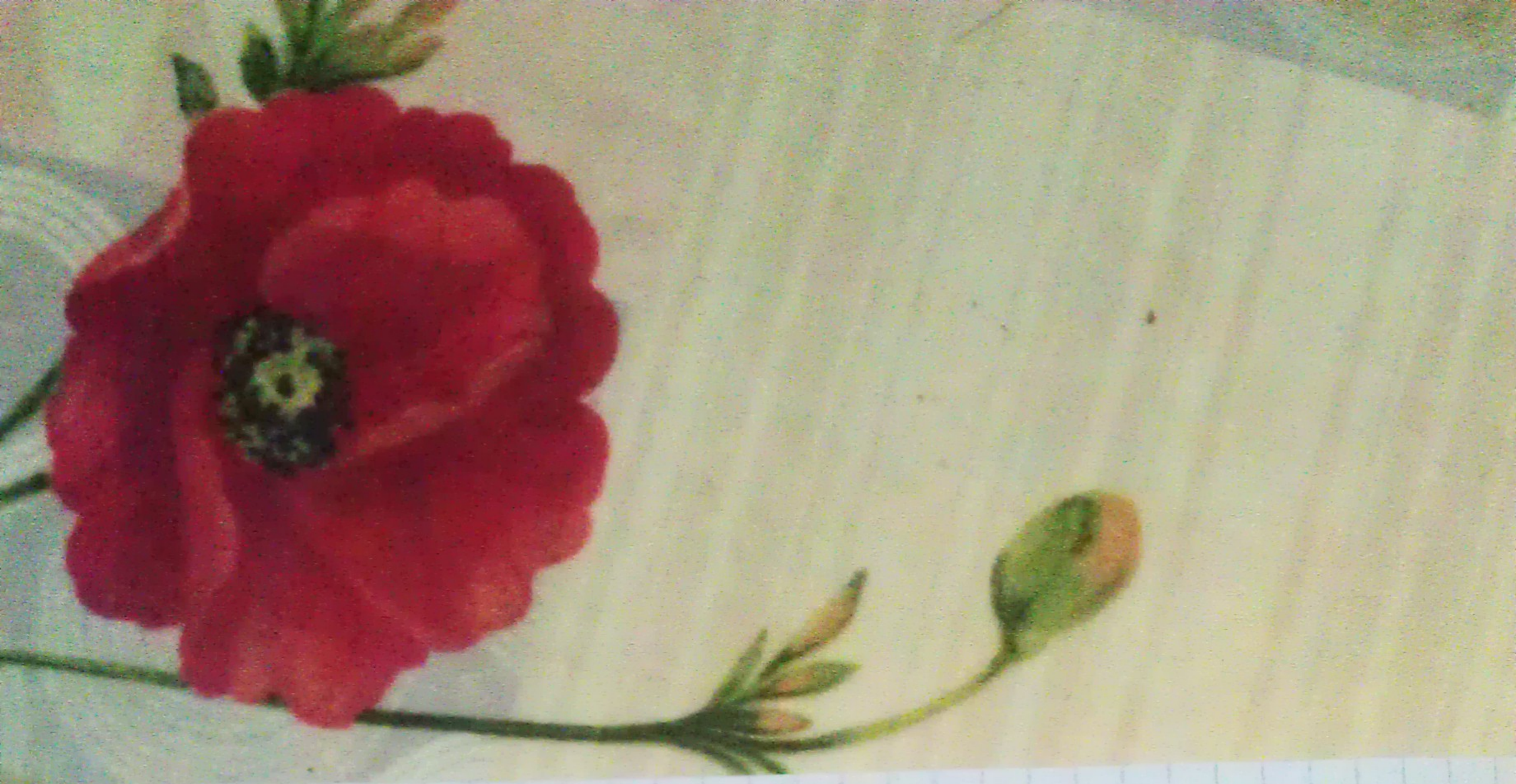


$$B = \frac{I \mu_0}{2\pi x}$$

$$\Phi = 2 \int_d^{d+h} dx \int_0^{\operatorname{ctg} \frac{x-d}{d}} \frac{I \mu_0}{2\pi x} dy =$$

$$2 \int_d^{d+h} \frac{I \mu_0 (x-d)}{2\pi x} dx = \frac{I \mu_0}{\pi} \left(h - d \ln \frac{d+h}{d} \right)$$

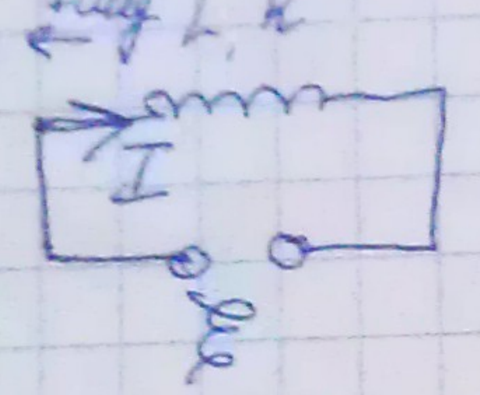
$$\mathcal{E}_i = -\frac{\partial \Phi}{\partial t} = \left(d \ln \frac{d+h}{d} - h \right) \cdot \frac{\mu_0}{\pi} \cdot \frac{\partial I}{\partial t} =$$



$$= I_0 e^{-(t/\tau)^2} \cdot \frac{1}{\tau} \frac{2t}{\tau} \frac{L\omega}{\pi} \left(h - d\ln \frac{d+h}{d} \right) = \mathcal{E}_u$$

$$I_u = \frac{\mathcal{E}_u}{R}$$

№ 10.14

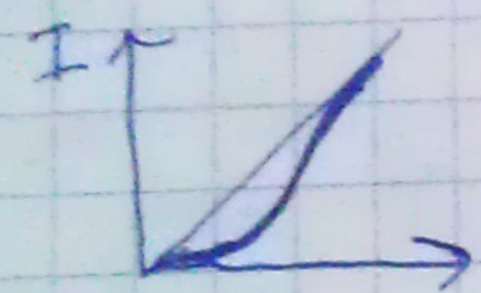


Дано: $I = I_0 e^{-t/\tau}$

L, R

$\mathcal{E}$ - ?

Зав. самоиндукции:



$$\mathcal{E}_{инд} = -\frac{\partial \Phi}{\partial t} = -L \frac{\partial I}{\partial t} =$$

$$= L I_0 \left(-\frac{1}{\tau} \right) e^{-t/\tau}$$

$$\mathcal{E} = IR - \mathcal{E}_u = I_0 e^{-t/\tau} + \frac{L I_0}{\tau} e^{-t/\tau}$$

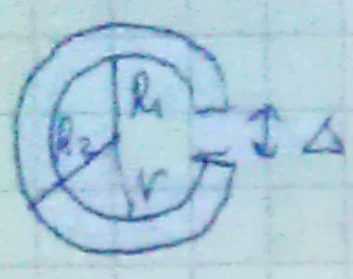
$$\Rightarrow \mathcal{E} = I_0 e^{-t/\tau} \left(1 + \frac{L}{\tau} \right)$$

Дз: 1) теория

2) 9. ~~4~~, ~~5~~, 7, 12, ~~13~~, 14

10. ~~2~~, 3, ~~5~~, ~~10~~, ~~11~~, 12, 13

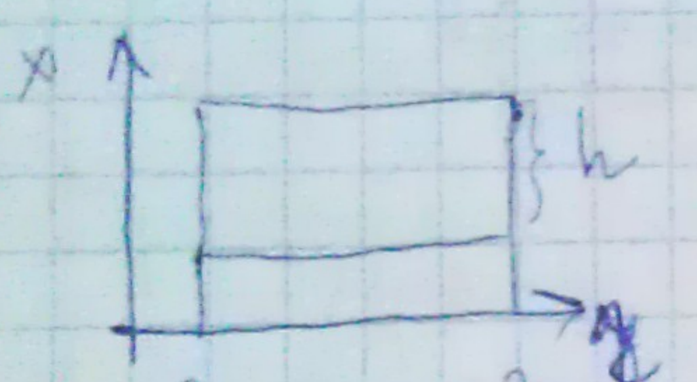
№ 9.7 Дано: $R_1, R_2, h, \Delta, \mu, N$. L - ?



$$B(r) = \frac{\mu\mu_0 N I}{2\pi r}$$

$$\ell = 2\pi R_1 = \Delta$$

$$H_1 \ell + H_2 \Delta = NI$$



$$H_1 = \frac{B}{\mu\mu_0}, \quad H_2 = \frac{B}{\mu_0}$$

$$\frac{B}{\mu\mu_0}(2\pi R_1 - \Delta) + \frac{B}{\mu_0}\Delta = NI$$

$B_1 = B_1 \mu\mu_0 H_1$
$B_1 = B, \; B_2 = \frac{B}{\mu}$

$$\Phi_2 N \oint \vec{B} d\vec{S} = \frac{\mu\mu_0 N^2 I}{2\pi}\left(\int_S \vec{B_1} d\vec{S} + \int_S \vec{B_2} d\vec{S}\right) =$$

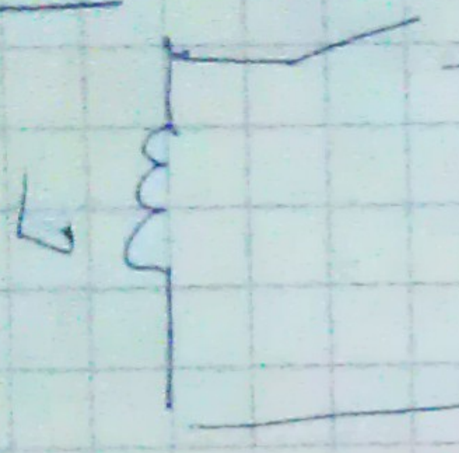
$$= \frac{\mu\mu_0 N^2 \pi h}{2\pi}\left(\ln(R_2 + \mu\Delta - \Delta) - \ln(R_1 + \mu\Delta - \Delta)\right) =$$

$$= \frac{\mu\mu_0 N^2 \pi h}{2\pi} \ln \frac{R_2 + \Delta(\mu - 1)}{R_1 + \Delta(\mu - 1)} = \Phi$$

$$\Rightarrow \boxed{L = \frac{\mu\mu_0 N^2 h}{2\pi} \ln \frac{R_2 + \Delta(\mu - 1)}{R_1 + \Delta(\mu - 1)}}$$

~~№ 931 (1998)~~

№ 913 Дано: C, U_0, L. I_{max} - ?



~~$I = I_0 \cos \omega t$, $\omega = \sqrt{\frac{1}{LC}}$~~

~~$U_0 = I_0 \sqrt{(\omega L - \frac{1}{\omega C})^2}$~~

~~$\omega L = \frac{1}{\sqrt{L} \cdot \sqrt{C}}$~~ $\cdot L = \frac{\sqrt{L}}{\sqrt{C}}$

~~$\omega C = \frac{\sqrt{C}}{\sqrt{L}}$~~

$W_{конд} = \frac{CU^2}{2} \Rightarrow W_0 = \frac{CU_0^2}{2}$

$W_{катушки} = \frac{LI^2}{2} \Rightarrow W_1 = \frac{LI_{max}^2}{2}$

ЗСЭ: $W_0 = W_1 \Rightarrow$ ~~$\frac{CU_0^2}{2} = \frac{LI_{max}^2}{2}$~~

$I_{max}^2 = \frac{C}{L} U_0^2$

$$\boxed{I_{max} = U_0 \sqrt{\frac{C}{L}}}$$

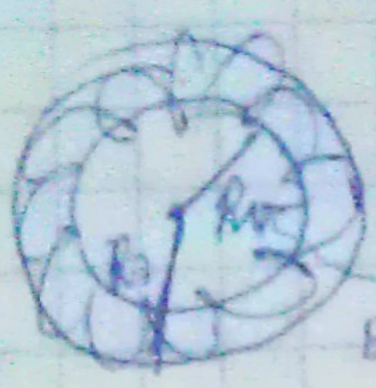
№ 914 Дано: L_1, L_2, N_1. L, N_2 - ?

