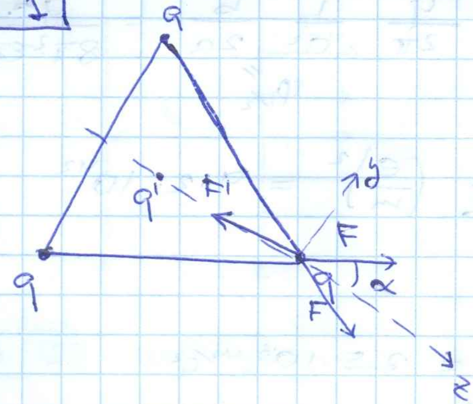
1.1

1



1) Заметим, каким бы ни был заряд q', сумма сил, действ. на него равна 0.

2) $R = \frac{2}{3} a \frac{\sqrt{3}}{2} = \frac{a\sqrt{3}}{3}$

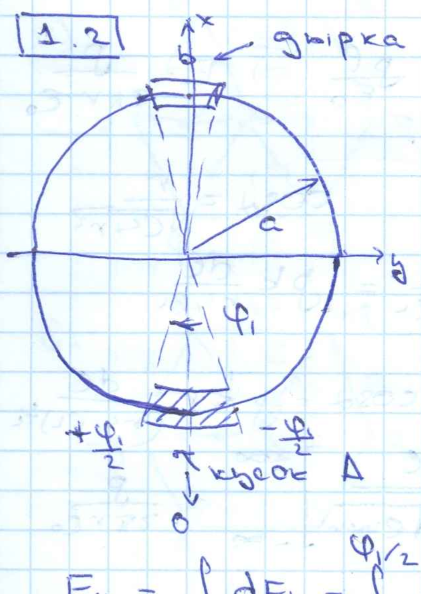
$F = \frac{kq^2}{a^2}$

$F' = \frac{kqq'}{R^2} = \frac{3kqq'}{a^2}$

$$\begin{cases} z: & F' + 2F\cos\alpha = 0 \\ y: & F\sin\alpha - F\sin\alpha = 0 \end{cases}$$

$$\frac{3kqq'}{a^2} + \frac{2kq^2}{a^2}\frac{\sqrt{3}}{2} = 0 \Rightarrow q' = -\frac{\sqrt{3}}{3} q$$

1.2



В силу симметрии, легко видеть, что напряжённость в центре кольца будет создаваться только куском A.

$2\pi a - b = (2\pi - \frac{b}{a}) a = \varphi_0 a$

$\varphi_1 = \frac{b}{a}$ $\qquad dq = \frac{q}{\varphi_0} d\varphi$

$dE = \frac{k\,dq}{a^2} = \frac{kq}{\varphi_0 a^2} d\varphi$

$dE_x = dE\cos\varphi \qquad dE_y = dE\sin\varphi$

$$E_y = \int dE_y = \int_{-\varphi_1/2}^{\varphi_1/2} \frac{kq}{\varphi_0 a^2} \sin\varphi\, d\varphi = 0.$$

$$E_x = \int dE_x = \int_{-\varphi_1/2}^{\varphi_1/2} \frac{kq}{\varphi_0 a^2} \cos\varphi\, d\varphi = \frac{kq}{\varphi_0 a^2} \sin\varphi \Big|_{-\varphi_1/2}^{+\varphi_1/2} =$$

$$= \frac{2kq}{\varphi_0 a^2} \sin\frac{\varphi_1}{2} \approx 2 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2\pi} \cdot \frac{1}{a^2} \cdot \underbrace{\frac{b}{2a}}_{\varphi_1/2} = \frac{qb}{8\pi^2 \varepsilon_0 a^3}$$

$\boxed{1.3}$

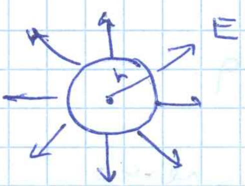
$$\frac{F_k}{F_r} = \frac{\frac{ke^2}{a^2}}{\frac{Gm_e^2}{a^2}} = \frac{1}{4\pi\varepsilon_0 G}\left(\frac{e}{m}\right)^2 = 4.2 \cdot 10^{42}$$

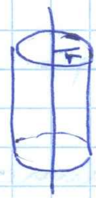
$\boxed{1.4}$

$$a = \frac{F_k}{m} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{ma^2} = 2.5 \cdot 10^{8}\ м/с^2$$

$\boxed{1.5}$ 1) Выведем напряжённость электр. поля нити на расстоянии r.

способ 1: (т. Гаусса)





$$\Phi = 2\pi r l \cdot E$$

$$q = \rho l$$

$$2\pi r l E = \frac{\rho l}{\varepsilon_0} \Rightarrow E = \frac{\rho}{2\pi r \varepsilon_0}$$

способ 2:

$$dq = \rho\, dl \qquad \cos\alpha = \frac{r}{\sqrt{l^2+r^2}}$$

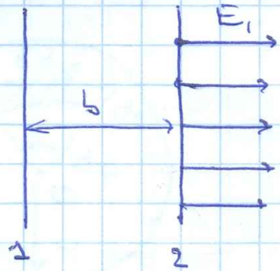
$$dE = \frac{k\,dq}{r^2+l^2} = \rho k \frac{dl}{r^2+l^2}$$

$$dE_z = dE\cos\alpha = \rho k r \frac{dl}{(r^2+l^2)^{3/2}}$$

$$E_z = \frac{\rho r}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{dl}{(r^2+l^2)^{3/2}} = \frac{\rho r}{4\pi\varepsilon_0}\left[\frac{l}{r^2\sqrt{l^2+r^2}}\right]_{-\infty}^{+\infty} = \frac{\rho}{2\pi r\varepsilon_0}$$

проекц. $E_x = 0$ в силу симметрии

г)

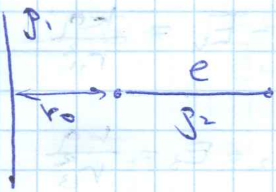


$$F = \rho l \cdot E_1$$

$$\frac{F}{l} = \frac{\rho^2}{2\pi b \varepsilon_0}$$

$\boxed{1.6}$ из задачи 1.5 имеем, что $E(l) = \dfrac{\rho_1}{2\pi l \varepsilon_0}$

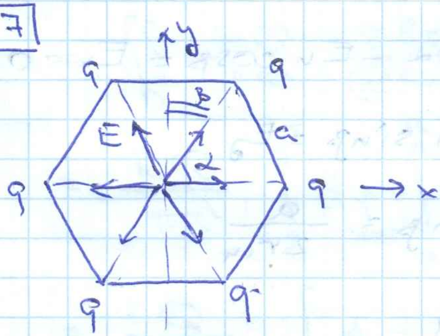
т.о.
$$F = \int E(l)\, dq = \int_{r_0}^{r_0+l} \frac{\rho_1}{2\pi l \varepsilon_0} \rho_2 dl =$$



$$= \frac{\rho_1 \rho_2}{2\pi\varepsilon_0} \ln l \Big|_{r_0}^{r_0+l} = \frac{\rho_1 \rho_2}{2\pi\varepsilon_0} \ln\left(1 + \frac{l}{r_0}\right).$$

$\boxed{1.7}$

а)



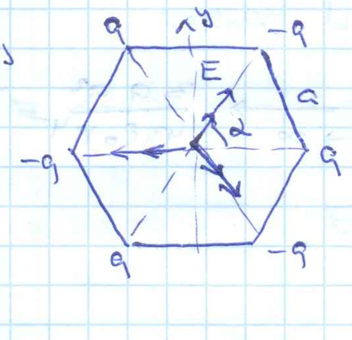
$$E = \frac{kq}{a^2}$$

x: $E + 2E\cos\alpha - 2E\cos\alpha - E = 0$

y: $2E\cos\beta - 2E\cos\beta = 0.$

$E_\Sigma = 0.$

б)



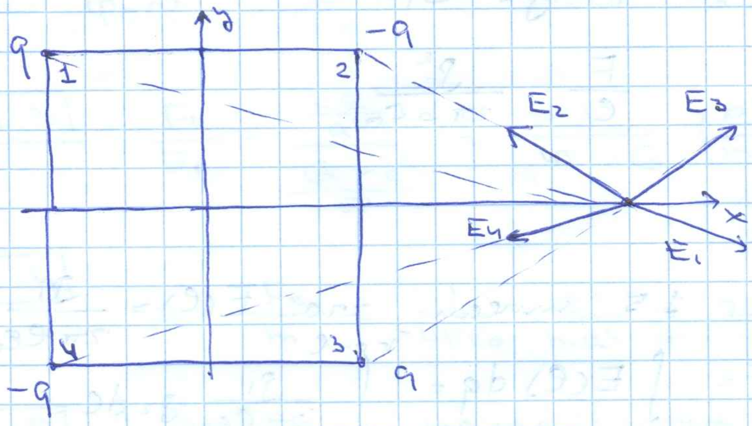
$$E = \frac{kq}{a^2}$$

x: $-2E + 4E \underbrace{\cos\alpha}_{=1/2} = 0$

y: $2E\sin\alpha - 2E\sin\alpha = 0.$

$E_\Sigma = 0.$

$\boxed{1.8}$



$$r_1 = r_4 = \sqrt{\frac{a^2}{4} + \left(\frac{a}{2} + x\right)^2} = r_{14} \qquad E_1 = E_4 = \frac{kq}{r_{14}^2}$$

$$r_2 = r_3 = \sqrt{\frac{a^2}{4} + \left(x - \frac{a}{2}\right)^2} = r_{23} \qquad E_2 = E_3 = \frac{kq}{r_{23}^2}$$

$$x: \quad E_1 \cos\beta + E_3 \cos\alpha - E_2 \cos\alpha - E_4 \cos\beta = E_x = 0$$

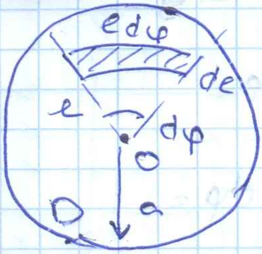
$$y: \quad (E_2 + E_3)\sin\alpha - (E_1 + E_4)\sin\beta = E_y$$

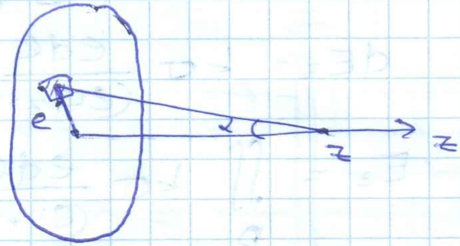
$$\sin\alpha = \frac{a}{2r_{23}} \qquad \sin\beta = \frac{a}{2r_{14}}$$

$$E_y = \frac{2kq}{r_{23}^2}\frac{a}{2r_{23}} - \frac{2kq}{r_{14}^2}\frac{a}{2r_{14}} =$$

$$= kqa\left(\frac{1}{r_{23}^3} - \frac{1}{r_{14}^3}\right) \xrightarrow[x \to \infty]{} \frac{3qa^2}{4\pi\varepsilon_0 r^4}$$

$\boxed{1.9}$





$$dq = \frac{Q}{\pi a^2}\,\rho\, d\varphi\, d\rho \qquad \cos\alpha = \frac{z}{\sqrt{z^2+\rho^2}}$$

$$dE = \frac{k\,dq}{\rho^2+z^2} = \frac{kQ}{\pi a^2}\,\frac{\rho\, d\rho}{\rho^2+z^2}\,d\varphi$$

$$dE_z = dE\cos\alpha = \frac{kQ}{\pi a^2}\,\frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\,d\varphi$$

$$E_z = \iint_D \frac{kQ}{\pi a^2}\,\frac{z\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\,d\varphi =$$

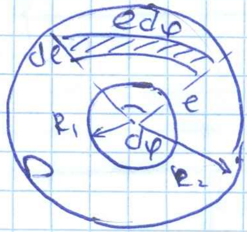
$$= \frac{kQ}{\pi a^2}\int_0^{2\pi} d\varphi \int_0^a \frac{z\rho\, d\rho}{(\rho^2+z^2)^{3/2}} = \frac{Q}{2\pi\varepsilon_0 a^2}\cdot\frac{z}{z}\left[-\frac{z}{\sqrt{\rho^2+z^2}}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 a^2}\left[1 - \frac{z}{\sqrt{a^2+z^2}}\right]$$

$z \ll a$: $\quad E_z = \dfrac{Q}{2\pi\varepsilon_0 a^2}$

$z \gg a$: $\quad E_z = \dfrac{Q}{4\pi\varepsilon_0 z^2}$

1.10 по аналогии с задачей 1.9



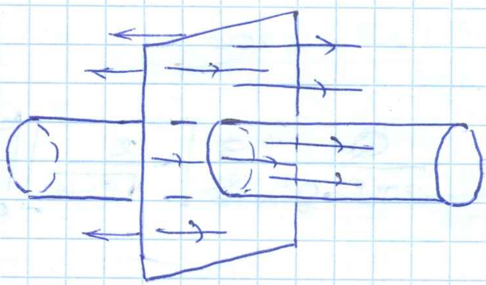
$$dE_z = k\sigma \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\, z\, d\varphi$$

$$E_z = \iint\limits_D k\sigma \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\, z\, d\varphi =$$

$$= k\sigma \int_0^{2\pi} d\varphi \int_{R_1}^{R_2} \frac{z\rho\, d\rho}{(\rho^2+z^2)^{3/2}} =$$

$$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{z}{\sqrt{\rho^2+z^2}}\right]_{R_1}^{R_2} = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{\sqrt{R_1^2+z^2}} - \frac{z}{\sqrt{R_2^2+z^2}}\right]$$

1.11 Способ 1: (т. Гаусса)

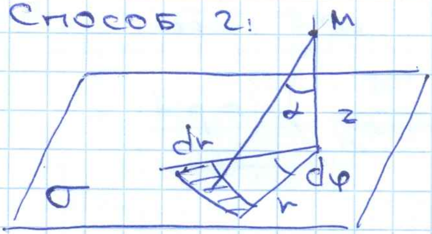


$$\Phi = 2SE$$

$$q = \sigma S$$

$$2SE = \frac{\sigma S}{\varepsilon_0} \rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Способ 2:



$$dq = \sigma r\, d\varphi\, dr$$

$$\cos\alpha = \frac{z}{\sqrt{z^2+r^2}}$$

$$dE = \frac{k\, dq}{r^2+z^2} = \sigma k \frac{r\, dr}{z^2+r^2}\, d\varphi$$

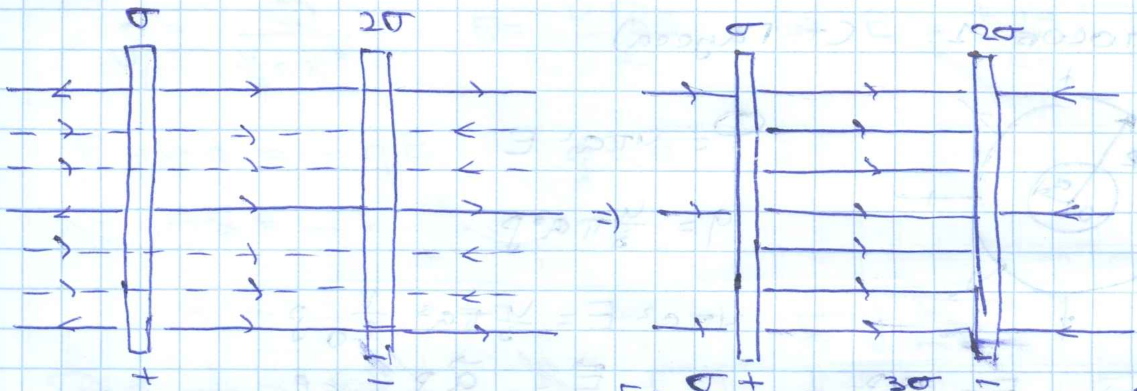
$$dE_z = dE\cos\alpha = \sigma k z \frac{r\, dr}{(z^2+r^2)^{3/2}}\, d\varphi$$

$$E_z = \sigma k \int_0^{2\pi} d\varphi \int_0^{\infty} \frac{r\, dr}{(z^2+r^2)^{3/2}}$$

$$E_z = \pi\sigma k \cdot z \int_0^{+\infty} \frac{dr^2}{(z^2+r^2)^{3/2}} = \pi\sigma k z \left[-\frac{2}{\sqrt{r^2+z^2}}\right]_0^{\infty} =$$