$L \sim N^2 \Rightarrow \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} \Rightarrow \boxed{N_2 = \sqrt{\frac{L_2}{L_1}} N_1}$

$B = B_1 + B_2$

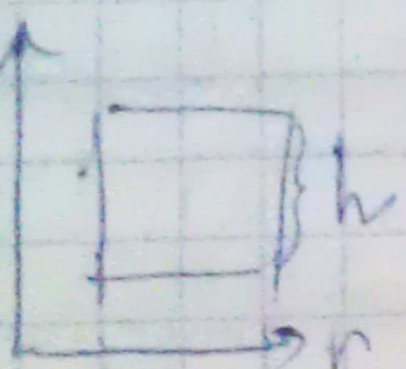
$\Phi_1 = L_1 I$, $\Phi_2 = L_2 I \Rightarrow \boxed{L = L_1 + L_2}$

№9.5 Дано: N, R_2, R_1, h, μ. ν_{12} - ?



$B_{(1)} = \frac{\mu_0 I}{2\pi r}$, внутри соленоида $B_1 = \frac{N \mu_0 I}{2\pi r}$

Но один виток: $\Phi = \int_S B_1 dS_2 = \frac{\mu N_0 I h}{2\pi} \int_0^h dx \int_{R_1}^{R_2} \frac{dr}{r} =$

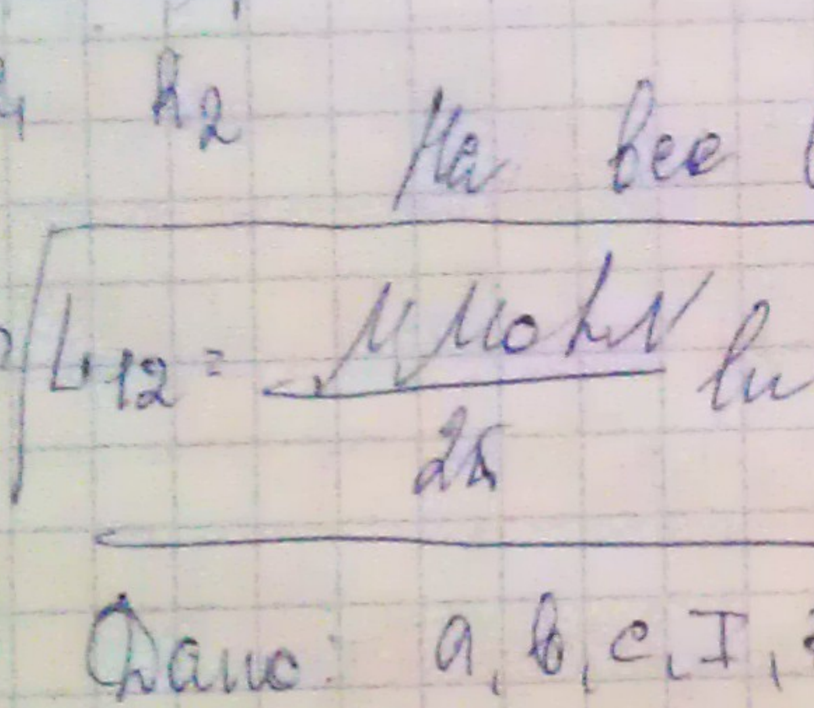


$= \frac{\mu \mu_0 I h}{2\pi} \ln \frac{R_2}{R_1}$

На все витки: $\Psi_{12} = \frac{N \mu \mu_0 I h N}{2\pi} \ln \frac{R_2}{R_1}$

$$\Rightarrow \boxed{L_{12} = \frac{N \mu \mu_0 h N}{2\pi} \ln \frac{R_2}{R_1}}$$

№10.5 Дано: $a, b, c, I, R, \Delta q$ - ?



$$\Delta q = -\frac{1}{R} \Delta \Phi$$

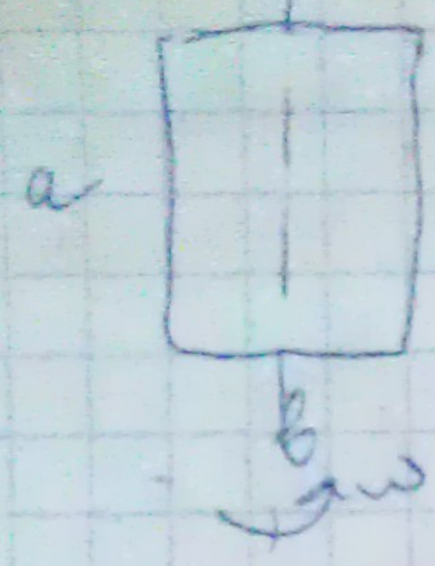
$$B(r) = \frac{\mu \mu_0 I}{2\pi r} \quad \mu = 1$$

$$\Phi = \int \vec{B} d\vec{S} = \frac{\mu_0 I}{2\pi} \int_0^a dx \int_c^{b+c} \frac{dr}{r} =$$

$$= \frac{\mu_0 I a}{2\pi} (\ln(b+c) - \ln c) = \frac{\mu_0 I a}{2\pi} \ln(1 + b/c)$$

$$\Rightarrow \boxed{\Delta q = -\frac{\mu_0 I a}{2\pi R} \ln(1 + b/c)}$$

№10.10 Дано: a, b, ω, Ω, B_0, ξ - ?



$$\xi = -\frac{\partial \Phi}{\partial t}$$

$$B = B_0 \cos \Omega t$$

$$S = ab\cos\omega t$$

$$\Phi = BS = \Phi(t) = B_0 ab \cos\Omega t \cos\omega t$$

$$\xi = -\frac{\partial \Phi}{\partial t} = -B_0 ab\left(-\Omega \sin\Omega t\cos\omega t - \omega \sin\omega t \cos\Omega t\right) =$$

$$\Rightarrow \boxed{\xi = B_0 ab\left(\Omega \sin\Omega t\cos\omega t + \omega \sin\omega t\cos\Omega t\right)}$$

№10.11 Дано: a, b, B, $\omega = \omega_0(1 - \exp(-\delta t))$

$\xi(t_2)$ - ?

$\omega = \omega_0(1 - \exp(-\delta t))$, $\omega_{max} = \omega_0$

$\omega' = \omega_0 \delta \exp(-\delta t)$

$\frac{\omega_0}{2} = \omega_0(1 - \exp(-\delta t))$

$\exp(-\delta t) = \frac{1}{2}$, $\exp(\delta t) = 2$, $\delta t = \ln 2$, $\Rightarrow \boxed{t_2 = \frac{\ln 2}{\delta}}$

$S = ab\cos\omega t$

$\Phi = BS = B_0 ab \cos(\omega(t) \cdot t)$

$\xi = \frac{\partial \Phi}{\partial t} = -B_0 ab\, \omega'(t)(-\sin\omega(t))$ =

$= B_0 ab\, \omega_0 \delta \exp(-\delta t) \sin\omega_0(1 - \exp(-\delta t))$

$\xi(t_2) = B_0 ab\, \omega_0 \delta \exp(-\ln 2) \sin\omega_0(1 - \exp(-\ln 2))$

$\frac{\partial \varphi}{\partial t} = Ba\omega\left(-\sin\omega(t)\cdot t\right)\left(\omega(t)\cdot t\right)' = -Ba\omega\sin(\omega t)\cdot t)\cdot$

$\cdot\left(\omega'(t)\cdot t + \omega(t)'\right) = -Ba\omega\sin\omega t\left(1-\exp(-\delta t)\right)\cdot$

$\cdot\left(\omega_0\delta\exp(-\delta t)\cdot t + \omega_0(1-\exp(-\delta t)\right)\cdot$

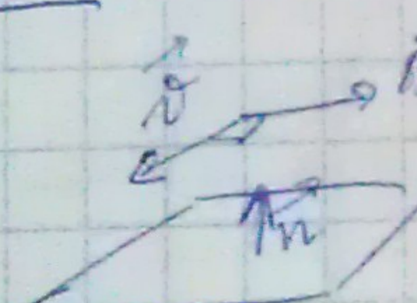
$\xi = Ba\omega\sin\omega_0(t-\exp(-\delta t))t\left(\omega_0\delta\exp(-\delta t)t + \omega_0(1-\exp(-\delta t)\right)$

$\xi(t_2) = Ba\omega\sin\left(\omega_0(1-\exp(-\ln 2)),\frac{\ln 2}{\delta}\right)\left(\omega_0\delta\cdot\frac{\ln 2}{\delta}\exp(\ln 2) + \omega_0(1-\exp(-\ln 2))\right) = Ba\omega\sin\left(\omega_0\cdot\frac{1}{2}\cdot\frac{\ln 2}{\delta}\right)\left(\omega_0\frac{\ln 2}{2} + \omega_0\cdot\frac{1}{2}\right)$

$= Ba\omega\sin\frac{\omega_0\ln 2}{2\delta}\cdot\frac{\omega_0}{2}(\ln 2+1)$

$\Rightarrow \boxed{\xi(t_2) = Ba\omega\frac{\omega_0}{2}(\ln 2+1)\sin\frac{\omega_0\ln 2}{2\delta}}$

№10.2 Дано: B, v σ-?



$\vec{F_л} = q[\vec{v},\vec{B}] = qvB\vec{n}$

$F_л = qvB = qE$

$\Rightarrow E = vB$

Вблизи пластины $E = \frac{\sigma}{\varepsilon\varepsilon_0} \Rightarrow \boxed{\sigma = \varepsilon\varepsilon_0 vB}$

B I



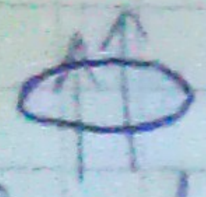
B II



$\vec{j} = \chi\vec{H}$;

$\int\vec{j}\,d\vec{l} = I' = \iint\vec{j}\,d\vec{s}$

I



$dl = r\,d\varphi$

$dS = r\,dr\,d\varphi$

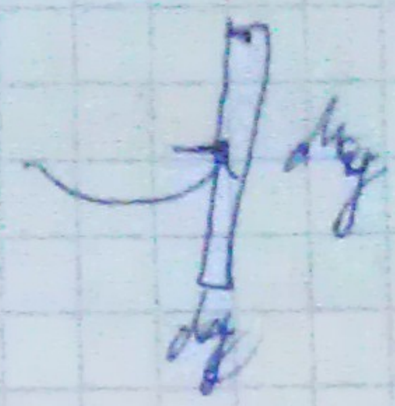
$\oint \vec{j} r\,d\varphi = \iint j'' r\,dr\,d\varphi$

$J_r = \oint j' r\,dr$

$\frac{\partial (J_r)}{\partial r} = \left(\oint j' r\,dr \right)'_r$

$r \frac{\partial J}{\partial r} + J = \vec{j}' r$

II



$\oint J\,dy = \iint j\,dr\,dy$

$J = \oint j\,dr$

$\frac{\partial J}{\partial r} = j$

Уравнения Максвелла

1. $\operatorname{rot} \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$
2. $\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
3. $\operatorname{div} \vec{B} = 0$
4. $\operatorname{div} \vec{D} = \rho$ ($\operatorname{rot} \vec{E} = 0$) - поле неподвижных зарядов

$\vec{B} = \mu \mu_0 \vec{H}$

$\vec{D} = \varepsilon \varepsilon_0 \vec{E}$ } ур-я среды

$\vec{j} = \lambda (\vec{E} + \vec{E}_{ст})$ - з-н Ома

$\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{j} = 0$ - з-н сохр. заряда

$\operatorname{div}\vec{D} = \rho$ - теор. Гаусса. Источником т. е. поля явл. свободные заряды, поле исч. на зарядах, линии не замкн.