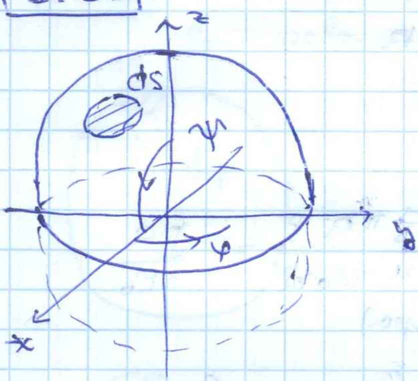
$$= \frac{\sigma}{2\varepsilon_0}$$

$\boxed{1.12}$



$E_1 = \frac{\sigma}{2\varepsilon_0}$ $\quad E_2 = \frac{3\sigma}{2\varepsilon_0}$ $\quad E_3 = \frac{\sigma}{2\varepsilon_0}$

$\boxed{1.13}$



$$dE = \frac{k\,dq}{R^2} = \frac{k\sigma}{R^2}\,dS$$

$$dE_z = \frac{k\sigma}{R^2}\cos\psi\,dS$$

$$dE_x = \frac{k\sigma}{R^2}\sin\psi\cos\varphi\,dS$$

$$dE_y = \frac{k\sigma}{R^2}\sin\psi\sin\varphi\,dS$$

$$E_z = \frac{k\sigma}{R^2}\iint_S \cos\psi\,dS = \frac{k\sigma}{R^2}\iint_S \cos\psi\,R^2\sin\psi\,d\varphi\,d\psi$$

$$= -k\sigma\int_0^{2\pi} d\varphi\int_0^{\pi/2}\cos\psi\,d\cos\psi = -\frac{2\pi\sigma}{4\pi\varepsilon_0}\cdot\frac{1}{2}\left[\cos^2\frac{\pi}{2} - \cos^2 0\right]$$

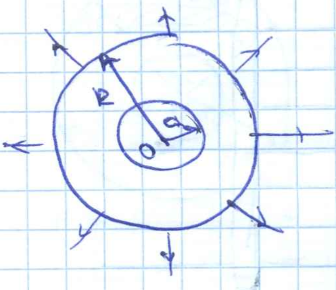
$$= \frac{\sigma}{4\varepsilon_0}$$

$$E_y = \iint_S dE_y = 0 \qquad E_x = \iint_S dE_x = 0.$$

$$\vec{E} = \vec{k}\,E_z, \qquad k = (0,0,1).$$

1.14 1) Рассмотрим шар без полости:

способ 1. (т. Гаусса)



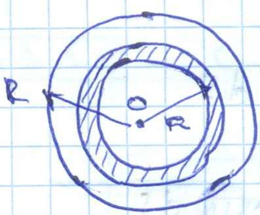
$$\Phi = 4\pi a^2 E$$

$$q = \frac{4}{3}\pi a^3 \rho$$

$$4\pi a^2 E = \frac{4}{3}\pi a^3 \frac{1}{\varepsilon_0}\rho$$

$\Rightarrow$ $E = \frac{a\rho}{3\varepsilon_0}$ или $\vec{E} = \frac{\vec{a}\rho}{3\varepsilon_0}$ (внутри шара)

способ 2:



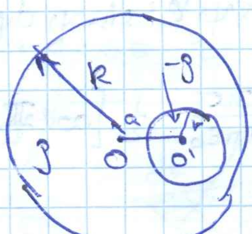
$$dq = \rho \cdot 4\pi a^2\, da$$

$$dE = \frac{k\,dq}{a^2}$$

$$E = \frac{Q'}{4\pi\varepsilon_0 a^2} = \frac{\frac{4}{3}\pi a^3 \rho}{4\pi\varepsilon_0 a^2} = \frac{\rho a}{3\varepsilon_0}$$

или $\vec{E} = \frac{\vec{a}\rho}{3\varepsilon_0}$ (внутри шара)

2) рассмотрим суперпозицию шаров



$$O': \vec{E}_б + \vec{E}_п = \frac{\rho\vec{a}}{3\varepsilon_0} - \frac{\rho\vec{0}}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}.$$

напряж. от большого шара (без полости)

напряж. от „полости"

$\boxed{1.15}$ из задачи 1.14 имеем:

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \qquad E = \frac{kQa}{R^3} \Rightarrow E = \frac{kqa}{R^3}$$

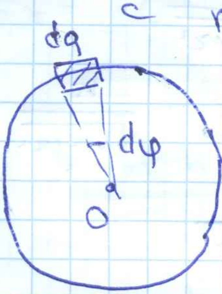
$$F = ma = -qE \qquad q = e$$

$$m\ddot{x} = -k\frac{q^2}{R^3}x$$

$$\ddot{x} + \frac{kq^2}{mR^3}x = 0 \Rightarrow \omega = \frac{q}{R}\sqrt{\frac{k}{mR}}$$

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{q}{4\pi R}\sqrt{\frac{1}{\pi\varepsilon_0 R}}$$

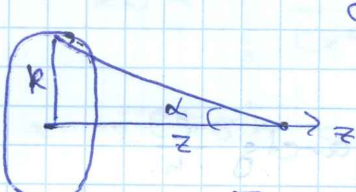
$\boxed{1.16}$ 1) Найдём напряжённость от кольца с равномерно распредел. зарядом q_0.



$$dq = \frac{q_0}{2\pi}d\varphi$$

$$dE = k\,dq\,\frac{1}{z^2+R^2}$$

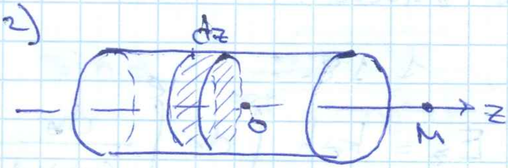
$$\cos\alpha = \frac{z}{\sqrt{z^2+R^2}}$$



$$dE_z = dE\cos\alpha = k\frac{q_0}{2\pi}d\varphi\frac{z}{(z^2+R^2)^{3/2}}$$

$$E_z = \int_0^{2\pi} dE_z = k\frac{q_0 z}{(z^2+R^2)^{3/2}}$$

2)



Положим: $q_0 = dq = \sigma\, dz\, 2\pi R$

$E_z = dE_z$

O – середина цилиндра

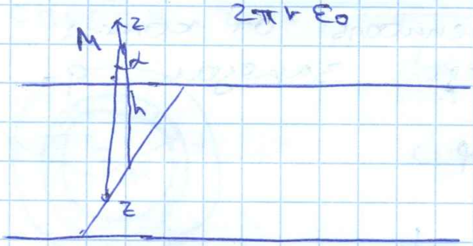
$$E_z^{n} = \int_{a-\ell}^{a+\ell} dE_z = 2\pi k R\sigma \int_{a-\ell}^{a+\ell} \frac{z\,dz}{(z^2+R^2)^{3/2}} =$$

$$= \frac{R\sigma}{2\pi\varepsilon_0}\left[-\frac{1}{\sqrt{R^2+z^2}}\right]\Bigg|_{a-\ell}^{a+\ell} =$$

$$= \frac{R\sigma}{2\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+(a-\ell)^2}} - \frac{1}{\sqrt{R^2+(a+\ell)^2}}\right]$$

$\boxed{1.17}$ из задачи 1.5 имеем:

$$dE = \frac{\rho}{2\pi r\varepsilon_0}; \quad \rho = \sigma\,dz; \quad r = \sqrt{h^2+z^2}$$



$$\cos\alpha = \frac{h}{\sqrt{h^2+z^2}}$$

$$dE_z = \frac{\sigma\,dz}{2\pi\sqrt{h^2+z^2}\,\varepsilon_0}\cdot\frac{h}{\sqrt{h^2+z^2}} =$$

$$= \frac{\sigma}{2\pi\varepsilon_0}\,\frac{h\,dz}{h^2+z^2}$$

$$E_z = \int_{-\ell}^{\ell} dE_z = \frac{\sigma}{2\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{h\,dz}{h^2+z^2} =$$

$$= \frac{\sigma}{2\pi\varepsilon_0}\,\mathrm{arctg}\left(\frac{z}{h}\right)\Bigg|_{-\ell}^{\ell} = \frac{\sigma}{\pi\varepsilon_0}\,\mathrm{arctg}\,\frac{\ell}{h}.$$

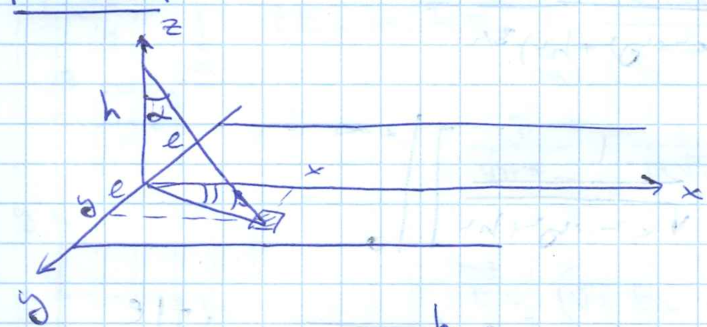
Возможно решение через разбиение полосы на прямоугольники, а не «нити».

Пример решения в задаче 1.18.:

$$dE_z = k\sigma h\,\frac{dx\,dy}{(x^2+y^2+h^2)^{3/2}} \qquad E_z = k\sigma h\int_{-\infty}^{+\infty}dx\int_{-\ell}^{\ell}\frac{dy}{(x^2+y^2+h^2)^{3/2}}$$

$$= \frac{\sigma}{\pi\varepsilon_0}\,\mathrm{arctg}\,\frac{\ell}{h}$$

$\boxed{1.18}$



$dq = \sigma\, dx\, dy$

$r_0 = \sqrt{x^2+y^2+h^2}$

$dE = \frac{k\,dq}{r_0^2} =$

$= k\sigma \frac{dx\,dy}{x^2+y^2+h^2}$

$\cos\alpha = \frac{h}{\sqrt{x^2+y^2+h^2}}$

$E_y = 0$ (в силу симметрии)

$$dE_z = k\sigma h \frac{dx\,dy}{(x^2+y^2+h^2)^{3/2}}$$

$$E_z = k\sigma h \int_0^{+\infty} dx \int_{-l}^{l} \frac{dy}{(x^2+y^2+h^2)^{3/2}} =$$

$$= k\sigma h \int_0^{+\infty} dx \left[\frac{y}{(x^2+h^2)\sqrt{x^2+y^2+h^2}}\right]\Bigg|_{-l}^{l} =$$

$$= k\sigma h \int_0^{+\infty} \frac{2l\,dx}{(x^2+h^2)\sqrt{x^2+l^2+h^2}} =$$

$$= 2lk\sigma h \operatorname{arctg}\left(\frac{lx}{h\sqrt{x^2+h^2+l^2}}\right)\Bigg|_0^{+\infty} \frac{1}{hl} =$$

$$= \frac{\sigma}{2\pi\varepsilon_0} \operatorname{arctg}\frac{l}{h}$$

$\sin\alpha = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+h^2}}$ $\qquad \cos\beta = \frac{x}{\sqrt{x^2+y^2}}$

$$dE_x = k\sigma \frac{x\,dx\,dy}{(x^2+y^2+h^2)^{3/2}}$$

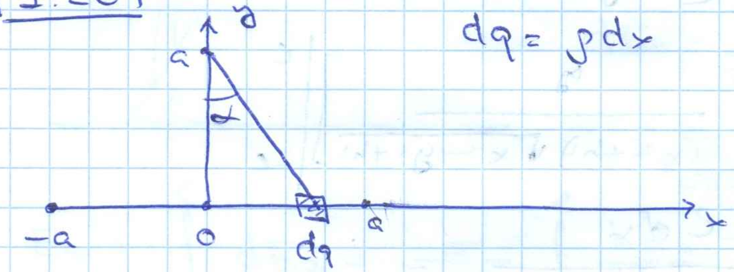
$$E_x = k\sigma \int_{-\ell}^{\ell} dy \int_0^{+\infty} \frac{x\,dx}{(x^2+y^2+h^2)^{3/2}} =$$

$$= k\sigma \int_{-\ell}^{\ell} dy \left[-\frac{1}{\sqrt{x^2+y^2+h^2}} \right]\Bigg|_0^{+\infty} =$$

$$= k\sigma \int_{-\ell}^{\ell} \frac{dy}{\sqrt{y^2+h^2}} = k\sigma \left[\ln\left(\sqrt{h^2+y^2}+y\right)\right]\Bigg|_{-\ell}^{\ell} =$$

$$= k\sigma \ln \frac{\sqrt{h^2+\ell^2}+\ell}{\sqrt{h^2+\ell^2}-\ell} \quad ; \quad k = \frac{1}{4\pi\varepsilon_0}$$

$\boxed{1.20}$



$$dq = \rho\, dx \qquad \cos\alpha = \frac{a}{\sqrt{a^2+x^2}}$$

$$\sin\alpha = \frac{x}{\sqrt{a^2+x^2}}$$

$$dE = \frac{k\,dq}{x^2+a^2} = k\rho \frac{dx}{x^2+a^2}$$

$$dE_y = dE\cos\alpha = k\rho a \frac{dx}{(x^2+a^2)^{3/2}}$$

$$dE_x = dE\sin\alpha = k\rho \frac{x\,dx}{(x^2+a^2)^{3/2}}$$

$$E_y = \int_{-a}^{+\infty} dE_y = k\rho a \int_{-a}^{+\infty} \frac{dx}{(x^2+a^2)^{3/2}} =$$

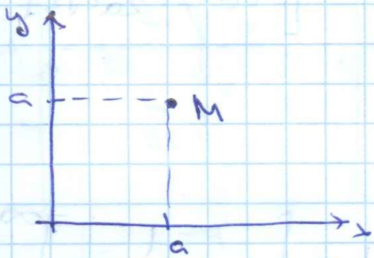
$$= k\rho a \left[\frac{x}{\sqrt{x^2+a^2}\; a^2} \right]_{-a}^{\infty} = k\rho a \left[\frac{1}{a^2} + \frac{1}{a^2\sqrt{2}} \right] =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\sqrt{2}+1}{a\sqrt{2}}$$

$$E_x = \int_a^{+\infty} dE_x = k\rho \int_a^{+\infty} \frac{x\,dx}{(x^2+a^2)^{3/2}} = k\rho\left[-\frac{1}{\sqrt{x^2+a^2}}\right]\Bigg|_a^{\infty} =$$

в силу симметрии

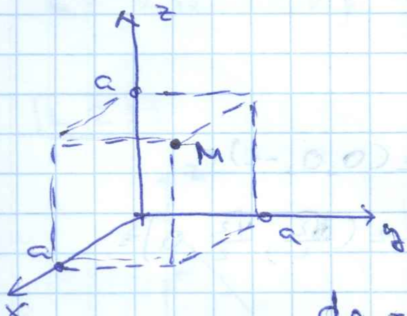
$$= \frac{\rho}{4\pi\varepsilon_0}\,\frac{1}{a\sqrt{2}}$$

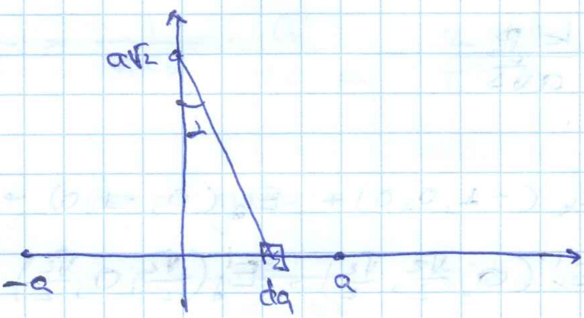


$$\vec{E} = (\sqrt{2}\,E_x - \sqrt{2}\,E_y)\,\vec{e} = \frac{\sqrt{2}\rho}{4\pi\varepsilon_0 a}\,\vec{e}$$

$$\vec{e} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$

$\boxed{1.21}$





$$dq = \rho\,dx$$

$$dE = \frac{k\,dq}{x^2+2a^2} \qquad \cos\alpha = \frac{a\sqrt{2}}{\sqrt{x^2+2a^2}} \qquad \sin\alpha = \frac{x}{\sqrt{x^2+2a^2}}$$

$$dE'_x = k\rho a\sqrt{2}\,\frac{dx}{(x^2+2a^2)^{3/2}}$$

$$dE_x = k\rho\,\frac{x\,dx}{(x^2+2a^2)^{3/2}}$$

$$E_x' = \int_{-a}^{+\infty} dE_x' = k\rho a\sqrt{2} \int_{-a}^{+\infty} \frac{dx}{(x^2+2a^2)^{3/2}} =$$

$$= k\rho a\sqrt{2} \left[\frac{x}{2a^2\sqrt{x^2+2a^2}} \right]_{-a}^{\infty} = k\rho a\sqrt{2} \left[\frac{1}{2a^2} - \frac{1}{2a^2\sqrt{3}} \right] =$$

$$= \frac{k\rho}{a\sqrt{3}\sqrt{2}} (\sqrt{3}+1)$$

$$E_x = \int_{a}^{+\infty} dE_x = k\rho \int_{a}^{+\infty} \frac{x\,dx}{(x^2+2a^2)^{3/2}} = k\rho \left[-\frac{1}{\sqrt{2a^2+x^2}} \right]_{a}^{\infty} =$$

$$= \frac{k\rho}{a\sqrt{3}}$$

$$\vec{E} = E_x(-1,0,0) + E_y(0,-1,0) + E_z(0,0,-1) +$$

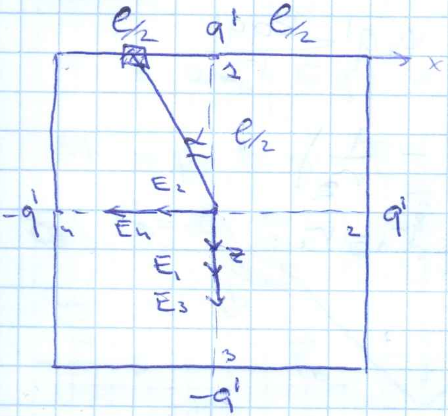
$$+ E_x'\left(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) + E_y'\left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right) + E_z'\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right) =$$

$$= E_x(-1,-1,-1) + E_x'(\sqrt{2},\sqrt{2},\sqrt{2}) =$$

$$= -\sqrt{3}E_x\vec{e} + \sqrt{2}\sqrt{3}E_x'\vec{e} = \left[-\sqrt{3}\frac{k\rho}{a\sqrt{3}} + \sqrt{2}\sqrt{3}\frac{k\rho}{a\sqrt{3}\sqrt{2}}(\sqrt{3}+1) \right]\vec{e}$$

$$= \frac{\sqrt{3}k\rho}{a}\vec{e}, \quad \vec{e} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

$\boxed{1.19}$



$a \equiv \ell/2 \qquad \rho = \frac{q'}{\ell} \qquad q' \equiv \frac{q}{2}$

$dq' = \rho\, dx \qquad dE = \frac{k\, dq'}{x^2+a^2} \qquad \cos\alpha = \frac{a}{\sqrt{x^2+a^2}}$

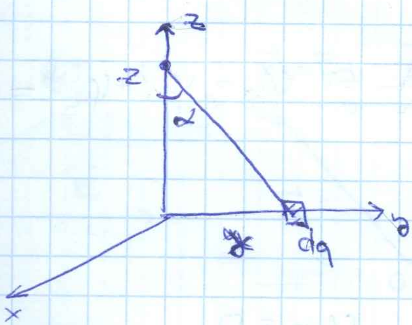
$$dE_z = k\rho a \frac{dx}{(x^2+a^2)^{3/2}}$$

$$E_z = k\rho a \int_{-a}^{a} \frac{dx}{(x^2+a^2)^{3/2}} =$$

$$= k\rho a \left[\frac{x}{a^2\sqrt{x^2+a^2}}\right]\Bigg|_{-a}^{a} = 2k\rho \frac{1}{a\sqrt{2}}$$

$$E = 2\sqrt{2}\cdot 2k\rho \frac{1}{a\sqrt{2}} = \frac{1}{\pi\varepsilon_0}\cdot\frac{q'}{2a\cdot a} = \frac{2q'}{\pi\varepsilon_0\ell^2} = \frac{q}{\pi\varepsilon_0 \ell^2}$$

$\boxed{1.21}$



$$dq = \rho\, dx$$

$\cos\alpha = \frac{z}{\sqrt{x^2+z^2}} \qquad \sin\alpha = \frac{x}{\sqrt{x^2+z^2}}$

$$dE = \frac{k\, dq}{x^2+z^2}$$

$$dE_z = k\rho z \frac{dx}{(x^2+z^2)^{3/2}}$$

$$dE_y = k\rho \frac{x\, dx}{(x^2+z^2)^{3/2}}$$

$$E_z = \int_0^{+\infty} k\rho z \frac{dx}{(x^2+z^2)^{3/2}} = k\rho z \left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]\Bigg|_0^{+\infty} = \frac{k\rho}{z} = E_0$$

$$E_y = \int_0^{+\infty} k\rho \frac{x\, dx}{(x^2+z^2)^{3/2}} = k\rho \left[-\frac{1}{\sqrt{x^2+z^2}}\right]\Bigg|_0^{+\infty} = \frac{k\rho}{z} = E_0$$

$$\vec{E} = 2E_z(0,0,1) + E_x(-1,0,0) + E_y(0,-1,0) =$$

$$= E_0(-1,-1,2) = \frac{\sqrt{6}\,\rho}{4\pi\varepsilon_0 z}\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$\boxed{2}$ $\boxed{2.1}$ В силу потенциальн. поля

$$W_1 = \varphi_1 q' = \frac{kqq'}{r_1} \qquad W_2 = \varphi_2 q' = \frac{kqq'}{r_2}$$

$$A_{12} = W_1 - W_2 = \frac{qq'}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

В случае б) $A = 0$, т.к. $r_1 = r_2$.

$\boxed{2.2}$

$$\varphi_0 = 2\sum_{\alpha=1}^{\infty} \frac{k(-1)^\alpha q}{\alpha a} = \frac{2kq}{a}\sum_{\alpha=1}^{\infty}\frac{(-1)^\alpha}{\alpha} =$$

$$= -\frac{q}{2\pi\varepsilon_0 a}\ln 2.$$

$\boxed{2.3}$ а) $\varphi_0^{(1)} = \frac{kq}{a/2}$ $\quad \varphi_0^{(2)} = -\frac{kq}{a/2} \Rightarrow \varphi_0 = \varphi_0^{(1)} + \varphi_0^{(2)} = 0$

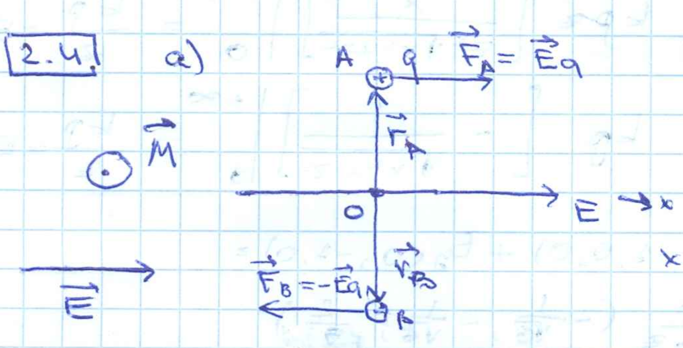
$$\varphi_\infty = 0$$

тогда $A = q'(\varphi_0 - \varphi_\infty) = 0$.

б) $\varphi_0^{(1)} = \varphi_0^{(2)} = \frac{kq}{a/2}$ $\quad \varphi_0 = \frac{kq}{a}$ $\quad \varphi_\infty = 0$

$$A = q'(\varphi_0 - \varphi_\infty) = +\frac{kqq'}{a}.$$

$\boxed{2.4}$ а)



$$W = q(\varphi_A - \varphi_B) =$$
$$= q\int_A^B \vec E\, d\vec r = -q\vec E\vec l = 0.$$

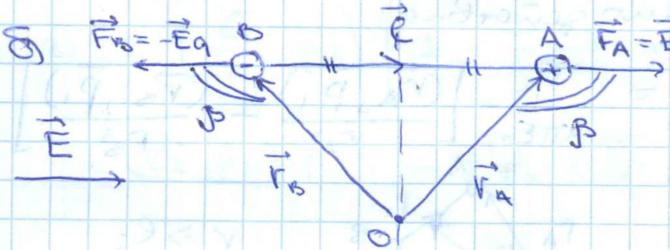
$$x:\ F_A - F_B = Eq - Eq = 0.$$
$$\vec F_\Sigma = 0$$

$$\vec M = [\vec r_A \times \vec F_A] + [\underset{-\vec r_A}{\vec r_B} \times \underset{-\vec F_A}{\vec F_B}] = [\vec r_A \times \vec E q] + [\vec r_A \times \vec E q] =$$

$$= 2[\vec r_A \times \vec E q] = [\vec p_e \times \vec E]$$ (направлено на нас)

$$M = p_e E \sin\frac{\pi}{2} = p_e E \qquad (2\vec r_A = \vec \ell)$$

б)



$\longrightarrow x$

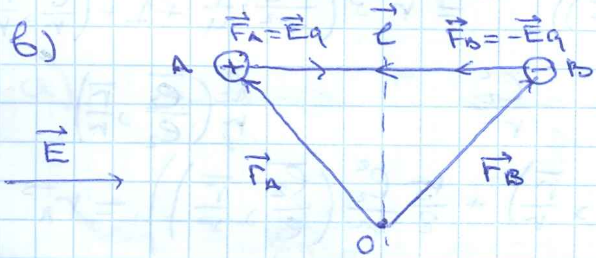
x: $F_A - F_B = Eq - Eq = 0$

$\vec F_\Sigma = 0$

$$\vec M = [\vec r_A \times \vec F_A] + [\vec r_B \times \underset{-\vec F_A}{\vec F_B}] = [\underset{\vec e}{\vec r_A - \vec r_B}, \vec F_A] = 0$$

$$W = q(\varphi_A - \varphi_B) = q\int_A^B \vec E\, d\vec r = -q\underset{\cos\alpha = \cos 0 = 1}{\vec E \vec \ell} = -Ep_e$$

в)



$\longrightarrow x$

x: $F_A - F_B = Eq - Eq = 0$

$\vec F_\Sigma = 0$

$$\vec M = [\vec r_A \times \vec F_A] + [\vec F_B \times \underset{-\vec F_A}{\vec F_B}] = [\underset{\vec e}{\vec r_A - \vec r_B}, \vec F_A] = 0$$

$$W = q(\varphi_A - \varphi_B) = q\int_A^B \vec E\, d\vec r = -q\underset{\cos\alpha = \cos\pi = -1}{\vec E \vec \ell} = Ep_e$$

$\boxed{2.5}$ Потенциал диполя в т. M (1-ый способ)

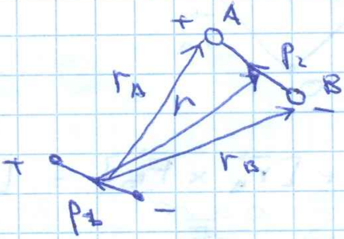
$$\varphi(M) = \frac{(\vec{r}, \vec{p}_e)}{4\pi\varepsilon_0 \cdot r^3}$$

Тогда энергия взаимодействия:

$$W_2 = q_2(\varphi_A - \varphi_B) = \frac{q_2}{4\pi\varepsilon_0}\left[\frac{(\vec{r}_A, \vec{p}_1)}{r_A^3} - \frac{(\vec{r}_B, \vec{p}_1)}{r_B^3}\right]$$

$$\vec{r}_A = \vec{r} + \frac{\vec{l}_2}{2}$$

$$\vec{r}_B = \vec{r} + \frac{\vec{l}_2}{2}$$



$r \gg l$

$$r_A^2 = \left(\frac{\bar{l}_2}{2} + \bar{r}\right)^2 = \frac{l_2^2}{4} + r^2 + (\bar{l}_2, \bar{r})$$

$$r_A^{-3} = \left(\frac{l_2^2}{4} + r^2 + (\bar{l}_2, \bar{r})\right)^{-3/2} = r^{-3}\left(\frac{l_2^2}{r^2}\cdot\frac{1}{4} + 1 + \left(\frac{\bar{l}}{r}, \frac{\bar{r}}{r}\right)\right)^{-3/2}$$

раскладываем до $\frac{l_2^2}{r^2}$ $\qquad \frac{l_2}{r}\left(\frac{\bar{l}_2}{l_2}, \frac{\bar{r}}{r}\right)$

$$= r^{-3}\left(1 - \frac{3}{8}\frac{l_2^2}{r^2} - \frac{3}{2}\frac{l_2}{r}\left(\frac{\bar{l}_2}{l_2}, \frac{\bar{r}}{r}\right) + \frac{15}{8}\frac{l_2^2}{r^2}\left(\frac{\bar{l}_2}{l_2}, \frac{\bar{r}}{r}\right)\right) = r_A^{-3}$$

$$r^{-3}\left(1 - \frac{3}{8}\frac{l_2^2}{r^2} + \frac{3}{2}\frac{l_2}{r}\left(\frac{\bar{l}_2}{l_2}, \frac{\bar{r}}{r}\right) + \frac{15}{8}\frac{l_2^2}{r^2}\left(\frac{\bar{l}_2}{l_2}, \frac{\bar{r}}{r}\right)\right) = r_B^{-3}$$

$$(\bar{r}_A, \bar{p}_1) = (\bar{r}, \bar{p}_1) + \left(\frac{\bar{l}_2}{2}, \bar{p}_1\right)$$

$$(\bar{r}_B, \bar{p}_1) = (\bar{r}, \bar{p}_1) - \left(\frac{\bar{l}_2}{2}, \bar{p}_1\right)$$