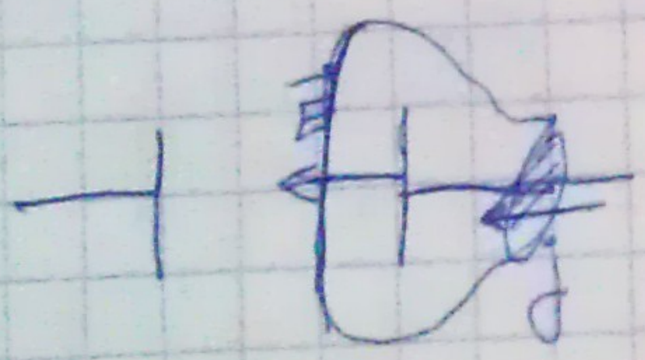
$\operatorname{div}\vec{B} = 0$ - усл. соленоид поля, магнитн. зарядов не сущ., поле вихревое, ~~линии не замкн.~~ нет точки, из кот. выходят магн. поле, линии замкн

$\operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ - 3-и Ф. индукции: перем. магн. поле порождает вихревое электр.

$\operatorname{rot}\vec{H} = \vec{j}$ - токи порожд. вихревое магн. поле

$\operatorname{rot}\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$ - перем. электр. поле порожд. магнитное

$\frac{\partial \vec{D}}{\partial t}$ ток смещения



№ 11.2



Дано: $\Delta B, R, R_1, R_2, B$ - возрас, Δ

Q - ?

$Q = \int D\,dW \cdot \int \lambda E^2\,dW$

$\operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

1) $\begin{vmatrix} \frac{1}{\rho}\vec{i} & \vec{j} & \frac{1}{\rho}\vec{k} \\ \rho & \rho E_\varphi & E_z \\ E\rho & \rho E_\varphi & E_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \end{vmatrix} = -\vec{k}(\omega B_0 \cos\omega t)$

$$\frac{1}{\rho}\left(\frac{\partial E_\rho}{\partial \varphi}^{=0} - \frac{\partial(\rho E_\varphi)}{\partial \rho}\right) = -\omega B_0 \cos\omega\varphi$$

$E \neq E(\varphi, z)$, $\vec{E} = E_0 \vec{e}_\varphi$

$$\vec{i}\left(\frac{\partial \rho E_\varphi}{\partial z}^{=0} - \frac{\partial E_z}{\partial \varphi}^{=0}\right) = 0$$

$$\vec{j}\left(\frac{\partial}{\partial z}(E_\rho)^{=0} - \frac{\partial E_z}{\partial \rho}^{=0}\right) = 0$$

$$\int \partial \rho E_\varphi = \int \rho \omega B_0 \cos\omega t \cdot \partial\rho$$

$$\rho E_\varphi = \frac{\rho^2}{2} \omega B_0 \cos\omega t + C$$

$$E_{\varphi in} = \frac{\rho}{2} \omega B_0 \cos\omega t$$

2) $$\frac{1}{\rho}\frac{\partial \rho E_\varphi}{\partial \rho} = 0$$

$$\rho E_\varphi = C$$

$$E_\varphi = \frac{C}{\rho} = \frac{const}{\rho}$$

$$E_{\tau 1} = E_{\tau 2}$$

$$E_\varphi^{in}\Big|_{\rho=R} = E_\varphi^{out}\Big|_{\rho=R}$$

$$\frac{R}{2}\omega B_0 \cos\omega t = \frac{const}{R}$$

$$\text{const} = \frac{R^2}{2\rho} \omega_0 B_0 \cos\omega t$$

$$E_\varphi^{\text{инд}} = \frac{R^2}{2\rho} \omega_0 B_0 \cos\omega t$$

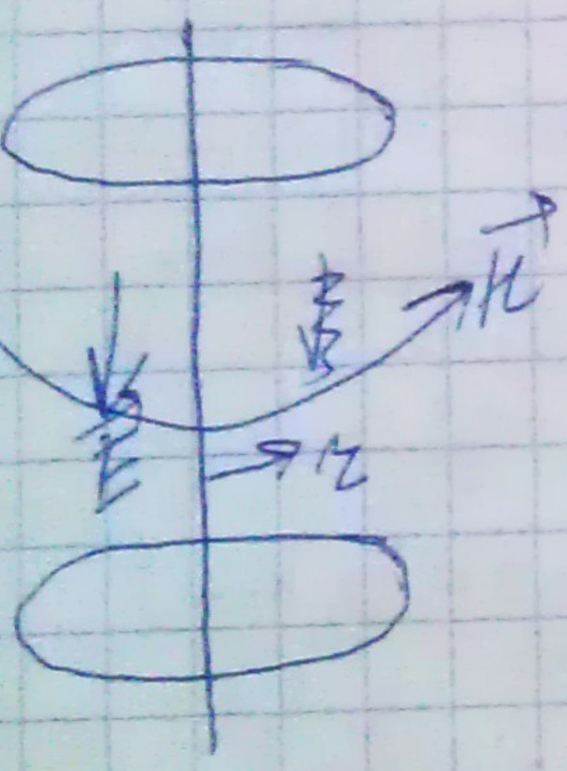
$$\Phi = \int_0^h \int_0^{R_2} \int \frac{R^4 \omega^2 B_0^2 \cos^2\omega t}{4\rho^2} \rho\, d\rho\, d\varphi\, dz$$

$$\Phi = \frac{1}{2} R^4 \pi h \omega^2 B_0^2 \cos^2\omega t \ln\frac{R_2}{R_1}$$

№11.8 Дано: d, $V = V_0 \cos\omega t$, η, (ε, μ).

H_0, B_0 (а) — ?

$$\operatorname{rot}\vec{H} = \frac{\partial \vec{D}}{\partial t}$$



$H \neq H(\varphi, z)$

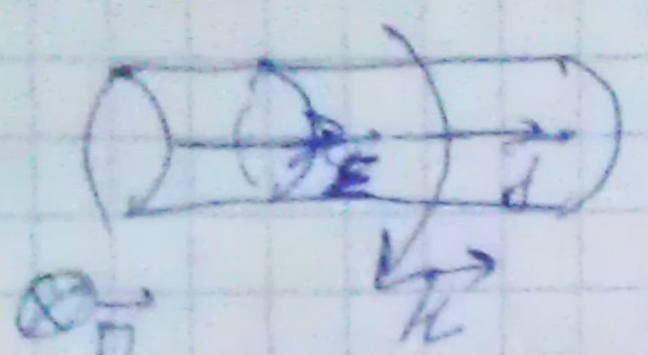
$\vec{H} = H_0 \vec{e}_\varphi$

$$\frac{1}{\rho} \frac{\partial \rho H_\varphi}{\partial \rho} = \frac{\varepsilon_0}{d} \omega \sin\omega t$$

$$K_\varphi = \frac{-\rho \varepsilon_0}{2d} \omega \sin\omega t, \quad K_0 = \frac{-\rho \varepsilon \varepsilon_0 \omega}{2d}$$

$$B_0 = \frac{-\mu\mu_0 \rho \varepsilon \varepsilon_0 \omega}{2d}$$

№ 11.11 Дано: a, j, ~~N~~ $\vec{\Pi}$ - ?, g-то. $\Pi S = N$



$\vec{\Pi} \overset{def}{=} [\vec{E}, \vec{H}]$ - вектор Пойнтинга

~~N~~ $E H \sin 90^\circ = E H \ell \cdot 2\pi a$

$H = \frac{I}{2\pi a}$

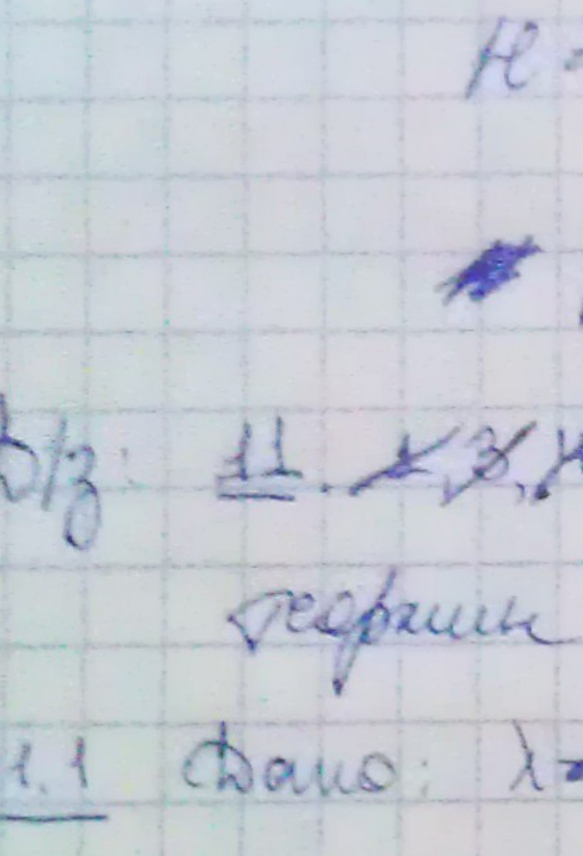
~~N~~ $\Pi S = E \ell I = U I = N$

Д/З: 11. ~~2, 3, 4, 5, 6, 7, 9~~, 10, 12, 13, 14

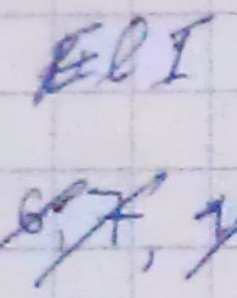
Лекция 4

№ 11.1 Дано: λ ~~=0~~, ℓ, R, B, R_1, R, k N - ?

$$B(t) = \begin{cases} kt, & 0 \le r \le R_1 \\ 0, & r > R_1 \end{cases}$$



B $\{r, \varphi, z\}$ $B = \{0, 0, kt\}$



$$Q = \int_V \gamma dV = \int_V \lambda E^2 dV$$

$$rot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{cases} \frac{1}{r}\frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} = 0 \\ \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = 0 \\ \frac{1}{r}\frac{\partial (r E_\varphi)}{\partial r} - \frac{\partial E_r}{\partial \varphi} = k \end{cases}$$

$dE_\varphi = dE_z = 0$

$$\begin{cases} \frac{\partial E_r}{\partial r} = 0 \\ \frac{\partial (r E_\varphi)}{\partial r} = -kr \end{cases} \qquad E_r = C_1$$

$$E_\varphi = -\frac{kr}{2} + \frac{C_2}{r}$$

$E_r = 0$, $E_\varphi = -\frac{kr}{2} + \frac{C_2}{r}$, $E_r = 0$ (при $r \to 0$)

$$\Rightarrow E = -\frac{kr}{2}$$

$$E^2 = \frac{k^2 r^2}{4}$$

$$\Phi = \lambda \int_0^{2\pi} d\varphi \int_0^{l} dz \int_0^{R} \frac{k^2 r^2}{4} \cdot r\, dr = \frac{\pi \lambda l k^2}{2} \int_0^{R} r^3 dr = \frac{\pi \lambda l k^2 R^4}{8}$$

$$\boxed{\Phi = \frac{\pi \lambda l k^2 R^4}{8}}$$

№ 11.3 $j(r,t) = kr e^{-t/\tau}$ $\quad B - ?$ $\quad B(t=0) = 0$

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{E} = \vec{j}/\lambda$$

$$\vec{j} = \frac{kr}{\lambda} e^{-t/\tau} = \left\{ \frac{kr}{\lambda} e^{-t/\tau}, 0, 0 \right\}$$

$$\operatorname{rot} \frac{\vec{j}}{\lambda} = \begin{vmatrix} \frac{1}{r}\vec{e}_r & \vec{e}_\varphi & \frac{1}{r}\vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ \frac{k}{\lambda} e^{-t/\tau} & 0 & 0 \end{vmatrix}$$

$$\vec{E} = \left\{ \frac{k^2 r}{\lambda} e^{-t/\tau}, 0, 0 \right\}$$

$$\begin{cases} \frac{1}{r}\frac{\partial E_x}{\partial \varphi} - \frac{\partial E_\varphi}{\partial x} = -\frac{\partial B_r}{\partial t} \\ \frac{\partial E_r}{\partial x} - \frac{\partial E_x}{\partial r} = -\frac{\partial B_\varphi}{\partial t} \\ \frac{1}{r}\frac{\partial (E_\varphi r)}{\partial r} - \frac{\partial E_r}{\partial \varphi} = -\frac{\partial B_x}{\partial t} \end{cases}$$

$$\frac{\partial B}{\partial t} = -\frac{k\tau}{\lambda} e^{-t/\tau} = (B$$

$$B = -\frac{k}{\lambda}\int_0^t e^{-t/\tau}\,dt =$$

$$= \frac{+k\tau}{\lambda} e^{-t/\tau}\Big|_0^t = \frac{k\tau}{\lambda}\left(e^{-t/\tau} - 1\right)$$

$$\boxed{B = \frac{k\tau}{\lambda}\left(e^{-t/\tau} - 1\right)}$$

№ 11.4 Дано: φ_0, d. $\rho(x)$ - ?