1) $(\bar r, \bar p_1)\left[-3\frac{\ell_2}{r}\left(\frac{\bar\ell_2}{\ell_2},\frac{\bar r}{r}\right)\right] r^{-3} = -3\frac{\ell_2}{r}\frac{(\bar r,\bar p_1)\left(\frac{\bar\ell_2}{\ell_2},\frac{\bar r}{r}\right)}{r^3} =$

$$= \frac{-3(\bar r,\bar p_1)(\bar\ell_2,\bar r)}{r^5}$$

2) $2\left(\frac{\bar\ell_2}{2},\bar p_1\right)\left[1-\frac{3}{8}\frac{\ell_2^2}{r^2}+\frac{15}{8}\frac{\ell_2^2}{r^2}\left(\frac{\bar\ell_2}{\ell_2},\frac{\bar r}{r}\right)\right]r^{-3} =$

$$= (\bar\ell_2,\bar p_1)\,r^{-3} - \frac{3}{8}\frac{\ell_2^3}{r^3}\left(\frac{\bar\ell_2}{\ell_2},\bar p_1\right)r^{-2} + \frac{15}{8}\frac{\ell_2^3}{r^3}\left(\frac{\bar\ell_2}{\ell_2},\frac{\bar r}{r}\right)\left(\frac{\bar\ell_2}{\ell_2},\bar p_1\right)r^{-2}$$

$$\approx (\bar\ell_2,\bar p_1)\,r^{-3}$$

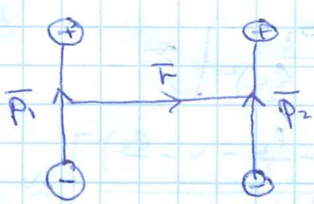
$$W_2 = \frac{q_2}{4\pi\varepsilon_0}\frac{(\bar\ell_2,\bar p_1)}{r^3} + \frac{q_2}{4\pi\varepsilon_0}\frac{-3(\bar r,\bar p_1)(\bar r,\bar\ell_2)}{r^5} =$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec p_1\cdot\vec p_2}{r^3} - 3\frac{(\vec r,\vec p_1)(\vec r,\vec p_2)}{r^5}\right]$$

a)

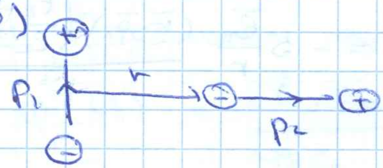
$$W_2 = -\frac{p_1 p_2}{2\pi\varepsilon_0 r^3} \qquad r \gg \ell_1;\ r \gg \ell_2$$

б)



$$W_2 = \frac{1}{4\pi\varepsilon_0}\frac{p_1 p_2}{r^3}$$

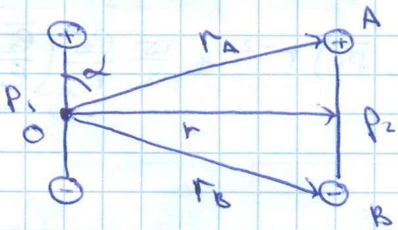
в)



$$W_2 = 0$$

2-ой способ:

б) $r_0 = \sqrt{r^2 + \frac{\ell_2^2}{4}} \approx r$



$$W_2 = q_2(\varphi_A - \varphi_B) =$$

$$= \frac{q_2}{4\pi\varepsilon_0}\left(\frac{\vec r_A \vec p_1}{r_A^3} - \frac{\vec r_B \vec p_1}{r_B^3}\right) =$$

$$= [r_A = r_B = r_0] \overset{1}{\underset{2}{=}}$$

$$\overset{1}{=} \frac{q_2}{4\pi\varepsilon_0}\left[\frac{r_0 p_1 \cos\alpha}{r_0^3} - \frac{r_0 p_1 \cos(\pi-\alpha)}{r_0^3}\right] =$$

$$= \frac{q_2}{4\pi\varepsilon_0}\left[\frac{2 r_0 p_1 \cos\alpha}{r_0^3}\right] = \frac{p_1 p_2}{4\pi\varepsilon_0 r_0^3} \approx \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}$$

$$\overset{2}{=} \frac{q_2}{4\pi\varepsilon_0 r_0^3}\,\vec p_1 \vec \ell_2 = \frac{p_1 p_2}{4\pi\varepsilon_0 r_0^3} \approx \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}$$

а)

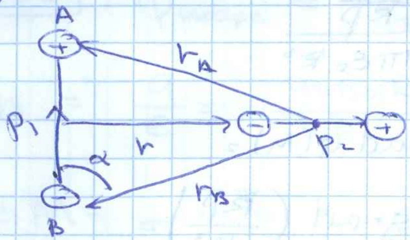
$$r = x$$

$$W_2 = \frac{q_2}{4\pi\varepsilon_0}\left[-\frac{p_1}{(x - \frac{\ell_2}{2})^2} + \frac{p_1}{(x + \frac{\ell_2}{2})^2}\right] \approx$$

$$= \frac{q_2 p_1}{4\pi\varepsilon_0}\left[\frac{(x + \frac{\ell_2}{2} + x - \frac{\ell_2}{2})(x - \frac{\ell_2}{2} - x - \frac{\ell_2}{2})}{(x^2 - \frac{\ell_2^2}{4})^2}\right] \approx$$

$$\approx \frac{q_2 p_1}{4\pi\varepsilon_0}\,\frac{2x\cdot(-\ell_2)}{x^4} \approx -\frac{p_1 p_2}{2\pi\varepsilon_0 x^3}$$

в)



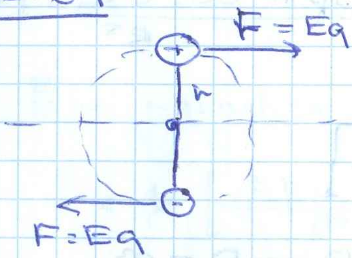
$$W_1 = q_1(\varphi_A - \varphi_B) =$$

$$= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{\overline{r_A}\,\overline{p_2}}{r_A^3} - \frac{\overline{r_B}\,\overline{p_2}}{r_B^3}\right) =$$

$$= [r_A = r_B = r_0] =$$

$$= \frac{q_1}{4\pi\varepsilon_0 \cdot r_0^3}\,\overline{p_2}(\overline{r_A} - \overline{r_B}) = \frac{\overline{p_1}\,\overline{p_2}}{4\pi\varepsilon_0 r_0^3} = 0.$$

$\boxed{2.6}$



$$J = mr^2 + mr^2 = 2mr^2, \quad r = \frac{l}{2}$$

$$J\varepsilon = 2Fr$$

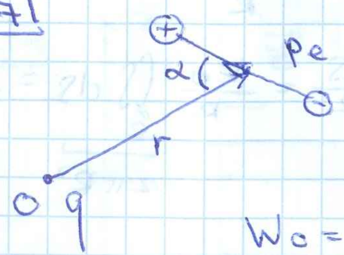
$$\varepsilon = \frac{\cancel{2}Eqr}{\cancel{2}mr^2} = \frac{Eq\frac{l}{2}}{m\frac{l^2}{4}} = \frac{2Ep_e}{ml^2} = \beta$$

из задачи 2.4 имеем: (ЗСЭ)

$$0 = \frac{J\omega^2}{2} - Ep_e \quad ; \quad p_e = ql = 2qr$$

$$\omega = \sqrt{\frac{2Ep_e}{J}} = \frac{2}{l}\sqrt{\frac{p_eE}{m}}$$

$\boxed{2.7}$

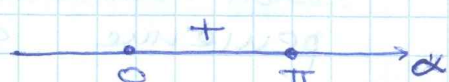


$$W = \varphi(O)q = q\frac{(-\overline{r}\,\overline{p_e})}{4\pi\varepsilon_0 r^3} =$$

$$\alpha \in [0, \pi] \qquad = \frac{-qrp_e\cos\alpha}{4\pi\varepsilon_0 r^3}$$

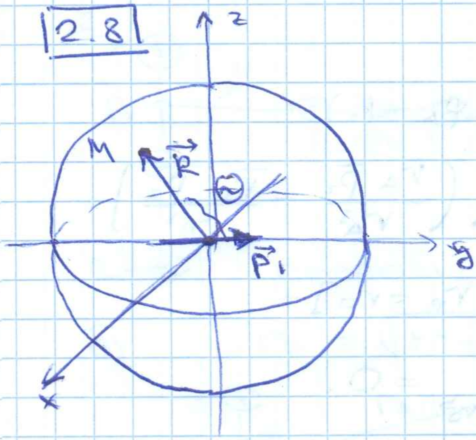
$$W_0 = 0 = \frac{-qrp_e\cos\alpha}{4\pi\varepsilon_0 r^3} \Rightarrow \cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$$

$$W' = \frac{qrp_e}{4\pi\varepsilon_0 r^3}(+\sin\alpha)$$



$W_{min} \sim \alpha = 0 \qquad W_{max} \sim \alpha = \pi.$

2.8



$$\varphi(M) = \frac{\bar{R}\bar{p}}{4\pi\varepsilon_0 R^3}$$

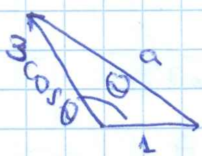
$$\bar{E} = -\operatorname{grad}\varphi(M) =$$

$$= -k \operatorname{grad}\left(\frac{\bar{R}\bar{p}}{R^3}\right) =$$

$$= k\left[\frac{3(\bar{p}\bar{R})\bar{R}}{R^5} - \frac{\bar{p}}{R^3}\right] =$$

$$= k\left[\frac{3pR\cos\Theta\,\bar{R} - R^2\bar{p}}{R^5}\right] = k\left[\frac{3pR^2\cos\Theta\,\bar{e}_1 - pR^2\bar{e}_2}{R^5}\right]$$

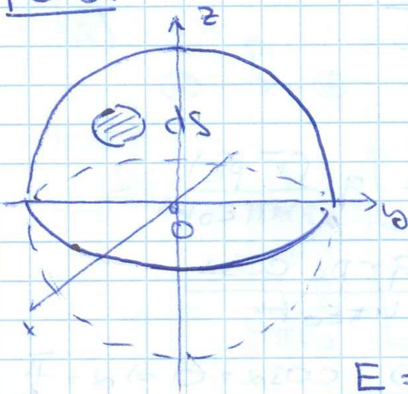
$$= \frac{p}{4\pi\varepsilon_0}\frac{1}{R^3}\left[3\cos\Theta\,\bar{e}_1 - \bar{e}_2\right]$$



$$1 + 9\cos^2\Theta - 6\cos\Theta\cos\Theta = a^2$$

$$E = \frac{p}{4\pi\varepsilon_0 R^3}\sqrt{1+3\cos^2\Theta}$$

2.9



$$dq = \sigma\, dS$$

$$d\varphi = \frac{k\,dq}{R} = \frac{k\sigma\, dS}{R}$$

$$\varphi = \iint_S k\sigma\frac{dS}{R} = \frac{k\sigma}{R}\underbrace{\iint_S dS}_{2\pi R^2} = \frac{R\sigma}{2\varepsilon_0}$$

$$E = -\operatorname{grad}\varphi$$

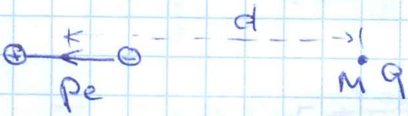
решение далее

$\boxed{2.10}$ $\varphi_{шара} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{R}$

$n = \dfrac{q}{e} = \dfrac{4\pi\varepsilon_0 R}{e} = 2 \cdot 10^{10}$

$\Delta M = m \cdot n = 2 \cdot 10^{-20}$ кг.

$\boxed{2.11}$



$\varphi(M) = \dfrac{\bar{r}\,\bar{p}}{4\pi\varepsilon_0 d^3}$

$\bar{E} = -\operatorname{grad}\varphi = k\left[\dfrac{3(\bar{p}\bar{r})\bar{r}}{d^5} - \dfrac{\bar{p}}{d^3}\right] =$

$= k\left[-\dfrac{3pd\bar{r}}{d^5} - \dfrac{\bar{p}}{d^3}\right]$

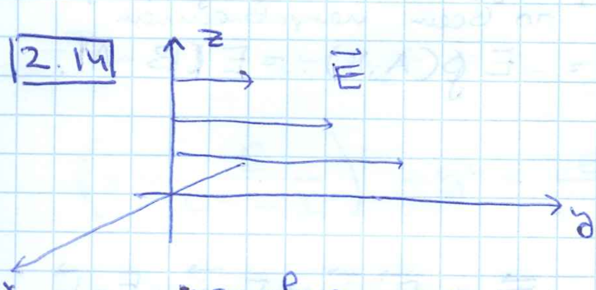
$E = k\left[-\dfrac{3pd^2}{d^5} + \dfrac{p}{d^3}\right] = -\dfrac{k\,2p}{d^3} = \dfrac{-p}{2\pi\varepsilon_0 d^3}$

$|F| = |Eq| = \dfrac{qp}{2\pi\varepsilon_0 d^3}$

$|F| = \dfrac{qp}{2\pi\varepsilon_0 d^3}$. в случае ⊕—⊖ $\cdot q$ притяжение

⊖—⊕ $\cdot q$ отталкиван.

$\boxed{2.14}$



$\vec{E} = (0, F(z), 0)$

$F(z) = \alpha z + \beta, \ \alpha \neq 0$

условие потенциальности поле:

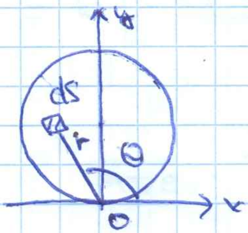
$\operatorname{rot}\vec{E} = 0$

$$\mathrm{rot}\,\vec{E} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & F(z) & 0 \end{vmatrix} = \bar{i}\left[0 - F'_z(z)\right] + \vec{j}\,0 + \vec{k}\,0$$

$$= -F'_z(z)\,\vec{i} \qquad F'_z = (\alpha z + \beta)' = \alpha \neq 0$$

$\Rightarrow$ поле непотенциально $\Rightarrow$ не может сущ.

$\boxed{2.15}$



$$r = [0; 2R\sin\Theta],\quad \Theta \in [0, \pi]$$

$$dq = \sigma\, dS \qquad d\varphi = \frac{k\sigma\, dS}{r}$$

$$\varphi = \frac{\sigma}{4\pi\varepsilon_0}\iint_S \frac{dS}{r} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{\pi} d\Theta \int_0^{2R\sin\Theta} \frac{1}{r}\, r\, dr =$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^{\pi} 2R\sin\Theta\, d\Theta = \frac{2R\sigma}{4\pi\varepsilon_0}\int_0^{\pi}\sin\Theta\, d\Theta =$$

$$= \frac{R\sigma}{2\pi\varepsilon_0}\left[-\cos\Theta\right]\Big|_0^{\pi} = \frac{R\sigma}{\pi\varepsilon_0}$$

$\boxed{2.12}$ Если предположить, что поле $\vec{E}$ однородное по всем направлениям

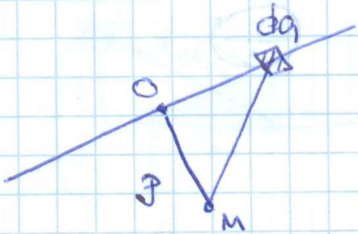
то $$\varphi_A - \varphi_B = \int_A^B E\, dr = E\,\rho(A, B) = E\,|B - A|.$$

тогда $$E_x = \frac{\varphi_4 - \varphi_1}{a}$$

$$E_y = \frac{\varphi_4 - \varphi_2}{a}, \qquad \vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$$

$$E_z = \frac{\varphi_4 - \varphi_3}{a}$$

2.13



$$dq = \lambda\, dx$$

$$d\varphi = \frac{k\,dq}{r} = \frac{k\lambda\, dx}{\sqrt{x^2+\rho^2}}$$

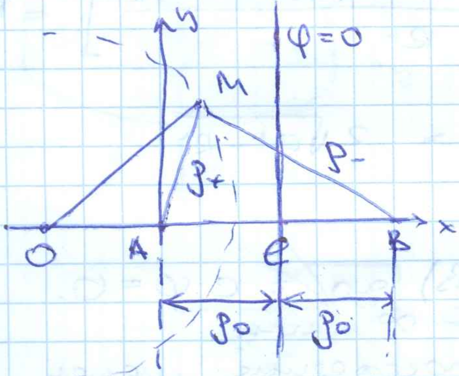
$$\varphi_+ = 2\int_{\rho}^{\rho_0} k\lambda \frac{dx}{\sqrt{x^2+\rho^2}} = 2k\lambda\left[\ln\left(\sqrt{x^2+\rho^2}+x\right)\right]_{\rho}^{\infty} =$$

$$= 2k\lambda \ln\frac{\rho_0}{\rho_+}$$

плоскость между проводник. имеет потенциал 0.

$$\varphi_- = -2k\lambda \ln\frac{\rho_0}{\rho_-}$$

ρ_0 – расстояние от плоскости до проводников



$$\varphi(M) = \varphi_+ + \varphi_- = 2k\lambda \ln\frac{\rho_-}{\rho_+}$$

$$\rho_+ = \sqrt{x^2+y^2}$$

$$\rho_- = \sqrt{(x-\ell)^2+y^2}$$

$$\frac{\rho_-}{\rho_+} = \sqrt{\frac{(x-\ell)^2+y^2}{x^2+y^2}} = const = c$$

$$x^2 - 2x\ell + \ell^2 + y^2 = c^2x^2 + c^2y^2$$

$$x^2 + y^2 + 2\frac{\ell}{c^2-1}x = \frac{\ell^2}{c^2-1}$$

$$\left(x + \frac{\ell}{c^2-1}\right)^2 + y^2 = \frac{\ell^2}{c^2-1} + \frac{\ell^2}{(c^2-1)^2} = \frac{c^2\ell^2}{(c^2-1)^2} = R^2$$

– окружность с центром $\left(-\frac{\ell}{c^2-1}, 0\right)$

и радиусом $R = \left|\frac{c\ell}{c^2-1}\right| \Rightarrow$ цилиндр.

$$\begin{cases} \ell + \dfrac{2\ell}{c^2-1} = \ell\,\dfrac{c^2+1}{c^2-1} = 2a \\ \ell\,\dfrac{c}{c^2-1} = R \qquad (2) \end{cases}$$

$\div$

$$\frac{c^2+1}{c} = \frac{2a}{R}$$

$$c^2 - 2\frac{a}{R}c + 1 = 0$$

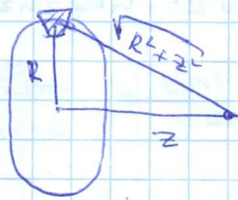
$$c = \frac{a}{R} + \sqrt{\frac{a^2}{R^2} - 1} = \frac{a + \sqrt{a^2 - R^2}}{R} \Rightarrow (2)$$

$$\ell\,\frac{a+\sqrt{a^2-R^2}}{R}\left[\frac{a^2 + 2a\sqrt{a^2-R^2} + a^2 - R^2 - R^2}{R^2}\right]^{-1} = R$$

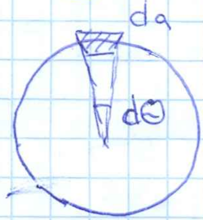
$$\ell = \frac{2a^2 + 2a\sqrt{a^2-R^2} - 2R^2}{a + \sqrt{a^2-R^2}} = 2\sqrt{a^2-R^2}$$

* ρ_0 — расстояние от $A(B)$ до т. с $\varphi = 0$.
можно положить $\rho_0 = \infty$ или
заметить, что плоскость посередине между
проводниками и паралл. им имеет $\varphi = 0$.

$\boxed{2.9}$ 1) найдём потенциал кольца в точке на оси $\perp$ плоскости кольца



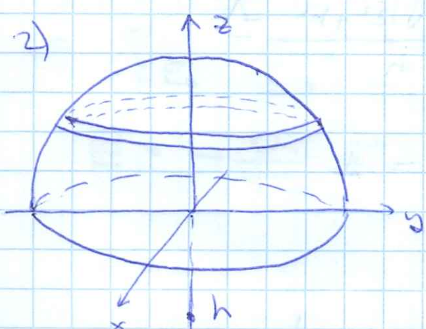
$$dq = \frac{Q}{2\pi}\,d\Theta$$

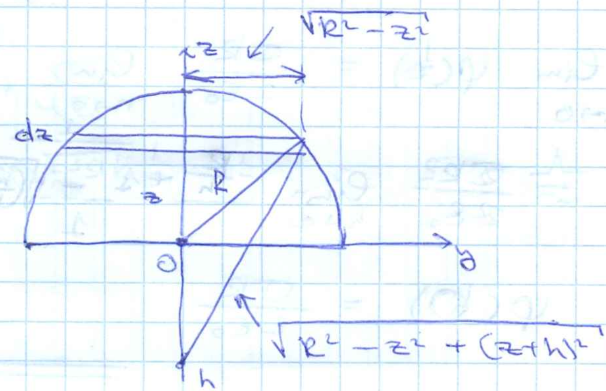


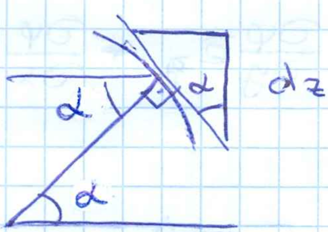
$$d\varphi = \frac{k\,dq}{\sqrt{R^2+z^2}} = \frac{k\frac{Q}{2\pi}\,d\Theta}{\sqrt{R^2+z^2}}$$

$$\varphi = \int_0^{2\pi} d\varphi = \frac{kQ}{2\pi}\cdot\frac{1}{\sqrt{R^2+z^2}}\int_0^{2\pi} d\Theta = \frac{kQ}{\sqrt{R^2+z^2}}$$

2)







$$\cos\alpha = \frac{\sqrt{R^2-z^2}}{R}$$

$$Q \equiv dq = \sigma\,dS\ ;\quad dS = 2\pi\sqrt{R^2-z^2}\,dz\,\frac{1}{\cos\alpha} =$$

$$= 2\pi\sqrt{R^2-z^2}\,\frac{R}{\sqrt{R^2-z^2}}\,dz = 2\pi R\,dz$$

$$d\varphi = \frac{k\,dq}{r} = 2\pi\sigma Rk\cdot\frac{dz}{\sqrt{R^2-z^2+(z+h)^2}}$$

$$\varphi(h) = \frac{\sigma R}{2\varepsilon_0}\int_0^R \frac{dz}{\sqrt{R^2 - z^2 + (z+h)^2}} = \frac{\sigma R}{2\varepsilon_0}\int_0^R \frac{dz}{\sqrt{R^2+h^2+2hz}} =$$

$$= \frac{\sigma R}{2\varepsilon_0}\cdot\frac{1}{+2h}\int_0^R \frac{d(R^2+h^2+2hz)}{\sqrt{R^2+h^2+2hz}} =$$

$$= \frac{\sigma R}{2\varepsilon_0}\frac{1}{2h}\left[2\sqrt{R^2+h^2+2hz}\right]\Big|_0^R =$$

$$= \frac{\sigma R}{2\varepsilon_0}\cdot\frac{1}{h}\left[R+h-\sqrt{R^2+h^2}\right]$$

$$\lim_{h\to 0}\varphi(h) = \frac{\sigma R}{2\varepsilon_0}\lim_{h\to 0}\frac{R+h-\sqrt{R^2+h^2}}{h} \overset{\Lambda}{=}$$

$$\overset{\Lambda}{=} \frac{\sigma R}{2\varepsilon_0}\lim_{h\to 0}\frac{1-\frac{1}{2}\frac{1}{\sqrt{R^2+h^2}}\cdot 2h}{1} = 1$$

$$\varphi(0) = \frac{\sigma R}{2\varepsilon_0}$$

$$h \equiv z$$

$$\vec{E} = -\operatorname{grad}\varphi = -\frac{\partial\varphi}{\partial z}\vec{e}_z - \frac{\partial\varphi}{\partial y}\vec{e}_y - \frac{\partial\varphi}{\partial x}\vec{e}_x;$$

$$\frac{\partial\varphi}{\partial y} = 0 \quad \frac{\partial\varphi}{\partial x} = 0$$

$$\frac{\partial\varphi}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\sigma R}{2\varepsilon_0}\frac{R+z-\sqrt{R^2+z^2}}{z}\right) =$$

$$= \frac{\sigma R}{2\varepsilon_0}\cdot\frac{\left(1-\frac{2h}{\sqrt{R^2+h^2}\cdot 2}\right)h - \left(R+h-\sqrt{R^2+h^2}\right)}{h^2} =$$

$$= \frac{\sigma R}{2\varepsilon_0}\cdot\frac{1}{h^2}\left[\cancel{h} - \frac{h^2}{\sqrt{R^2+h^2}} - R - \cancel{h} + \sqrt{R^2+h^2}\right] =$$

$$= \frac{\sigma R}{2\varepsilon_0} \frac{1}{h^2} \left[-R + \frac{R^2 + h^2 - h^2}{\sqrt{R^2+h^2}} \right]$$

$$\lim_{h\to 0} \varphi'(h) = \frac{\sigma R}{2\varepsilon_0} \lim_{h\to 0} \left(\frac{-R}{h^2} + \frac{R^2}{\sqrt{R^2+h^2} \cdot h^2} \right) = \left\{ t = \frac{h}{R} \right\}$$

$$= \frac{\sigma R}{2\varepsilon_0} \lim_{t\to 0} \left(\frac{-R + \frac{R^2}{R\sqrt{1-t^2}}}{R^2 t^2} \right) =$$

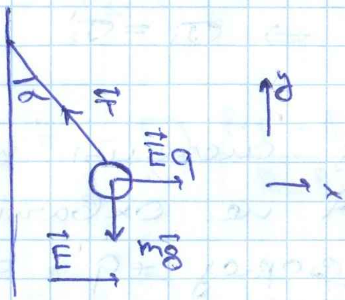
$$= \frac{\sigma R}{2\varepsilon_0} \lim_{t\to 0} \left(\frac{-R + R\left[1 + t^2 \cdot \frac{1}{2}\right]}{R^2 t^2} \right) =$$

$$= \frac{\sigma}{2\varepsilon_0} \lim_{t\to 0} \frac{+\frac{1}{2} t^2}{t^2} = + \frac{\sigma}{4\varepsilon_0}$$

$$\vec{E} = -\frac{\sigma}{4\varepsilon_0} \vec{e_z}$$

3.1

$\boxed{3}$



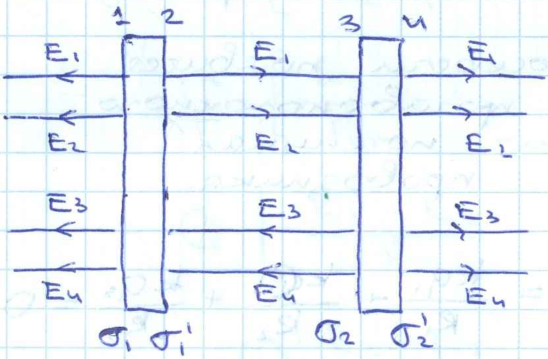
$$y:\quad T\cos\alpha = mg$$

$$x:\quad T\sin\alpha = Eq = \frac{\sigma}{2\varepsilon_0} q$$

$$\operatorname{tg}\alpha = \frac{\sigma q}{2\varepsilon_0 mg}$$

$$q = \frac{2mg\varepsilon_0 \operatorname{tg}\alpha}{\sigma}$$

3.2



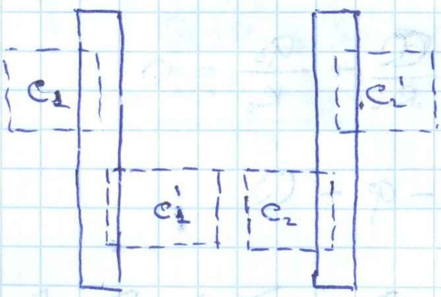
E'' E E'

$\sigma_1^0 = \sigma_1 + \sigma_1'$ $\quad \sigma_2^0 = \sigma_2 + \sigma_2'$

$$E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_1'}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_2'}{2\varepsilon_0} = \frac{\sigma_1^0 - \sigma_2^0}{2\varepsilon_0} = E_{внутр.}$$

$$E'' = -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_1'}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_2'}{2\varepsilon_0} = -\frac{\sigma_1^0 + \sigma_2^0}{2\varepsilon_0}$$

$$E' = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_1'}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_2'}{2\varepsilon_0} = \frac{\sigma_1^0 + \sigma_2^0}{2\varepsilon_0} = -E'' = E_{внеш.}$$



Запишем т. Гаусса для цилиндров c_1' и c_2:

$$\oint \vec{E}_{внутр.}\, d\vec{S} = \frac{\Sigma q}{\varepsilon_0}$$

$$\sigma_1' = -\sigma_2 \Leftarrow \quad E_{внутр.} S = \frac{\sigma_1' S}{\varepsilon_0} = \frac{-\sigma_2 S}{\varepsilon_0}$$

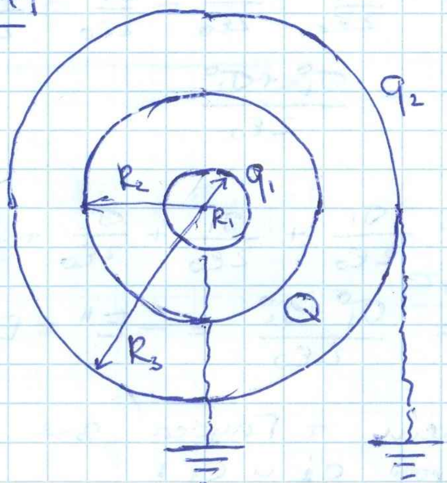
Запишем т. Гаусса для цилиндров C_1 и C_2'

$$E_{внеш} S = \frac{\sigma_1 S}{\varepsilon_0} = \frac{\sigma_2' S}{\varepsilon_0} \Rightarrow \sigma_1 = \sigma_2'$$

$\boxed{3.3}$ Т.к. суммарный заряд системы не равен 0, и проводник A не охватывает заряд со всех сторон заряд $+q$, то только часть силовых линий, исходящих от $+q$, заканчиваются на индуцированных зарядах проводника

$\boxed{3.4}$ На проводящей плоскости появится индуцированный заряд противоположного знака, который уменьшит потенциал проводника.