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho(x)}{\varepsilon_0}$$

$$\rho(x) = -\varepsilon_0 \frac{d^2\varphi}{dx^2}$$

$$\varphi(x) = \varphi_0\left(\frac{x}{d}\right)^{4/3} = \frac{\varphi_0}{d^{4/3}} \cdot x^{4/3}$$

$$\frac{d\varphi}{dx} = \frac{\varphi_0}{d^{4/3}} \cdot \frac{4}{3} x^{1/3}$$

$$\frac{d^2\varphi}{dx^2} = \frac{4}{3}\frac{\varphi_0}{d^{4/3}} \cdot \frac{1}{3} x^{-2/3} = \frac{4}{9}\varphi_0 \cdot \frac{1}{x^{2/3} d^{4/3}}$$

$$\boxed{\rho(x) = -\varepsilon_0 \frac{4}{9}\varphi_0 \cdot \frac{1}{x^{2/3} d^{4/3}}}$$

№ 11.5 Дано: u_0, d $\quad j - ?$

$$j = ev\,n(x), \qquad n(x) = -\frac{\rho(x)}{e}$$

$$j = \frac{ev\,(-\rho(x))}{e} = -\rho(x)v = +\varepsilon_0 \frac{4}{9} u_0 \frac{1}{x^{2/3} d^{4/3}} \cdot v$$

ЗСЭ: $\frac{mv^2}{2} - e\varphi(x) = \frac{mv_0^2}{2} - e\varphi(0) = 0$

$$v = \sqrt{\frac{2e\varphi(x)}{m}}$$

$$\varphi(x) = u_0 \left(\frac{x}{d}\right)^{4/3} \Rightarrow v = \sqrt{\frac{2e}{m}} \sqrt{u_0} \cdot \left(\frac{x}{d}\right)^{2/3}$$

$$j = \varepsilon_0 \cdot \frac{4}{9} u_0 \cdot \frac{\sqrt{u_0}}{x^{2/3} d^{4/3}} \cdot \frac{\sqrt{2e}}{\sqrt{m}} \cdot \frac{x^{2/3}}{d^{2/3}} = \frac{4}{9} \varepsilon_0 \frac{u \sqrt{u_0} \sqrt{2e}}{\sqrt{m} d^2}$$

$$\boxed{j = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{u_0^{3/2}}{d^2}}$$

№ 11.6 $\quad v(d/2) - ?$

$$v = \sqrt{\frac{2e}{m}} \sqrt{u_0} \left(\frac{x}{d}\right)^{2/3}$$

$$v(d/2) = \sqrt{\frac{2e}{m}} \sqrt{u_0} \cdot \frac{1}{2^{2/3}} = \sqrt{\frac{eu_0}{m}} 2^{\frac{1}{2} - \frac{2}{3}} = \sqrt{\frac{eu_0}{m}} \cdot \frac{1}{2^{1/6}}$$

$$\boxed{v(d/2) = \frac{1}{2^{1/6}} \sqrt{\frac{eu_0}{m}}}$$

№11.7 t – ?

$$v(x) = \frac{dx}{dt}$$

$$v(x) = \sqrt{\frac{2e}{m}} \cdot \sqrt{n_0} \cdot \frac{x^{2/3}}{d^{2/3}} = \frac{dx}{dt}$$

$$\sqrt{\frac{2e}{m}} \sqrt{n_0} \frac{dt}{d^{2/3}} = \frac{dx}{x^{2/3}}$$

$$\sqrt{\frac{2e}{m}} \sqrt{n_0} \frac{t}{d^{2/3}} = -\frac{3}{5} x^{-5/3}$$

$$\tau = \left| \frac{d}{\vartheta_{cp}} \right|$$

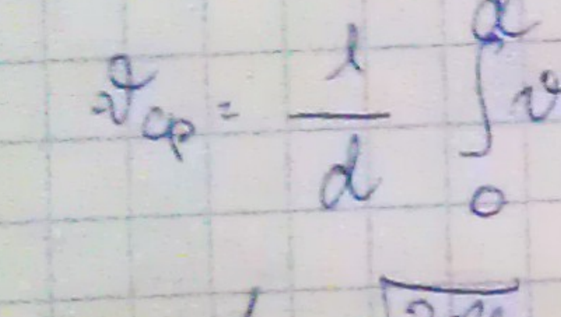
$$\vartheta_{cp} = \frac{1}{d} \int_0^d v(x)dx = \sqrt{\frac{2en_0}{m}} \frac{1}{d^{5/3}} \int_0^d x^{2/3} dx = \sqrt{\frac{2en_0}{m}} \frac{1}{d^{5/3}} d^{-1/3} \left(-\frac{1}{3}\right)$$

$$-\frac{1}{3d^2} \sqrt{\frac{2en_0}{m}}$$

$$\Rightarrow \boxed{\tau = 3d \sqrt{\frac{m}{2en_0}}}$$

№11.12 Дано: R, n, W. Π – ?

$$\vec{\Pi} = [\vec{E}, \vec{H}]$$



$$v = \sqrt{\frac{2W}{m}}$$

$$j = env = en\sqrt{\frac{2W}{m}}$$

$$I = \int_S j\,dS = en\sqrt{\frac{2W}{m}}\,\pi R^2,$$

Внутри $\oint Hdl = en\sqrt{\frac{2W}{m}}\pi r^2$

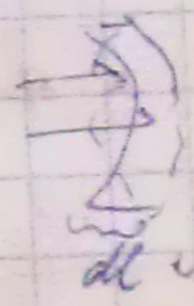
$H \cdot 2\pi r = en\sqrt{\frac{2W}{m}}\pi r^2$

$H = \frac{en}{2}\sqrt{\frac{2W}{m}}r$

Снаружи $H \cdot 2\pi r = en\sqrt{\frac{2W}{m}}\pi R^2$

$H = \frac{en}{2}\sqrt{\frac{2W}{m}}\frac{R^2}{r}$

$div\vec{E} = \frac{n e}{\varepsilon\varepsilon_0}$



$n = \frac{N}{V}$

$\rho = \frac{Q}{V}$

Внутри: $\oint E\,dS = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0} = \frac{neV}{\varepsilon_0}$

$E \cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0}$ $\quad \lambda = \frac{dQ}{dl} = \frac{\rho dV}{dl} = \frac{\rho \pi r^2 dl}{dl}$

$= \rho\pi r^2$

Неск. ур. ур. 2: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$\Rightarrow E = \frac{ne\rho\pi r^2}{2\pi\varepsilon_0 r} = \frac{ner}{2\varepsilon_0}$

$\Pi_{in} = \frac{en}{2}\sqrt{\frac{2W}{m}}\,r\frac{ner}{2\varepsilon_0} = \boxed{\frac{(en)^2}{4\varepsilon_0}\cdot r^2\sqrt{\frac{2W}{m}} = \Pi_{in}}$

Снаружи: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, $\lambda = \frac{\rho dV}{dl} = \rho\pi R^2 = ne\pi R^2$

$E = \frac{ne\rho\pi R^2}{2\pi\varepsilon_0 r} = \frac{neR^2}{2\varepsilon_0 r} \Rightarrow \boxed{\Pi_{out} = \frac{(ne)^2}{4\varepsilon_0}\cdot\frac{R^4}{r^2}\sqrt{\frac{2W}{m}}}$

№1110 Дано: v, d 1) $q = const$ 2) $U = const$ $j_{см} - ?$

$$j_{см} = \frac{\partial D}{\partial t}$$

$$D = \varepsilon\varepsilon_0 E$$

1) $q = const$, $E = \frac{q}{2S} = const \Rightarrow D = const$

$$\Rightarrow \boxed{j_{см} = \frac{\partial D}{\partial t} = 0}$$

2) $U = const$ $\quad d(t) = d + vt$

$$C = \varepsilon\varepsilon_0 \frac{S}{d} \quad E = \frac{CU}{\varepsilon_0 S} = \frac{\varepsilon_0 S}{d(t)} \cdot \frac{U}{\varepsilon_0 S} = \frac{U}{d(t)} =$$

$$= \frac{U}{d+vt}$$

$$D = \frac{\varepsilon_0 U}{d+vt}$$

$$\boxed{j_{см} = \frac{\partial D}{\partial t} = \frac{-\varepsilon_0 U v}{(d+vt)^2}} \rightarrow \frac{-\varepsilon_0 U v}{d^2}$$

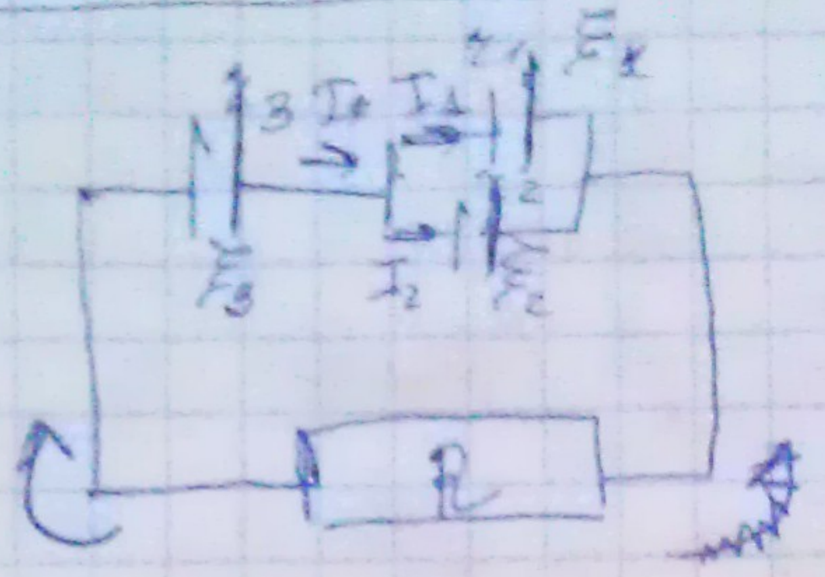
Объёмный элем. тока: $j\,dV = \frac{dI}{dS} dV = dI dl$ (dI dl — лин. элем. тока)

$\vec{i} = \frac{dI}{dl}$ } лин. и объёмн.

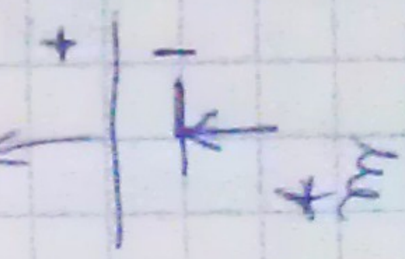
$\vec{j} = \frac{dI}{dS}$ } плотности.

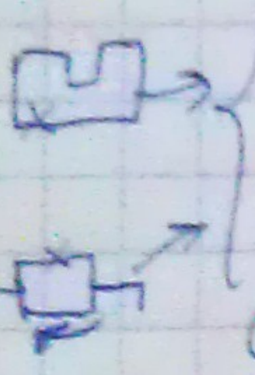
Правила Кирхгофа

№ 12.8



I_R - ?





$$\begin{cases} I = I_1 + I_2 & (1) \\ IR + Ir_3 + I_2 r_2 = \mathcal{E}_3 + \mathcal{E}_2 & (2) \\ I_1 r_1 - I_2 r_2 = \mathcal{E}_1 - \mathcal{E}_2 & (3) \end{cases}$$

(1) в (2):

$$I_1 (R + r_3) + I_2 (R + r_3 + r_2) = \mathcal{E}_3 + \mathcal{E}_2$$

$$I_2 = \frac{\mathcal{E}_3 + \mathcal{E}_2 - I_1 (R + r_3)}{R + r_3 + r_2} \Rightarrow \text{в (3)}, \text{ выразить } I_1$$

$$I_1 = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_3 r_2 + \mathcal{E}_1 R - \mathcal{E}_2 r_3 - \mathcal{E}_2 R - \mathcal{E}_2 r_3}{R r_1 + r_3 r_1 + r_1 r_2 + r_2 r_3 + R r_2}$$

$\Rightarrow$ ~~Получим~~ I_1 и I_2 через $\mathcal{E}_i$, r_i, R. $I = I_1 + I_2$

$$\Rightarrow I = \frac{(\mathcal{E}_2 - \mathcal{E}_1)(R r_1 + R r_2 + r_3 r_1 + r_1 r_2 + r_2 r_3) +}{(R r_1 + r_3 r_1 + r_1 r_2 + r_2 r_3 + R r_2) r_2}$$

$$+ \frac{(\mathcal{E}_1 - \mathcal{E}_2)(r_1 + r_2)(r_3 + R) + r_2 (r_1 + r_2)(\mathcal{E}_1 + \mathcal{E}_2)}{(\ldots) r_2}$$

$$I = \frac{\mathcal{E}_2 r_1 + \mathcal{E}_1 r_2 + \mathcal{E}_3 r_1 + \mathcal{E}_3 r_2}{(\ldots) \text{ числ}}$$

Переходные процессы

$u_R = IR$ $\quad I_R = \frac{u_R}{R}$

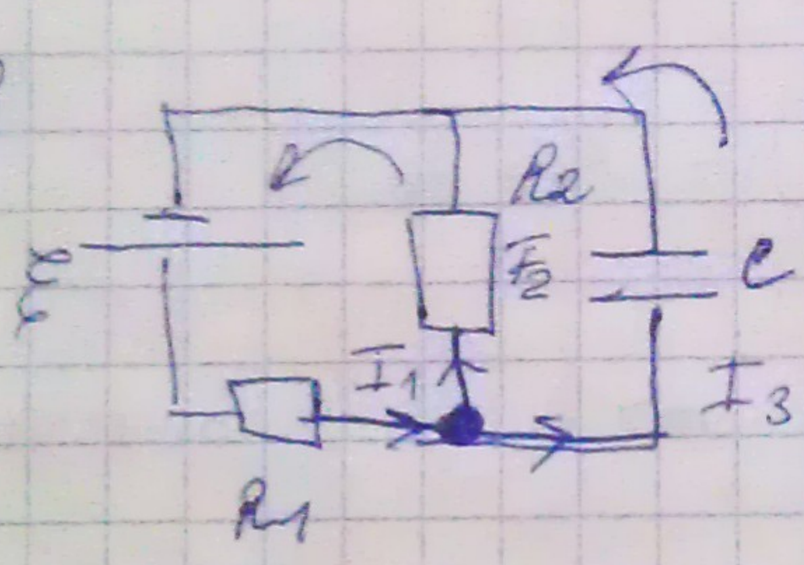
$u_L = L\frac{dI}{dt}$ $\quad I_L = \frac{1}{L}\int u dt$

$u_C = \frac{1}{C}\int I dt$ $\quad I_C = C\frac{du}{dt}$

— через I

⊣⊢ через u

№ 12.18



$u(t) - ?$

$u_3 = u_2$

$u_1 = \varepsilon - u_3$

⇒ в точке.

$I_1 = \frac{u_1}{R_1}, \quad I_2 = \frac{u_2}{R_2}$

$I_3 = C\frac{du_3}{dt}$

$\begin{cases} I_1 = I_2 + I_3 \\ \underbrace{I_1 R_1}_{u_1} + \underbrace{I_2 R_2}_{u_2} = \varepsilon \\ u_3 - u_2 = 0 \end{cases}$

$\frac{\varepsilon - u_3}{R_1} = \frac{u_3}{R_3} + \frac{C du_3}{dt}$

$\frac{du_3}{dt} + \frac{1}{C}\left(\frac{1}{R_2} + \frac{1}{R_1}\right)u_3 = \frac{\varepsilon}{CR_1}$ — неоднор ДУ

$$\Rightarrow u_3 = Ae^{-\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t}$$

$A = A(t)$

$$A'e^{-\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t} = \frac{\mathcal{E}}{CR_1}$$

$$A = \frac{\mathcal{E}}{CR_1}\cdot\frac{CR_1R_2}{R_1+R_2}e^{\pm\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t} + A_1$$

$$u_3 = \frac{\mathcal{E}R_2}{R_1+R_2} + A_1 e^{-\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t}$$

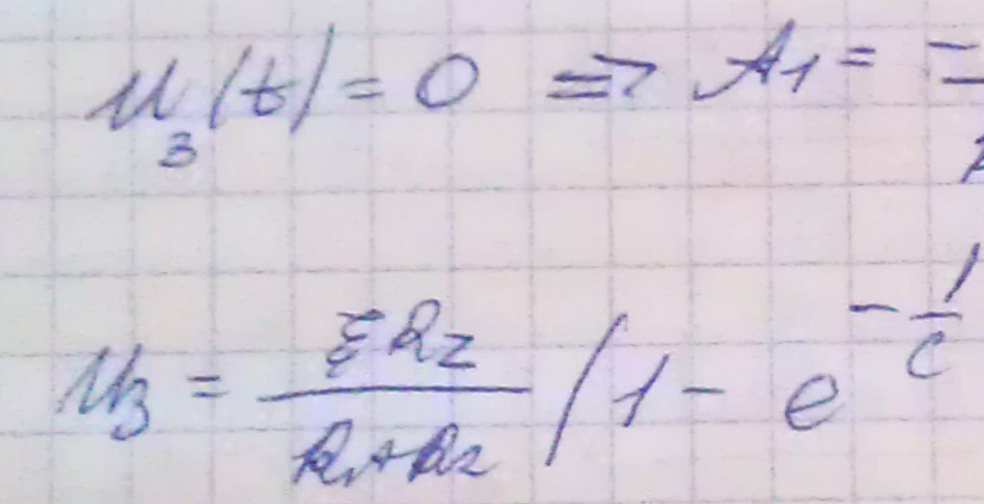
$$u_3(t)=0 \Rightarrow A_1 = -\frac{\mathcal{E}R_2}{R_1+R_2}$$

$$u_3 = \frac{\mathcal{E}R_2}{R_1+R_2}\left(1 - e^{-\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t}\right)$$

Д/З: теория

№ 12. 16, 7, ~~10~~, ~~11~~, ~~20~~, 21, 24, 26

№ 12. 10



$r, \mathcal{E}$ - ?

$$r = r_1' + r_2' + r_3', \qquad r_1' = r_1, \quad r_3' = r_2$$

$$\frac{1}{r_2'} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1+r_2}{r_1 r_2} \Rightarrow r_2' = \frac{r_1 r_2}{r_1+r_2}$$

$$\Rightarrow \boxed{r = r_1 + r_2 + \frac{r_1 r_2}{r_1+r_2}}$$

$$\begin{cases} I = I_1 + I_2 \\ I_1 r_1 - I_2 r_2 = \mathcal{E}_1 - \mathcal{E}_2 \\ IR + I r_2 + I_1 r_1 + I r_1 = 2\mathcal{E}_2 + \mathcal{E}_1 \end{cases}$$

Надо IR

$$I_1 R + I_2 R + I_1 r_2 + I_2 r_2 + I_1 r_1 + I_1 r_1 + I_2 r_1 = 2\mathcal{E}_2 + \mathcal{E}_1$$

$$I_1 R + I_2 R + I_1 r_2 + I_2 r_2 + 2 I_1 r_1 + I_2 r_1 = 2\mathcal{E}_2 + \mathcal{E}_1$$

$$I_1 (R + r_2 + 2r_1) + I_2 (R + r_2 + r_1) = 2\mathcal{E}_2 + \mathcal{E}_1$$

$$I_2 = \frac{2\mathcal{E}_2 + \mathcal{E}_1 - I_1 (R + r_2 + 2r_1)}{R + r_2 + r_1}$$

$$I_1 r_1 - \frac{2\mathcal{E}_2 + \mathcal{E}_1 - I_1 (R + r_2 + 2r_1)}{R + r_2 + r_1} \cdot r_2 = \mathcal{E}_1 - \mathcal{E}_2$$

$$I_1 \left(r_1 + \frac{r_2 (R + r_2 + 2r_1)}{R + r_2 + r_1} \right) = \mathcal{E}_1 - \mathcal{E}_2 + \frac{(2\mathcal{E}_2 + \mathcal{E}_1) r_2}{R + r_2 + r_1}$$

$$I_1 \left(\frac{r_1 R + r_1 r_2 + r_1^2 + r_2 R + r_2^2 + 2 r_1 r_2}{R + r_2 + r_1} \right) = \frac{(\mathcal{E}_1 - \mathcal{E}_2)(R + r_2 + r_1) + (2\mathcal{E}_2 + \mathcal{E}_1) r_2}{R + r_2 + r_1}$$

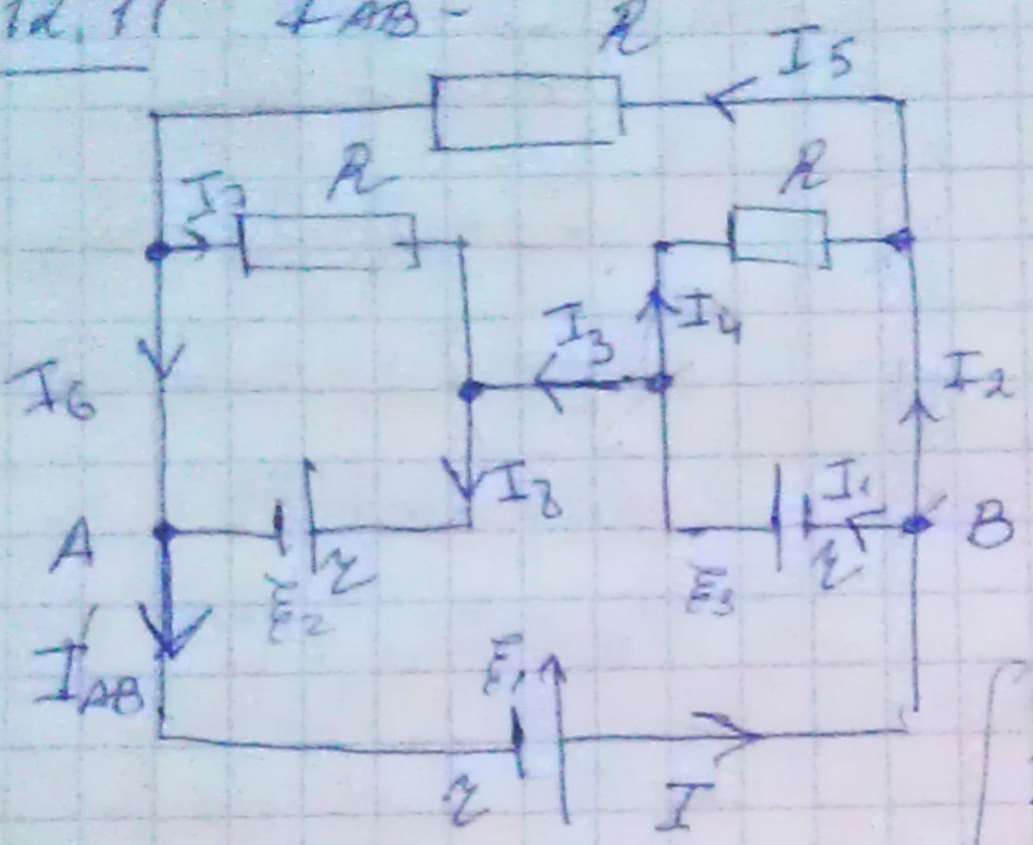
$$I_1 \left(\frac{r_1 R + r_2 R + 3 r_1 r_2 + r_1^2 + r_2^2}{R + r_2 + r_1} \right) = \frac{(\mathcal{E}_1 - \mathcal{E}_2)(R + r_2 + r_1) + (2\mathcal{E}_2 + \mathcal{E}_1) r_2}{R + r_2 + r_1}$$

$$= \mathcal{E}_1 R + \mathcal{E}_1 r_2 - \mathcal{E}_2 R - \mathcal{E}_2 r_2 + \mathcal{E}_1 r_2 + 2\mathcal{E}_2 r_2$$