$\boxed{3.7}$



$\varphi = 0$

$$\begin{cases} \varphi_1 = \dfrac{kq_1}{R_1} + \dfrac{kQ}{R_2} + \dfrac{kq_3}{R_3} = 0 \\[2ex] \varphi_3 = \dfrac{kq_1}{R_3} + \dfrac{kQ}{R_3} + \dfrac{kq_3}{R_3} = 0 \end{cases}$$

$$\begin{cases} q_1 + Q + q_3 = 0 \\[2ex] \dfrac{q_1}{R_1} + \dfrac{Q}{R_2} + \dfrac{q_3}{R_3} = 0 \end{cases}$$

$$q_3 = -q_1 - Q$$

$$\frac{q_1}{R_1} = -\frac{Q}{R_2} + \frac{q_1 + Q}{R_3} \Rightarrow \cancel{q_1\left(\frac{1}{R_1} - \frac{1}{R_3}\right) = Q}$$

$$\Rightarrow \quad q_1\left(\frac{1}{R_1}-\frac{1}{R_3}\right)=-Q\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

$$q_1=-Q\,\frac{R_3-R_2}{R_2\cancel{R_3}}\cdot\frac{R_1\cancel{R_3}}{R_3-R_1}=-Q\,\frac{R_1}{R_2}\cdot\frac{R_3-R_2}{R_3-R_1}$$

$$q_3=-Q+Q\,\frac{R_1}{R_2}\cdot\frac{R_3-R_2}{R_3-R_1}=$$

$$=Q\left[\frac{R_1}{R_2}\left(1-\frac{R_2-R_1}{R_3-R_1}\right)-1\right]=$$

$$=Q\left[\frac{R_1}{R_2}-1-\frac{R_1}{R_2}\cdot\frac{R_2-R_1}{R_3-R_1}\right]=$$

$$=Q\left[-\frac{R_2-R_1}{R_2}-\frac{R_2-R_1}{R_2}\cdot\frac{R_1}{R_3-R_1}\right]=$$

$$=-Q\,\frac{R_2-R_1}{R_2}\,\frac{R_3}{R_3-R_1}$$

$$q=\begin{cases}0, & r<R_1\\ q_1, & R_1\le r<R_2\\ Q+q_1, & R_2\le r<R_3\\ \underbrace{Q+q_1+q_3}_{=0}, & R_3\le r\end{cases}$$

тогда по т. Гаусса имеем:

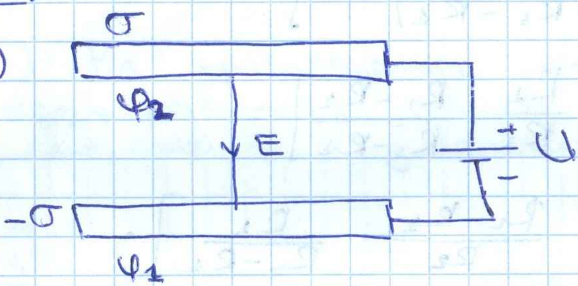
$$E\,4\pi r^2=\frac{q}{\varepsilon_0}$$

В итоге имеем:

$$E = \begin{cases} 0, & r < R_1 \\ \dfrac{-1}{4\pi\varepsilon_0} \cdot \dfrac{Q}{r^2} \cdot \dfrac{R_1}{R_2} \cdot \dfrac{R_3 - R_2}{R_3 - R_1}, & R_1 \le r < R_2 \\ \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q}{r^2} \dfrac{R_3}{R_2} \dfrac{R_2 - R_1}{R_3 - R_1}, & R_2 \le r < R_3 \\ 0, & R_3 \le r \end{cases}$$

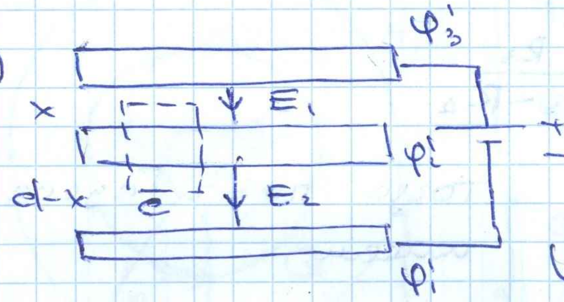
$\boxed{3.8}$

1)



$$\varphi_2 - \varphi_1 = U = \int_2^1 \vec{E}\,d\vec{r} = Ed = \frac{\sigma}{\varepsilon_0} d$$

2)



$$\begin{cases} \varphi_3' - \varphi_2' = E_1 x \\ \varphi_2' - \varphi_1' = E_2 (d - x) \end{cases} \oplus$$

$$U = \varphi_3' - \varphi_1' = E_1 x + E_2 (d - x)$$

Запишем т. Гаусса для цилиндра C:

$$-E_1 S + E_2 S = \frac{\sigma S}{\varepsilon_0} \Rightarrow -E_1 + E_2 = \frac{\sigma}{\varepsilon_0} = \frac{U}{d}$$

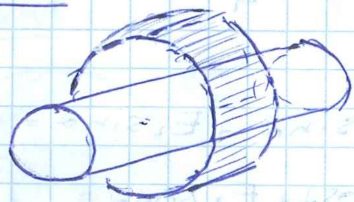
т.о. $\begin{cases} E_1 x + E_2 (d - x) = U \\ -E_1 + E_2 = \dfrac{U}{d} \end{cases}$

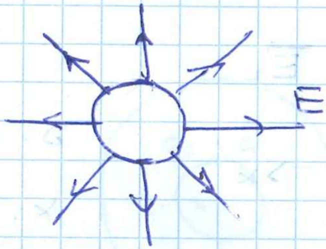
$$E_1 = E_2 - \frac{U}{d}$$

$$\cancel{E_2 x} - \frac{Ux}{d} + E_2 d - \cancel{E_2 x} = U \Rightarrow E_2 = U\frac{d+x}{d^2}$$

$$E_1 = U\frac{d+x}{d^2} - \frac{U}{d} = U\frac{x}{d^2}$$

$\boxed{3.11}$





1) $r \geq R$ по т. Гаусса: $E \cdot 2\pi r \cdot h = \frac{Q}{\varepsilon_0}$

$$Q = \pi R^2 \cdot h \cdot \rho$$

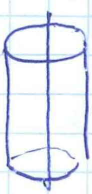
$$E = \frac{R^2 \rho}{2\varepsilon_0 r} \qquad \vec{E} = \frac{\rho R^2}{2\varepsilon_0 r^2}\vec{r}$$

2) $r \leq R$ по т. Гаусса:

$$E \cdot 2\pi r \cdot h = \frac{\pi r^2 h \rho}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0} \qquad \vec{E} = \frac{\rho}{2\varepsilon_0}\vec{r}$$

$\boxed{3.12}$ 1) Найдём напряжённость от провода

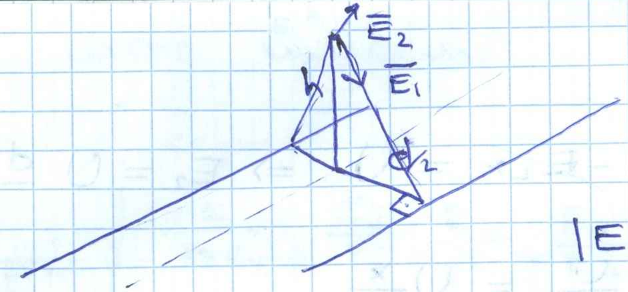


по т. Гаусса:

$$E \cdot 2\pi r \cdot \ell = \frac{\rho \ell}{\varepsilon_0}$$

$$E = \frac{\rho}{2\pi r \varepsilon_0}$$

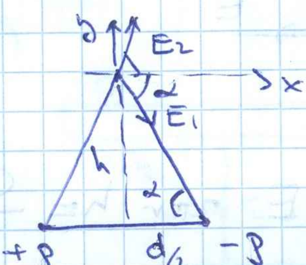
2)



$$|E_1| = |E_2| = \frac{\rho}{2\pi r \varepsilon_0} =$$

$$= \frac{\rho}{2\pi \sqrt{h^2 + \frac{d^2}{4}}\, \varepsilon_0}$$

$$\overline{E} = \overrightarrow{E}_1 + \overrightarrow{E}_2$$



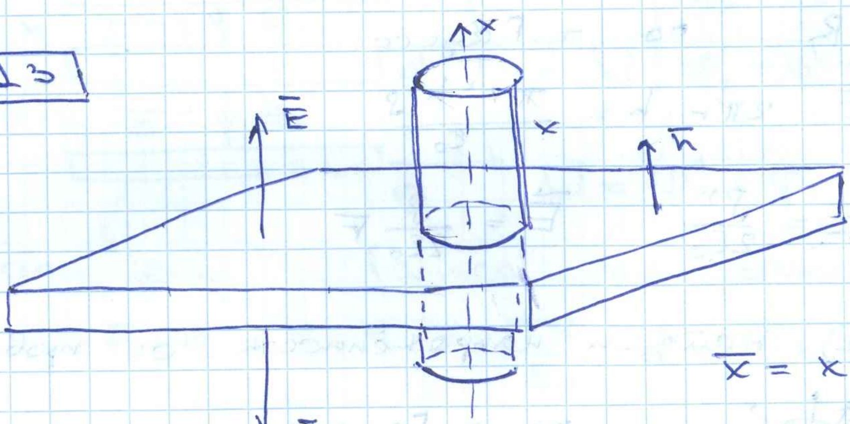
y: $-E_1 \sin\alpha + E_2 \sin\alpha = 0.$

x: $E_1 \cos\alpha + E_2 \cos\alpha =$

$$= \frac{2\rho}{2\pi\sqrt{h^2 + \frac{d^2}{4}}\, \varepsilon_0} \cdot \frac{d}{2\sqrt{h^2 + \frac{d^2}{4}}} =$$

$$= \frac{\rho\, d}{2\pi \varepsilon_0 \left(h^2 + \frac{d^2}{4}\right)} = \frac{2\rho d}{\pi \varepsilon_0 (4h^2 + d^2)}$$

$\boxed{3.13}$



$$\overline{x} = x \cdot \overline{n}$$

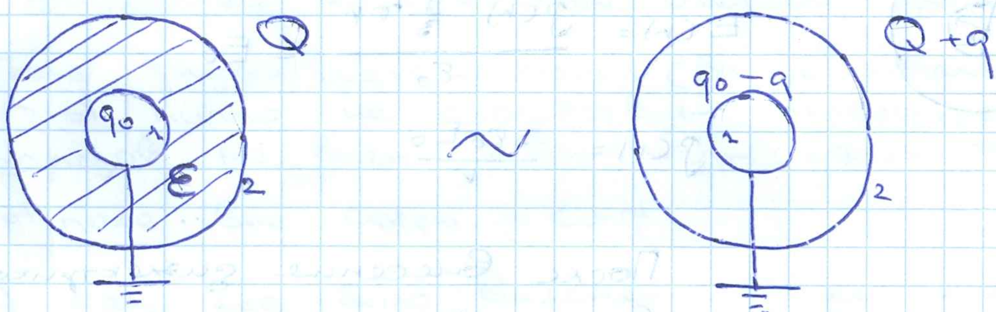
по т. Гаусса: (вне пластины)

$$2ES = \frac{\rho S 2d}{\varepsilon_0} \Rightarrow E = \frac{\rho d}{\varepsilon_0}; \; \overline{E} = \frac{\rho d}{\varepsilon_0}\overline{n}$$

по т. Гаусса (внутри пластины)

$$2ES = \frac{\rho S 2x}{\varepsilon_0} \Rightarrow E = \frac{\rho x}{\varepsilon_0}; \; \overline{E} = \frac{\rho \overline{x}}{\varepsilon_0}$$

$\boxed{3.14}$



с одной стороны $\varphi_1 = \frac{q_0}{4\pi\varepsilon\varepsilon_0 r}$

с другой $\varphi_1 = \frac{q_0-q}{4\pi\varepsilon_0 r}$ $\Rightarrow$

$$\Rightarrow \frac{q_0}{\varepsilon} = q_0 - q \Rightarrow q = q_0\left(1 - \frac{1}{\varepsilon}\right).$$

$$0 = \varphi_1 + \varphi_2 = \frac{q_0 - q}{4\pi\varepsilon_0 r} + \frac{Q-q}{4\pi\varepsilon_0 R}$$

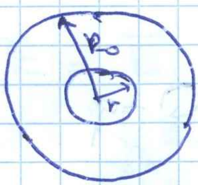
$$\frac{q_0}{\varepsilon r} + \frac{Q}{R} + \frac{q_0}{R} - \frac{q_0}{\varepsilon R} = 0$$

$$q_0\left(1 - \frac{1}{\varepsilon} + \frac{R}{r}\frac{1}{\varepsilon}\right) = -Q$$

$$q_0\left(\frac{\varepsilon + R/r - 1}{\varepsilon}\right) = -Q$$

$$q_0 = -Q\,\frac{\varepsilon}{\varepsilon - 1 + \frac{R}{r}}$$

$\boxed{3.15}$

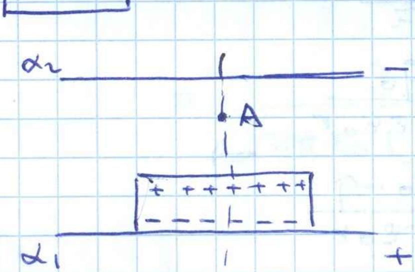


$$E \cdot 4\pi r^2 = \frac{\rho\,\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E(r) = \frac{\rho(r)\,\frac{1}{3}\pi r}{\varepsilon_0} = E_0$$

$$\rho(r) = \frac{3\varepsilon_0 E_0}{r}$$

$\boxed{3.6}$

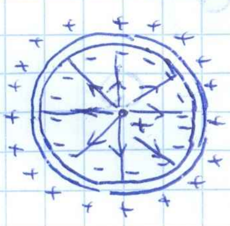


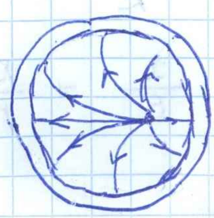
После внесение диэлектрика между пластин, поверхност. плотность заряда на пластине α_2 напротив диэлектрика увеличится $\Rightarrow$

$\Rightarrow$ напряжён. E_A увеличится.

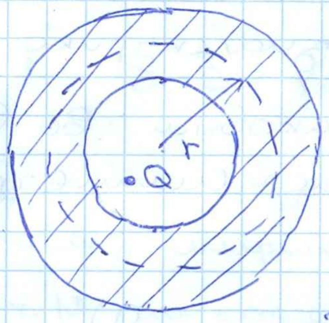
$\boxed{3.5}$ 2) Очевидно, что при перемещении зарядов внутри сферы заряд (индуцированный) на внутренней обкладке будет перераспределятся

$\Rightarrow$ поле изменится.





Рассмотрим следующую сферу (пунктир) радиуса r:



Запишем т. Гаусса:

$$\underbrace{\oint D\,dS}_{=0} = q \Rightarrow q = 0$$

$=0$, т.к. $D = \varepsilon\varepsilon_0 E = 0$.

Заряд индуцируемый на внутренней поверхности метал. сферы равен $-Q$.

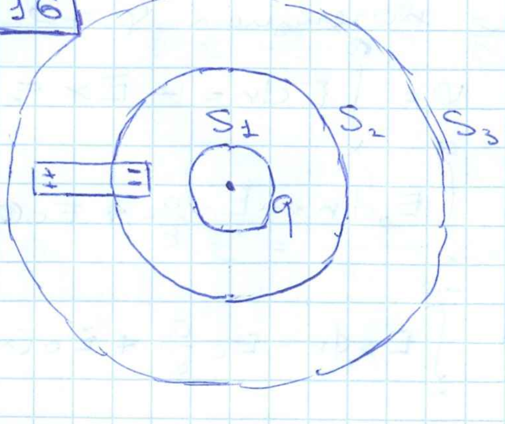
Поле внутри метал. сферы (внутри металла) равно $0 \Rightarrow$ ничто не способствует перераспределению зарядов на внеш. поверхности сферы $\Rightarrow$

$\Rightarrow$ поле вне сферы = const.

1) Как уже было выяснено выше поле внутри оболочки и вне её не зависят друг от друга. $\Rightarrow$ ~~[illegible]~~ внесение заряженного проводника повлияет только на внешнее поле, но не на внутреннее.

* рассуждения справедливы для $\forall$ оболочки, а не только для сферы.

$\boxed{3.16}$



По т. Гаусса имеем:

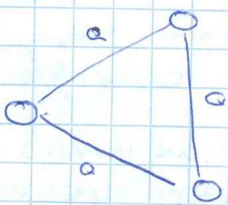
$\oint \Phi\, dS = q$, т.о.

$$\Phi_{D_1} = \Phi_{D_2} = \Phi_{D_3} = q$$

$$\Phi_{E_1} = \Phi_{E_3} = \frac{q}{\varepsilon_0} > \Phi_{E_2}$$

Теорема Гаусса не позволяет посчитать $\vec{D}(r)$, т.к. из-за влияния диэлектрика $D(r)$ перестает быть одинаковым во всех направлениях. (отсутствие симметрии).

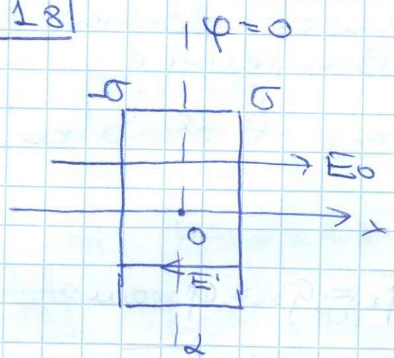
$\boxed{3.9}$



$$\begin{cases} \frac{kq_1}{R} = U & (1) \\ \frac{kq_2}{R} = \frac{kq_1}{a} + U & (2) \\ \frac{kq_3}{R} = \frac{kq_1}{a} + \frac{kq_2}{a} + U & (3) \end{cases}$$

$$\frac{(3)-(1)}{(2)-(1)} ; \quad \frac{q_3 - q_1}{q_2 - q_1} = \frac{q_1 + q_2}{q_1} \Rightarrow q_3 = \frac{q_2^2}{q_1}$$

$\boxed{3.18}$



(1) $D = \varepsilon_0 E_0 = \varepsilon \varepsilon_0 E$

$\Rightarrow E = \frac{E_0}{\varepsilon}$ (поле внутри диэлектр)

Положим, что плоскость α имеет потенциал $\varphi = 0$.

1) $|x| < a \quad \varphi = \int_x^0 E\,dr = -Ex = -\frac{E_0 x}{\varepsilon}$

2) $|x| > a \quad \varphi = \int_a^0 E\,dr + \int_x^a E_0\,dr = -E_0 \frac{a}{\varepsilon} - E_0 (x-a)$

$x < a \quad \varphi = \int_{-a}^0 E\,dr + \int_x^{-a} E_0\,dr = E_0 \frac{a}{\varepsilon} - E_0 (x+a)$

(2) $E = E_0 - E' = E_0 - \dfrac{\sigma}{\varepsilon_0} = \dfrac{E_0}{\varepsilon} \Rightarrow \sigma = \varepsilon_0 \left(\dfrac{\varepsilon - 1}{\varepsilon}\right) E_0$

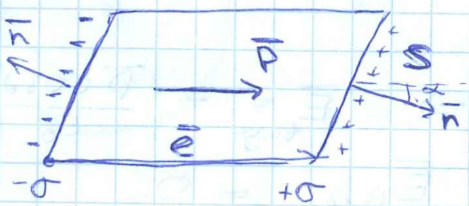
(3) $\overline{P} = \overline{D} - \overline{E} = \varepsilon_0(\varepsilon - 1)\overline{E} = \dfrac{\varepsilon - 1}{\varepsilon}\varepsilon_0 \overline{E_0}$.

$\boxed{3.20}$ Из задачи 1.17 известно, что полоса, шириной в L, создаёт напряж. в т. M, находящейся на расстоянии $l/2$ от пластины и равноудалённой от её границ, равное:

$$E = \frac{\sigma}{\pi\varepsilon_0} \operatorname{arctg} \frac{L}{l}$$

Докажем следующий факт:

$$\sigma = (\overline{P}, \overline{n}) = Pn\cos\alpha$$



$$\overline{P} = \frac{1}{\Delta V} \sum_{i=1}^{n} \overline{p_{e_i}} = \frac{\sigma S}{V}\overline{e} = \frac{\sigma S}{S l \cos\alpha}\overline{l}$$

$$\Rightarrow \sigma = (\overline{P}, \overline{n})$$

т.о. $E = \dfrac{P}{\pi\varepsilon_0} \operatorname{arctg} \dfrac{L}{l}$

т.к. у диэлектрика две поверхности, то
(разноимённо заряжен.)

$$E_0 = 2E = \frac{2P}{\pi \varepsilon_0} \operatorname{arctg} \frac{L}{l} \quad (\overline{E}_0 \uparrow\downarrow \overline{P})$$

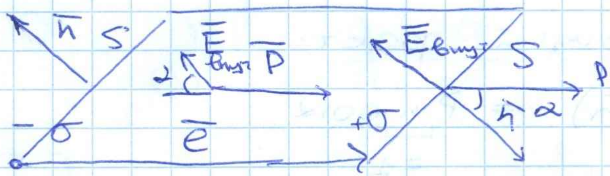
а) $l \ll L$: $\quad E_0 = \frac{2P}{\pi\varepsilon_0} \cdot \frac{\pi}{2} = \frac{P}{\varepsilon_0}$

$$\overline{D} = \varepsilon_0 \overline{E}_0 + \overline{P} = 0.$$

б) $l \gg L$: $\quad E_0 = \frac{2P}{\pi\varepsilon_0} \frac{L}{l} \quad (\overline{E}_0 \uparrow\downarrow \overline{P})$

$$\overline{D} = \varepsilon_0 \overline{E}_0 + \overline{P} = \overline{P}\left(1 - \frac{2}{\pi}\frac{L}{l}\right)$$

$\boxed{3.19}$

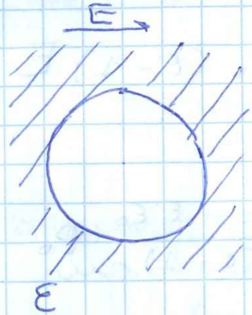


$$0 = \overline{D} = \varepsilon_0 \overline{E}_{внут} + \overline{P}\cos\alpha \Rightarrow \overline{E}_{внут} = -\frac{\overline{P}\cos\alpha}{\varepsilon_0}$$

$$0 = \overline{D} = \varepsilon_0 \overline{E}_{внеш} \Rightarrow \overline{E}_{внеш} = 0.$$

$\overline{D} = 0$ вследствие т. Гаусса.

3.21

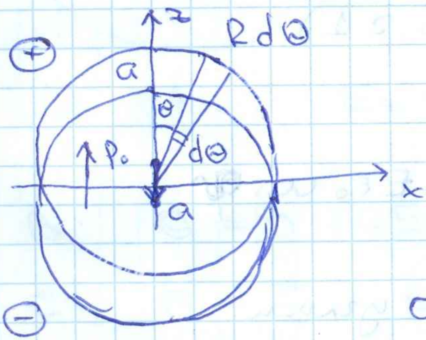


Напряжённость E' внутри сферы можно представить в виде разности однородного поля E и поля E_1, создаваем шаром (вместо полости) из диэлектрика.

Из примера 2.10 известно, что распред-ие заряда на поверхности шара удовл. соотнош.:

$$\sigma = \sigma_0 \cos\theta = P_0 \cos\theta. \quad (3.20)$$

Найдём E_1 (поле индуц. зарядов).

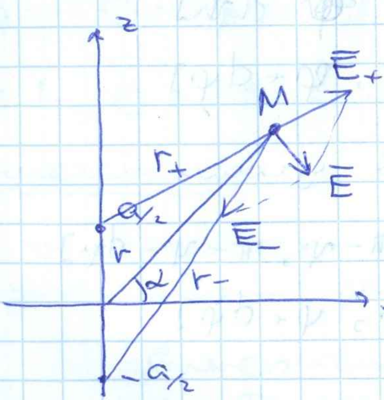


$$dq = \rho \cdot 2\pi R \sin\theta \cdot R d\theta \cdot a\cos\theta$$

$$dS = 2\pi R \sin\theta \cdot R d\theta$$

$$\Downarrow$$

$$\sigma = \underbrace{a\rho}_{\sigma_0} \cos\theta = P_0 \cos\theta$$



$$E_+ = \frac{\rho r_+}{3\varepsilon_0} \qquad E_- = \frac{\rho r_-}{3\varepsilon_0}$$

$$E_x = \frac{\rho}{3\varepsilon_0} r\cos\alpha - \frac{\rho}{3\varepsilon_0} r\cos\alpha = 0$$

$$E_z = \frac{\rho}{3\varepsilon_0}\left(r\sin\alpha - \frac{a}{2}\right) - \frac{\rho}{3\varepsilon_0}\left(r\sin\alpha + \frac{a}{2}\right) =$$

$$= -\frac{\rho a}{3\varepsilon_0} = -\frac{\sigma_0}{3\varepsilon_0} = -\frac{P_0}{3\varepsilon_0}$$

т.о. $\vec{E_1} = -\frac{\vec{P_0}}{3\varepsilon_0}$; $\vec{E} \uparrow\uparrow \vec{P_0}$

$$D = \varepsilon_0 E + P_0 = \varepsilon\varepsilon_0 E \Rightarrow P_0 = \varepsilon_0(\varepsilon - 1)E$$

тогда

$$E' = E + E_1 \overset{1}{=} \frac{P_0}{\varepsilon_0(\varepsilon-1)} + \frac{P_0}{3\varepsilon_0} = \frac{2\varepsilon_0 + \varepsilon\varepsilon_0}{3(\varepsilon-1)} P_0$$

$\boxed{\text{в ответе должно быть } E_1}$ $\overset{2}{=} E + \frac{(\varepsilon-1)E}{3} = \frac{\varepsilon+2}{3} E.$

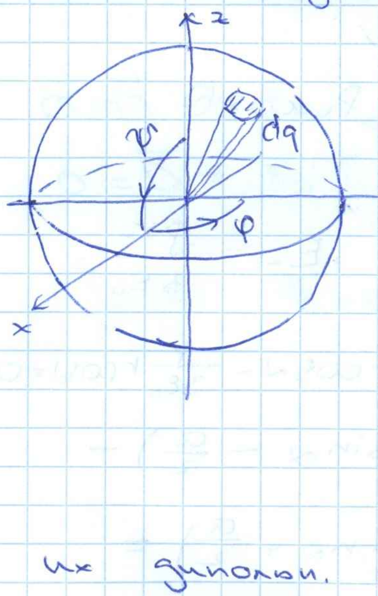
$\boxed{3.22}$ Найдём дипольный момент шара.

1) Поле создаваем инд. зарядами должно погасить внешнее поле:

$$E_0 = \frac{\sigma_0}{3\varepsilon_0} \quad (3.21)$$

из примера 2.10:

$$\sigma = \sigma_0 \cos\psi = 3E\varepsilon_0 \cos\psi$$



dq соотв. углам:

$\psi \in [\psi, \psi + d\psi]$

$\varphi \in [\varphi; \varphi + d\varphi]$

$-dq$ соотв. углам:

$\psi \in [\pi - \psi, \pi - \psi - d\psi]$

$\varphi \in [\varphi, \varphi + d\varphi]$

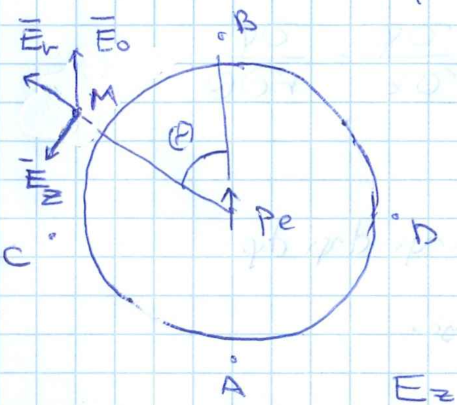
их дипольн. момент:

$$dp_e = \sigma(\psi) \cdot \underbrace{R^2 d\psi \sin\psi\, d\varphi}_{dS} \cdot \underbrace{2R\cos\psi}_{l}$$

$$p_e = 2R^3 \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sigma(\psi)\sin\psi\cos\psi\, d\psi =$$

$$= 4\pi\varepsilon_0 E_0 R^3$$

г) Внешнее поле будет определяться дипольн. моментом p_e и E_0.



$$\underset{\text{диполь}}{\varphi(M)} = \frac{\bar{p}\bar{r}}{4\pi\varepsilon_0 r^3} = \frac{p\cos\Theta}{4\pi\varepsilon_0 r^2}$$

$$E = -\operatorname{grad}\varphi: \qquad r \sim R$$

$$E_r = -\frac{\partial\varphi}{\partial r} = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\Theta}{r^3}$$

$$E_z = -\frac{\partial\varphi}{\partial z} = -\frac{\partial\varphi}{r\partial\Theta} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\Theta}{r^3}$$

B: $\Theta = 0$ $\quad E_B = E_0 + \frac{2p}{4\pi\varepsilon_0 R^3} = 3E_0$

C: $\Theta = \frac{\pi}{2}$ $\quad E_C = E_0 - \frac{p}{4\pi\varepsilon_0 R^3} = 0$

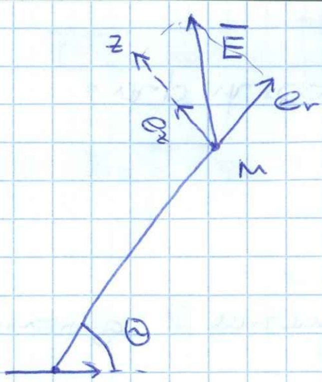
D: $\Theta = \frac{3\pi}{2}$ $\quad E_D = E_C = 0$

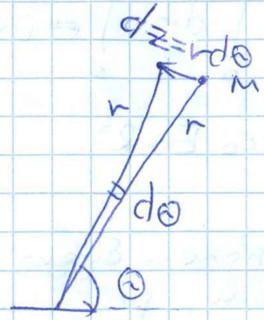
A: $\Theta = \pi$ $\quad E_B = E_A = 3E_0$

Замечание 1:

E_r определяется движением точки M, вызванным изменением расстояния r при фикс. Θ.

E_z – движением точки M, вызв. измен. Θ при фиксиров. r.