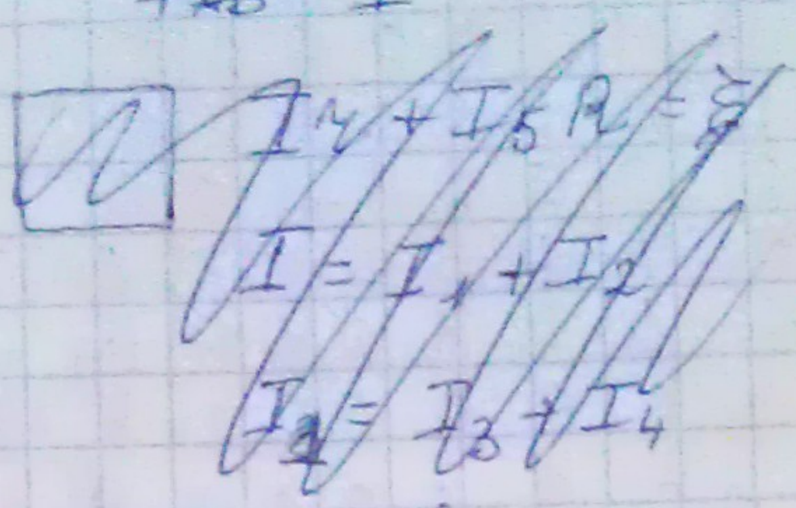
$$I_1 = \frac{(\mathcal{E}_1 - \mathcal{E}_2)(R + r_2 + r_1) + (2\mathcal{E}_2 + \mathcal{E}_1) r_2}{r_1 R + r_2 R + 3 r_1 r_2 + r_1^2 + r_2^2} \Rightarrow \text{Найдем } I_2.$$

после логического сокращения $\mathcal{E} = \mathcal{E}_2 \frac{r_2}{r_1 + r_2} + \mathcal{E}_1 \frac{r_1 - 2r_2}{r_1 + r_2}$

№ 12.11 I_{AB} - ?



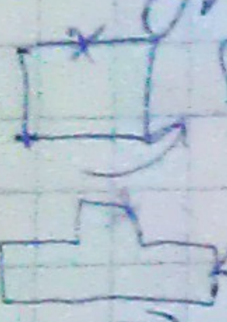
$I_{AB} = I$



Для узлов:

$$\begin{cases} I = I_1 + I_2 \\ I_1 = I_3 + I_4 \\ I_5 = I_2 + I_4 \\ I_5 = I_6 + I_7 \\ I_8 = I_7 + I_3 \\ \sout{I = I_6 + I_8} \end{cases}$$

Контуры:



$$\begin{cases} I_4 + I_5 R = \mathcal{E}_1 \\ I_2 + I_1 r + I_8 r = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\ I_1 r + I_4 R = \mathcal{E}_3 \\ I_7 r + I_8 R = -\mathcal{E}_2 \end{cases}$$

$$\begin{cases} I = I_1 + I_2 \\ I_1 = I_3 + I_4 \\ I_5 = I_2 + I_4 \\ I_5 = I_6 + I_7 \\ I_8 = I_7 + I_3 \\ \sout{I = I_6 + I_8} \\ I_2 + I_5 R = \mathcal{E}_1 \\ I_2 + I_1 r + I_8 r = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\ I_1 r + I_4 R = \mathcal{E}_3 \end{cases}$$

$I_1 = I - I_2$

$I - I_2 = I_3 + I_4$

$I_2 = I - I_3 - I_4$

0	1	2	3	4	5	6	7	8	
I	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	
1	-1	-1	0	0	0	0	0	0	0
0	1	0	-1	-1	0	0	0	0	0
0	0	-1	0	-1	1	0	0	0	0
0	0	0	0	0	1	-1	-1	0	0
0	0	0	-1	0	0	0	-1	1	0
r	0	0	0	R	0	0	0		$\mathcal{E}_1$
r	r	0	0	0	0	0	0	R	$\mathcal{E}_1 + \mathcal{E}_3$
0	r	0	0	R	0	0	0	0	$\mathcal{E}_2$

$$\left[\begin{array}{ccccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & r & r & 0 & 0 & R & 0 & 0 & 0 & \mathcal{E}_1 \\
0 & 2r & r & 0 & 0 & 0 & 0 & 0 & r & \mathcal{E}_1+\mathcal{E}_3-\mathcal{E}_2 \\
0 & r & 0 & 0 & R & 0 & 0 & 0 & 0 & \mathcal{E}_3
\end{array}\right]$$

$$\sim \left[\begin{array}{ccccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & r & 0 & -R & R & 0 & 0 & 0 & \mathcal{E}_1-\mathcal{E}_3 \\
0 & 0 & r & 0 & -2R & 0 & 0 & 0 & 0 & \mathcal{E}_1-\mathcal{E}_3-\mathcal{E}_2 \\
0 & 0 & 0 & 0 & R & 0 & 0 & 0 & 0 & \mathcal{E}_3
\end{array}\right]$$

$$\sim \left[\begin{array}{ccccccccc|c}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & r & 0 & -R & R & 0 & 0 & 0 & \mathcal{E}_1-\mathcal{E}_3 \\
0 & 0 & 0 & 0 & -2R & 0 & 0 & 0 & 0 & \mathcal{E}_2 \\
0 & 0 & 0 & 0 & R & 0 & 0 & 0 & 0 & \mathcal{E}_3
\end{array}\right]$$

$$\begin{cases}
I_2 = I - I_3 - I_4 \\
I_7 r + I_7 R + I_3 R = -\mathcal{E}_2 \\
I_1 r + I_6 R + I_7 R = \mathcal{E}_1 \\
2 I_1 r - I_2 r + I_7 R + I_3 R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\
I_1 r - I_2 r + I_4 R = \mathcal{E}_3
\end{cases}$$

$$\begin{cases}
I_1 = I - I_3 = I_6 + I_7 \\
I_7 r + I_7 R + I_3 R = -\mathcal{E}_2 \\
I_1 r + I_6 R + I_7 R = \mathcal{E}_1 \\
2 I_1 r - I_2 r + I_7 R + I_3 R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\
I_1 r - I_2 r + I_4 R = \mathcal{E}_3
\end{cases}$$

$$\begin{cases}
I_1 = I_6 + I_7 + I - I_3 \\
I_7 r + I_7 R + I_3 R = -\mathcal{E}_2 \\
I_1 r + I_6 R + I_7 R = \mathcal{E}_1 \\
2 I_1 r - (I - I_3 - I_4) r + I_7 R + I_3 R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\
I_1 r - (I - I_3 - I_4) r + I_4 R = \mathcal{E}_3
\end{cases}$$

$$\begin{cases}
I = I_6 + I_7 + I_3 = I_6 + I_7 - I \\
I_7 r + I_7 R + (-I + I_6 + I_7) R = -\mathcal{E}_2 \\
I_1 r + I_6 R + I_7 R = \mathcal{E}_1 \\
2 I_1 r - (I - I_6 - I_7 + I - I_4) r + I_7 R + (I_6 + I_7 - I) R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\
I_1 r - (I - I_6 - I_7 + I - I_4) r + I_4 R = \mathcal{E}_3
\end{cases}$$

$$I_1 = I - I_2$$

$$I_2 = I - I_3 - I_4$$

$$\sim \begin{cases}
I_5 = I_2 + I_4 = I - I_3 \\
I_5 = I_6 + I_7 \\
I_8 = I_7 + I_3
\end{cases}$$

$$I_7 r + I_7 R + I_3 R = -\mathcal{E}_2$$

$$I_1 r + I_6 R + I_7 R = \mathcal{E}_1$$

$$I_1 + I_1 - I_2 r + I_7 R + I_3 R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2$$

$$I r - I_2 r + I_4 R = \mathcal{E}_3$$

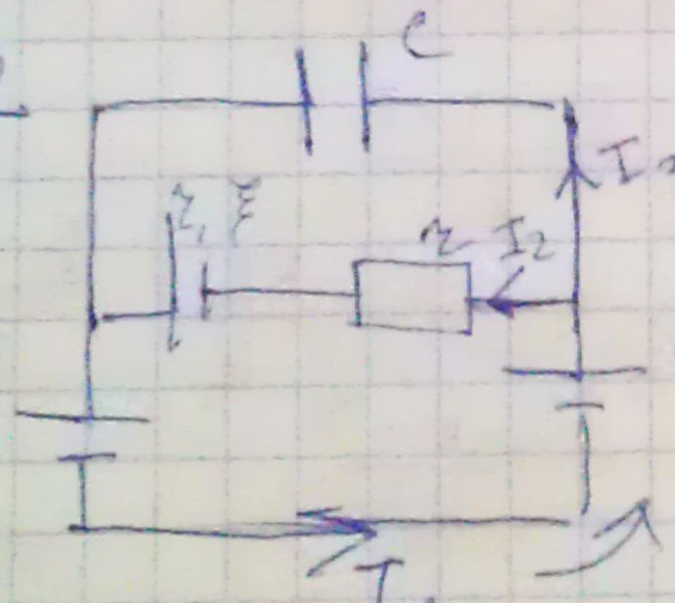
$$\begin{cases}
I = I_6 + I_7 + I_3 \\
I_7 r + I_7 R + I_3 R = -\mathcal{E}_2 \\
I_1 r + I_6 R + I_7 R = \mathcal{E}_1 \\
2 I_1 r - (I - I_3 - I_4) r + I_7 R + I_3 R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\
I_1 r - (I - I_3 - I_4) r + I_4 R = \mathcal{E}_3
\end{cases}$$

$$\begin{cases} I_7(r+2R) + I_6R - IR = -\mathcal{E}_2 \\ I_2 + I_6R + I_7R = \mathcal{E}_1 \\ 2I_2 - (2I - I_6 - I_2 - I_4)r + I_7R + (I_6 + I_7)R = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\ I_2 - (2I - I_6 - I_2 - I_4)r + I_4R = \mathcal{E}_3 \end{cases}$$

$$\begin{cases} I_7(r+2R) + I_6R - IR = -\mathcal{E}_2 \\ I_2 + I_6R + I_7(R) = \mathcal{E}_1 \\ I(\cancel{2r - 2r} - R) + I_6(r+R) + I_7(r+2R) + I_4r = \mathcal{E}_1 + \mathcal{E}_3 - \mathcal{E}_2 \\ I(r - 2r) + I_6r + I_7r + I_4(R+r) = \mathcal{E}_3 \end{cases}$$

$$\Rightarrow \boxed{I = \frac{R(\mathcal{E}_1 - \mathcal{E}_2 + \mathcal{E}_3) + 3r\mathcal{E}_1}{3r(R+r)}}$$

№ 12.20



$$\begin{cases} I_1 = I_2 + I_3 \\ I_1r + I_2r + I_2r + I_1r = 6\mathcal{E} \\ I_1r + u_c + I_1r = 5\mathcal{E} \end{cases}$$

$$\begin{cases} I_1 = I_2 + I_3 \\ u_1 + u_2 = 3\mathcal{E} \\ 2u_1 + u_c = 5\mathcal{E} \end{cases}$$

$$\begin{cases} I_1 = I_2 + I_3 \\ I_1r + I_2r = 3\mathcal{E} \\ 2I_1r + u_c = 5\mathcal{E} \end{cases}$$

$$I_1 = \frac{u_1}{r}, \quad I_2 = \frac{u_2}{r}, \quad I_3 = C\frac{du_c}{dt}$$

$$u_1 = \frac{5\mathcal{E} - u_c}{2}, \quad u_2 = 3\mathcal{E} - \frac{5\mathcal{E}}{2} + \frac{u_c}{2} = \frac{\mathcal{E} + u_c}{2}$$

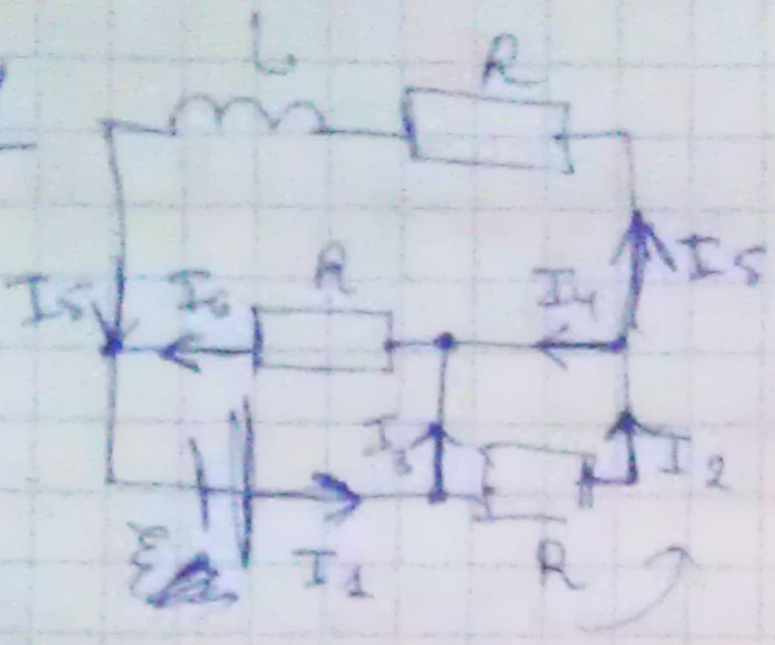
$$\frac{5\mathcal{E} - u_c - (\mathcal{E} + u_c)}{2r} = C\frac{du_c}{dt}$$

$$\frac{4\mathcal{E} - 2u_c}{2r} = C\frac{du_c}{dt}$$

$$2\mathcal{E} - u_c = Cr\frac{du_c}{dt}$$

$$\mathcal{L}_L \frac{dU_c}{dt} + U_c = 2\mathcal{E} \Rightarrow \boxed{U_c = \mathcal{E} e^{-t/\tau_c}}$$

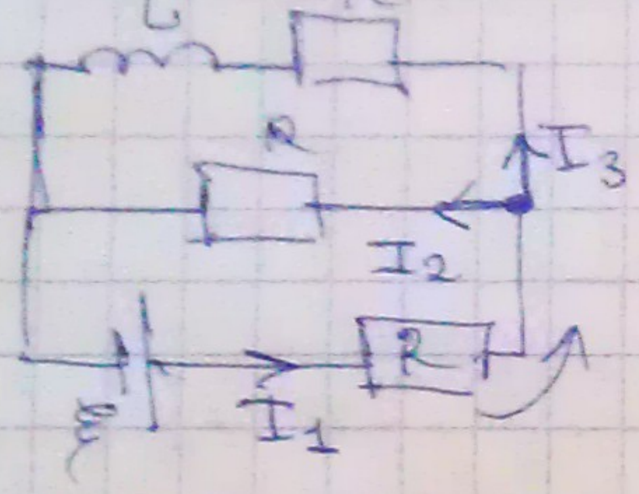
№12.21



$$I_1 = I_3 + I_2$$
$$I_2 = I_4 + I_5$$
$$I_1 = I_5 + I_6$$
$$I_2 R + I_5 R + U_L = \mathcal{E}$$
$$I_6 R = \mathcal{E}$$
$$I_2 R + I_6 R = \mathcal{E}$$

$$U_L = L \frac{dI_3}{dt}$$

1)



$$\begin{cases} I_1 = I_2 + I_3 \\ I_1 R + I_3 R + U_L(0) = \mathcal{E} \\ I_1 R + I_2 R = \mathcal{E} \end{cases}$$

$$I_2 = \frac{\mathcal{E} - I_1 R}{R}$$

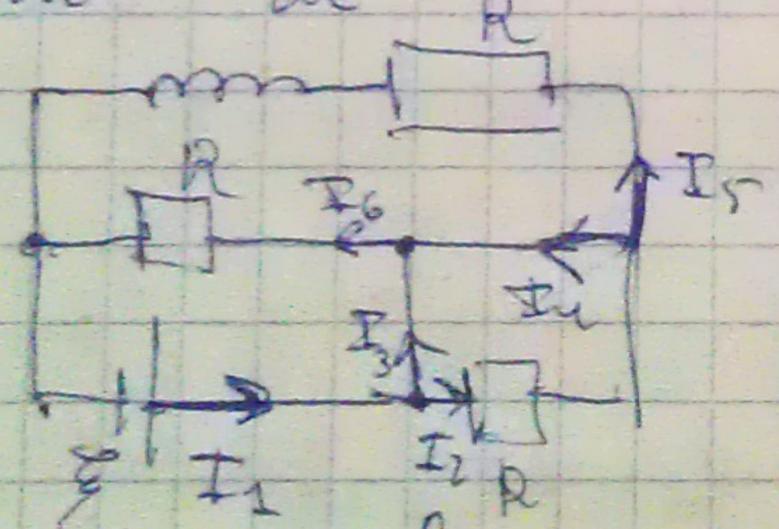
$$I_3 = I_1 - I_2 = \frac{2I_1}{R} - \frac{\mathcal{E} - I_1 R}{R} =$$

$$= \frac{2I_1 R - \mathcal{E}}{R}$$

$$U_L(0) = \mathcal{E} - I_1 R - 2I_1 R + \mathcal{E} =$$

$$= 2\mathcal{E} - 3I_1 R$$

2) $U_L = L \frac{dI_5}{dt}$



$$I_1 = I_2 + I_3$$
$$I_2 = I_4 + I_5$$
$$I_6 = I_3 + I_4$$

$$I_2 = 0$$

$$I_1 = I_2 + I_3$$
$$\mathcal{E} = I_3 R$$
$$\mathcal{E} = I_2 R + U_L$$

$$U_L = L \frac{dI_2}{dt}$$

$$\xi = RI_2 + L\frac{dI_2}{dt}$$

$$L\frac{dI}{dt} + RI = 0$$

$$\frac{dI}{I} = -\frac{R}{L}dt$$

$$I = e^{-\frac{R}{L}t} \cdot C(t)$$

$$\frac{dI}{dt} = -\frac{R}{L}C(t)e^{-\frac{R}{L}t} + C'(t)e^{-\frac{R}{L}t}$$

$$\xi = R\cancel{e^{-\frac{R}{L}t}}\,C - R\cancel{C(t)e^{-\frac{R}{L}t}} + C'e^{-\frac{R}{L}t}$$

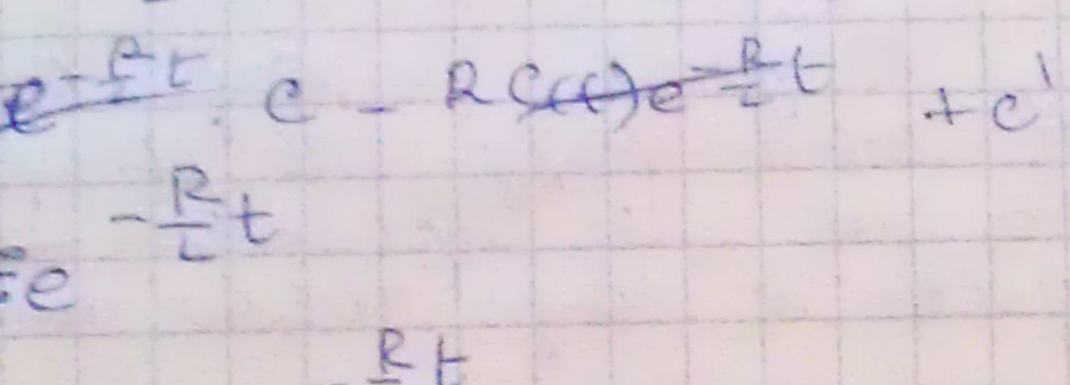
$$C' = \xi e^{-\frac{R}{L}t}$$

$$C = -\frac{L}{R}\xi e^{-\frac{R}{L}t} \Rightarrow I_L = -\frac{L}{R}e^{-\frac{R}{L}t}\,\xi$$

$$u_L = \frac{dI_L}{dt} = \xi e^{-\frac{R}{L}t}$$

Колебания в цепи

№ 12.27



$$\ddot{u}_2 + \underbrace{\frac{1}{6CR}}_{\text{затухание}}\dot{u}_2 + \frac{u}{6C^2R^2} = 0$$

$$u_2 = e^{-\delta t}\cos(\omega t + \varphi)$$

$$\delta = \frac{1}{6CR}$$

$$\omega = \frac{1}{\sqrt{6}CR}$$

A, φ - найти из нач. усл.

$u = u_0 e^{i(\omega t + \varphi)}$

$u = \underbrace{u_0 e^{i\varphi}} \cdot e^{i\omega t}$

комплексная амплитуда $\tilde{u}$

$Z_R = R$

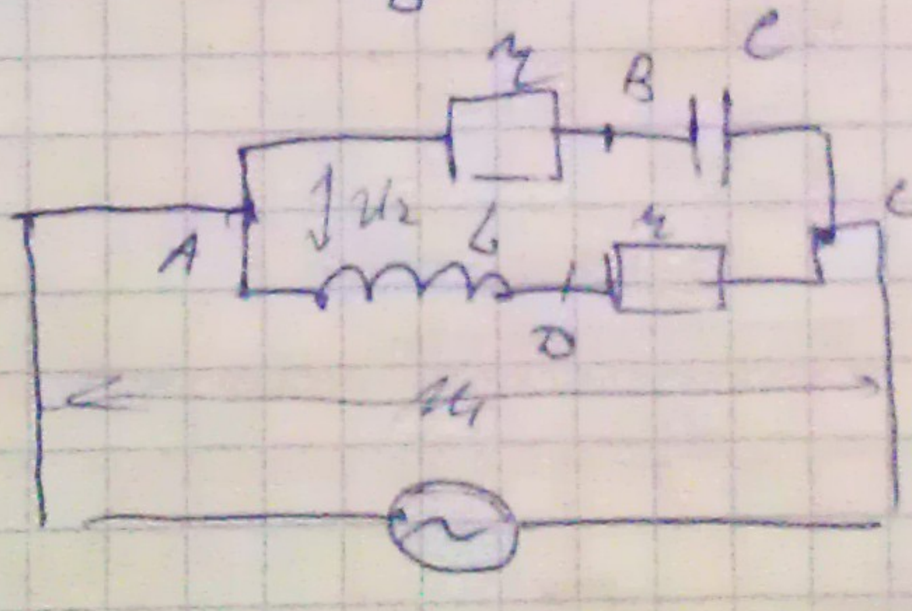
$Z_L = i\omega L$ — импеданс; не происх. выделение тепла

$Z_C = \frac{1}{i\omega C}$

катушка и конд. в цепи сдвигают фазу но потери тепла на них не происходят $\Leftrightarrow$ сверхпроводящие катушки и конд. чисто мнимые

$\tilde{u}_C = \tilde{I}_C Z_C$ $\quad \tilde{u}_L = \tilde{I}_L Z_L$

№ 12.42



$\mathcal{E} = \tilde{u}_1 = \varphi_C - \varphi_A$

$\tilde{u}_2 = \varphi_B - \varphi_D$

$\tilde{u}_1 - \tilde{u}_2 = ? = \mathcal{E} - (\varphi_B - \varphi_D)$

$\frac{\mathcal{E}}{Z} = \tilde{I}_1 + \tilde{I}_2 = \frac{\varphi_B - \varphi_A}{r} + \frac{\varphi_C - \varphi_D}{r}$

$\tilde{u}_2 = \varphi_B - \varphi_D = \frac{r\mathcal{E}}{Z} - \mathcal{E} = \mathcal{E}\left(\frac{r}{Z} - 1\right)$

Нар Z - обычное сопр-е

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{r + \frac{1}{i\omega C}} + \frac{1}{r + i\omega L} =$$

$$= \frac{i\omega C r - \omega^2 C L + 1 + i\omega C r}{r + i\omega C r^2 - \omega^2 L C r + i\omega L} = \frac{(1 - \omega^2 C L) + i \cdot 2\omega C r}{r(1 - \omega^2 C L) + i(\omega L + \omega C r^2)} = \frac{b}{a}$$

$$\tilde{U}_2 = \mathcal{E}\left(\frac{r - \frac{a}{b}}{\frac{a}{b}}\right) = \left(\frac{rb - a}{a}\right)\mathcal{E} =$$

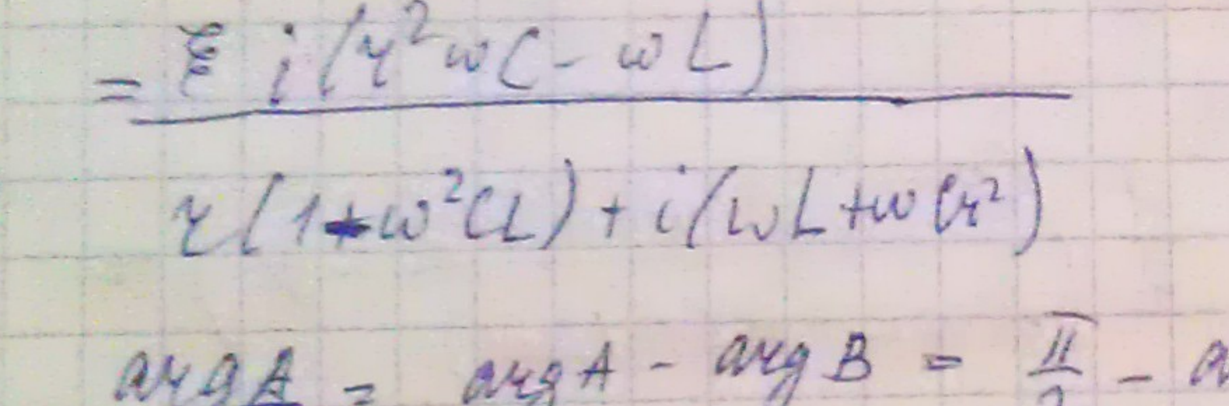
$$= \mathcal{E}\,\frac{r(1 - \omega^2 LC) + i r^2 2\omega C - r(1 - \omega^2 CL) - i\omega(L + Cr^2)}{r(1 - \omega^2 CL) + i(\omega L + \omega C r^2)} =$$

$$= \frac{\mathcal{E}\, i(r^2 \omega C - \omega L)}{r(1 + \omega^2 CL) + i(\omega L + \omega C r^2)}$$

$$\arg\frac{A}{B} = \arg A - \arg B = \frac{\pi}{2} - \operatorname{arctg}\frac{\omega L + \omega C r^2}{r(1 - \omega^2 CL)} =$$

$$= \operatorname{arctg}\frac{r(1 - \omega^2 CL)}{\omega L + \omega C r^2} = \operatorname{arctg}\frac{r(1 + \omega^2 CL)}{\omega L + \omega C r^2}$$

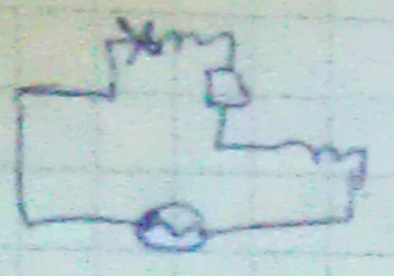
№ 45 Дано: U_0, R, C, L Надо: $\tilde{U}, \tilde{I}$



$\langle P \rangle = U_\varphi I_\varphi \cos\varphi$

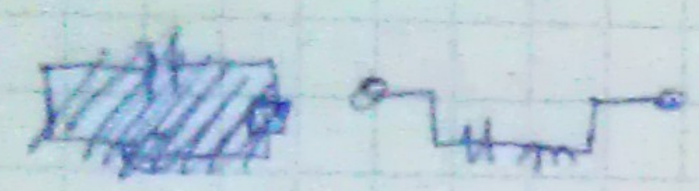
$\cos\varphi = 1$, $\varphi = 0$ } на R

U_m, I_m - амплитуды $U_{ср} = \frac{U_m}{\sqrt{2}}$, $I_{ср} = \frac{I_m}{\sqrt{2}}$

1) 

$$2i\omega L\cdot \tilde{I}_1 - \tilde{I}R = \mathcal{E} \quad (1)$$

$$i\omega L\tilde{I}_1 + \tilde{I}_2\cdot\frac{1}{i\omega C} = \mathcal{E} \quad (2)$$

2) 

$$\tilde{I}_2 = \tilde{I} + \tilde{I}_1 \quad (3)$$

Надо $\tilde{I}$, $\tilde{U} = \tilde{I}R$

$(3) \Rightarrow \tilde{I}_2 = i\omega C\left(\mathcal{E} - \tilde{I}_1 i\omega L\right) \Rightarrow$ это и (3) в (1)

$$\Rightarrow 2i\omega L\tilde{I}_1 - R\left(i\omega C\mathcal{E} + \tilde{I}_1\omega^2 CL - \tilde{I}_1\right) = \mathcal{E}$$

$$\Rightarrow \tilde{I}_1 = \frac{\mathcal{E}(1+i\omega C)}{2i\omega L - RCL\omega^2 + R}$$

$$(1)\Rightarrow \tilde{I} = \frac{2i\omega L\tilde{I}_1 - \mathcal{E}}{R} = \frac{1}{R}\left(\frac{2\pi i\omega L\mathcal{E}(1+iRC\omega)}{2i\omega L + R(1-CL\omega^2)} - \mathcal{E}\right) =$$

$$= \frac{-\mathcal{E}(\omega^2 CL + 1)}{2i\omega L + R(1-CL\omega^2)} = \underbrace{I_0 e^{i\varphi}}_{\text{амплитуда}}$$

ДЗ: 12.45, 37, 38, 32, 44, 48

Теория 71-82

↓

не будет 83, 84, 85, 86, 87, 88, 89

Волновое уравнение

$\rho = 0, \quad j = 0$

$$(rot \vec{H})_t' = \left(\frac{1}{c} \frac{\partial \vec{E}}{\partial t}\right)_t'$$

$$rot \frac{\partial \vec{H}}{\partial t} = \frac{1}{c} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\frac{\partial \vec{H}}{\partial t} = c \cdot rot \vec{E}$$

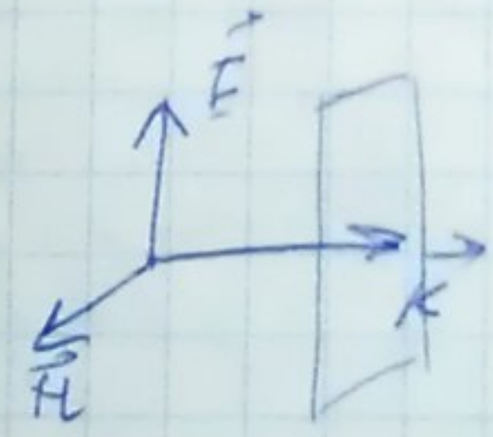
$$rot\, rot \vec{E} = \cancel{grad\, div \vec{E}}^{\,0} - \Delta E$$

$$\Delta E = -\frac{1}{c} \frac{\partial^2 E}{\partial t^2}$$

Волновое число $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

$$c = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu \mu_0}}, \quad c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

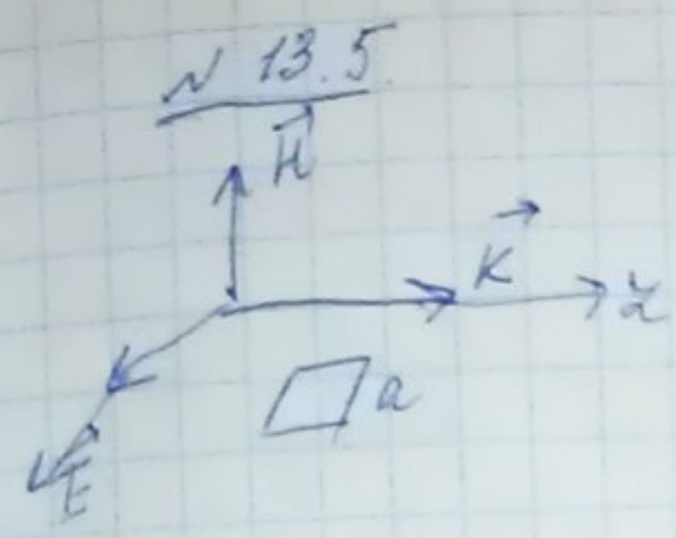
Излучает волну только ускоренный заряд



В плоской волне фронт $\perp \vec{k}$

У точ. заряда фронт - сфера

№ 13.5



Дано λ, a, E_0. ξ - ?

$$\sqrt{\varepsilon\varepsilon_0}\,[\vec{n}, \vec{E}] = \sqrt{\mu\mu_0}\,\vec{H}$$

$$\vec{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\vec{E}$$

$$H = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,E_0 \cos(\omega t - kz)$$

$$\xi = -\frac{\partial \Phi}{\partial t}$$

$$\Phi = \int B\,dS = \int_0^a \int_0^a \mu_0 \sqrt{\frac{\varepsilon_0}{\mu_0}}\,E_0 \cos(\omega t - kz)\,dx\,dz =$$

$$= \sqrt{\varepsilon_0\mu_0}\,E_0\,a\left(-\frac{1}{k}\right)\sin(\omega t - kz)\Big|_0^a =$$

$$= \sqrt{\varepsilon_0\mu_0}\,E_0\,a\,\frac{1}{k}\left(\sin\omega t - \sin(\omega t - ka)\right) =$$

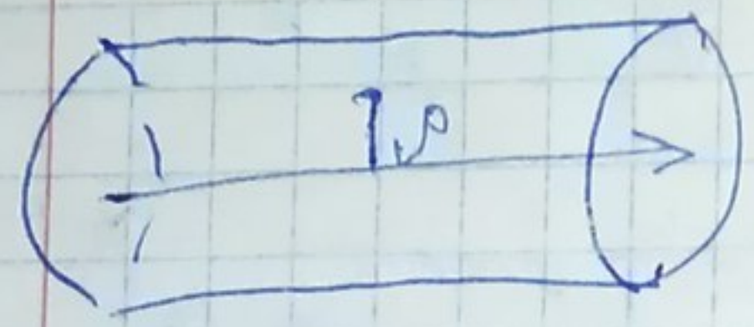
$$= 2\sin\frac{-ka}{2}\cos\frac{2\omega t + ka}{2}\cdot\left(-\frac{E_0 a \lambda}{c 2\pi}\right) =$$

$$= \frac{E_0 a \lambda}{\pi c}\sin\frac{ka}{2}\cos\frac{2\omega t + ka}{2}$$

$$\xi = -\frac{E_0 a \lambda}{\pi c}\,\omega \sin\frac{a\pi}{\lambda}\sin\left(\frac{2\pi c}{\lambda}t + \frac{\pi a}{\lambda}\right)$$

№ 13.3 $\quad \vec{E} = \vec{p}\,e^{-\frac{r^2}{a^2}}\cos(\omega t - kz)$

$< \int |\vec{\Pi}|\,ds >_T$ - ср. эн. в ед. времени



$$H = \sqrt{\frac{\varepsilon_0}{\mu_0}} E$$

$$\Pi = [\vec{E}, \vec{H}]$$

$$\oint \Pi ds = \iint \sqrt{\frac{\varepsilon_0}{\mu_0}} E^2 \rho \, d\varphi \, dz =$$

$$= \rho \iint \sqrt{\frac{\varepsilon_0}{\mu_0}} e^{-2\left(\frac{\rho}{\alpha}\right)^2} \cos^2(\omega t - kz) \, d\varphi \, dz$$