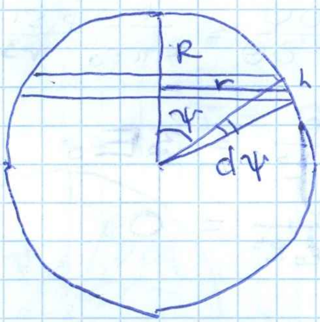


Именно вследствие этого: $\frac{\partial\varphi}{\partial z} = \frac{\partial\varphi}{r\partial\Theta}$

Замечание 2:

Обоснуем, что $dS = R^2 \sin\psi \, d\psi \, d\varphi$

1) посчитаем площадь кольца.



$$S_{кольца} = 2\pi \underbrace{R\sin\psi}_{r} \underbrace{R\,d\psi}_{h}$$

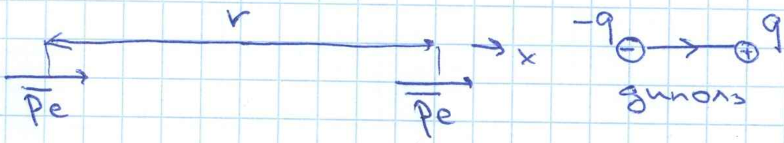
2) $dS = \frac{S_{кольца}}{2\pi} d\varphi =$

$$= R^2 \sin\psi \, d\psi \, d\varphi$$

$\boxed{3.17}$ Из задачи 3.22 известно, что сфере, помещ. в однородное поле E_0 имеет дипольный момент

$$\overline{p_e} = 4\pi\varepsilon_0 \overline{E_0} a^3$$

тогда:



$$F_x = 2\frac{kq^2}{r^2} - \frac{kq^2}{(r+l)^2} - \frac{kq^2}{(r-l)^2} = kq^2\left[\frac{2}{r^2} - \frac{1}{(r+l)^2} - \frac{1}{(r-l)^2}\right]$$

$$= kq^2\left[\frac{2}{r^2} - \frac{2(r^2+l^2)}{(r^2-l^2)^2}\right] =$$

$$= kq^2\left[\frac{\cancel{2r^4} - 4r^2l^2 + 2l^4 - \cancel{2r^4} - 2r^2l^2}{r^2(r^2-l^2)^2}\right] =$$

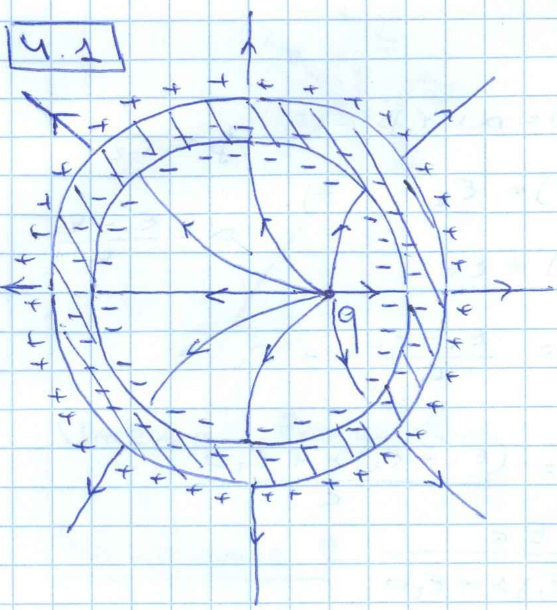
$$= kq^2\, 2l^2\left[\frac{l^2 - 3r^2}{r^2(r^2-l^2)^2}\right] = \{l \ll r\} =$$

$$= 2kq^2l^2 \cdot \frac{-3r^2}{r^6} = -\frac{6p_e^2}{4\pi\varepsilon_0 r^4}$$

$$F_x = -6\,\frac{(4\pi\varepsilon_0)^2 E_0^2 a^6}{\cancel{4\pi\varepsilon_0}\, r^4} = -\frac{24\pi\varepsilon_0 E_0^2 a^2}{r^4}$$

притягиваются, т.к. $F_x < 0$.

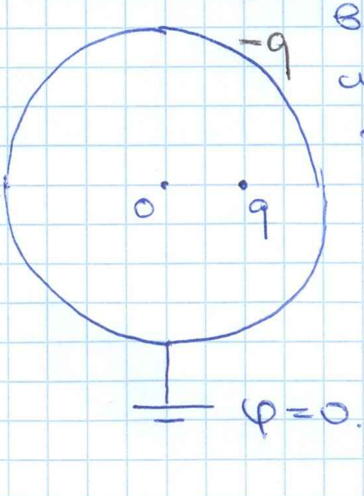
4.1



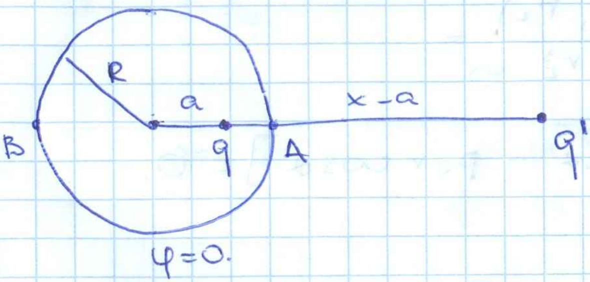
Заряд, находящ. внутри сферы вызовет перераспределен. своб. зарядов. Т.о. на внутр. оболочке будет индуц. заряд q_0 противополож. знака, а на внешней $-q_0$.

В силу т. Гаусса $q_0 = -q$. (см. 3.5)

Заряд, наход. на внутр. оболочке, погасит поле заряда q (поле внутри металла $= 0$). Заряд, наход. на внеш. оболочке не взаим. с зарядом q. Потенциал внеш. оболочки равен $\varphi_0 = \frac{kq}{R}$, как и внутренней. Т.о. образом убрав внешн. оболочку потенциал внутренней станет 0. Следовательно можно переформулировать задачу:



Найти силу, с которой взаимодейств заряд q внутри заземлённой сферы с этой сферой.



$$\begin{cases} \varphi_A = 0 = \dfrac{kq}{R-a} + \dfrac{kq'}{x-R} \\ \varphi_B = 0 = \dfrac{kq}{R+a} + \dfrac{kq'}{x+R} \end{cases}$$

$$\begin{cases} q'R - q'a + qx - qR = 0 & (1) \\ q'R + q'a + qx + qR = 0 & (2) \end{cases}$$

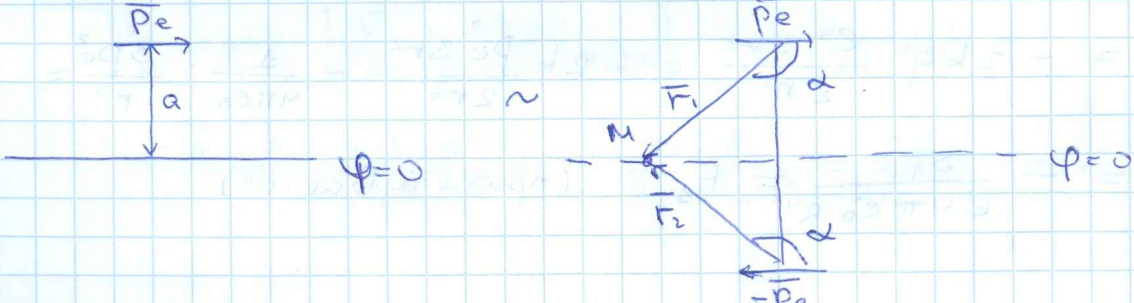
(1) + (2)

(2) − (1)

$$\begin{cases} q'R + qx = 0 & \Rightarrow x = \dfrac{R^2}{a} \\ q'a + qR = 0 & \Rightarrow q' = -q\dfrac{R}{a} \end{cases}$$

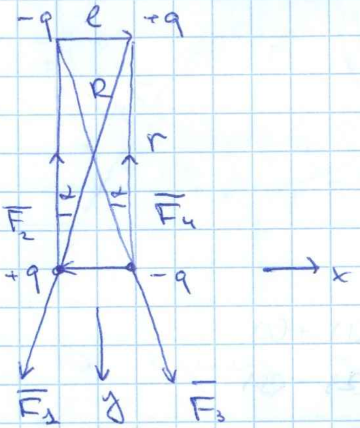
$$F = kq \cdot \frac{q'}{(x-a)^2} = kq^2 \frac{Ra}{(R^2-a^2)^2}$$

$\boxed{4.2}$ a)



$$\varphi(M) = \frac{\bar{p}_e \bar{r}_1}{4\pi\varepsilon_0 r_1^3} + \frac{(-\bar{p}_e, \bar{r}_2)}{4\pi\varepsilon_0 r_2^3} =$$

$$= \frac{1}{4\pi\varepsilon_0 r^3}\left[p_e r\cos\alpha - p_e r\cos\alpha\right] = 0.$$



$x:\ -F_1\sin\alpha + F_3\sin\alpha = 0$

$y:\ F_1\cos\alpha - F_2 + F_3\cos\alpha - F_4 =$

$$F_1 = \frac{kq^2}{R^2} = F_3$$

$$F_2 = \frac{kq^2}{r^2} = F_4$$

$$= \frac{2kq^2}{R^2}\cos\alpha - \frac{2kq^2}{r^2} =$$

$$\cos\alpha = \frac{r}{R} \qquad l^2 = R^2 - r^2$$

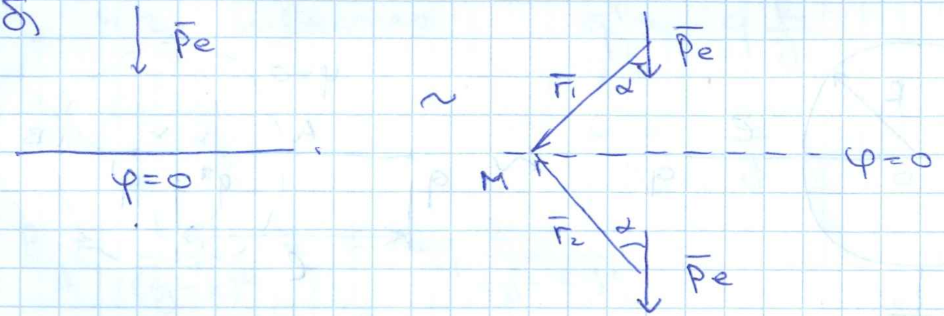
$$= 2kq^2\left[\frac{r}{R^3} - \frac{1}{r^2}\right] = 2kq^2\,\frac{(r-R)(R^2+Rr+r^2)}{R^3 r^2} =$$

$$= -2kq^2\,\frac{l^2(R^2+Rr+r^2)}{R^3 r^2 (R+r)} = \{R \approx r\} =$$

$$= -2kq^2\,\frac{l^2\, 3r^2}{2r^6} = -2k\,\frac{p_e^2\, 3r^2}{2r^6} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{3p_e^2}{r^4} =$$

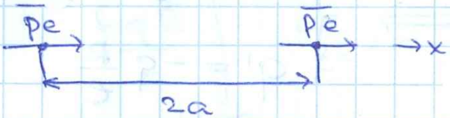
$$= -\frac{3p_e^2}{64\pi\varepsilon_0 a^4} = F_1$$ (притягиваются)

б)



$$\varphi(M)=\frac{\bar{p}_e\bar{r}_1}{4\pi\varepsilon_0 r_1^3}+\frac{\bar{p}_e\bar{r}_2}{4\pi\varepsilon_0 r_2^3}=\frac{1}{4\pi\varepsilon_0 r^3}\left[p_e r\cos\alpha+p_e r\cos(\pi-\alpha)\right.$$

$$=0.$$



$$x:\ F_2=2\cdot\frac{kq^2}{4a^2}-\frac{kq^2}{(2a+\ell)^2}-\frac{kq^2}{(2a-\ell)^2}=$$

$$=kq^2\left[\frac{1}{2a^2}-\frac{1}{(2a+\ell)^2}-\frac{1}{(2a-\ell)^2}\right]=$$

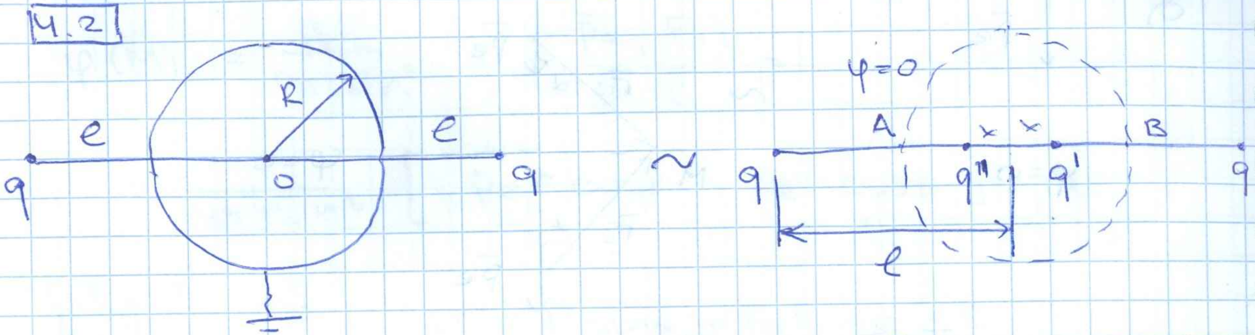
$$=kq^2\left[\frac{1}{2a^2}-\frac{2(4a^2+\ell^2)}{(4a^2-\ell^2)^2}\right]=$$

$$=kq^2\left[\frac{\cancel{16a^4}-8a^2\ell^2+\ell^4-\cancel{16a^4}-4a^2\ell^2}{2a^2(4a^2-\ell^2)^2}\right]=$$

$$=kq^2\ell^2\left[\frac{\ell^2-12a^2}{2a^2(4a^2-\ell^2)^2}\right]=\{\ell\ll a\}=$$

$$=kp_e^2\,\frac{-12a^2}{32a^6}=-\frac{3p_e^2}{32\pi\varepsilon_0 a^4}=2F_1\ \text{(отталкив).}$$

$\boxed{4.2}$



$$l = \frac{a}{2}$$

$$\begin{cases} \varphi_A = k\left[\dfrac{q}{l-R} + \dfrac{q'}{R-x}\right] = 0 \\ \varphi_B = k\left[\dfrac{q}{l+R} + \dfrac{q'}{R+x}\right] = 0 \end{cases} \xrightarrow{(4.1)} \begin{cases} x = \dfrac{R^2}{l} \\ q' = -q\dfrac{R}{l} \end{cases}$$

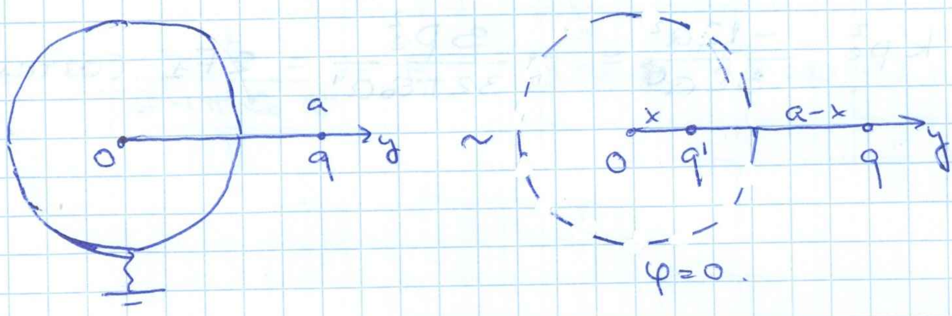
$$F = kq\left[\frac{q}{4l^2} - \frac{q'}{(l+x)^2} - \frac{q'}{(l-x)^2}\right] =$$

$$= kq^2\left[\frac{1}{4l^2} - \frac{Rl \cdot 2(l^4+R^4)}{(l^4-R^4)^2}\right] =$$

$$= \frac{kq^2}{4l^2}\left[1 - \frac{8Rl^3(l^4+R^4)}{(l^4-R^4)^2}\right] =$$

$$= \frac{kq^2}{a^2}\left[1 - 8\xi\frac{(1+\xi^4)}{(1-\xi^4)^2}\right], \quad \xi = \frac{2R}{l}$$

$\boxed{4.5}$ 1)



из ч.2 известно: $q' = -q\frac{R}{y}$

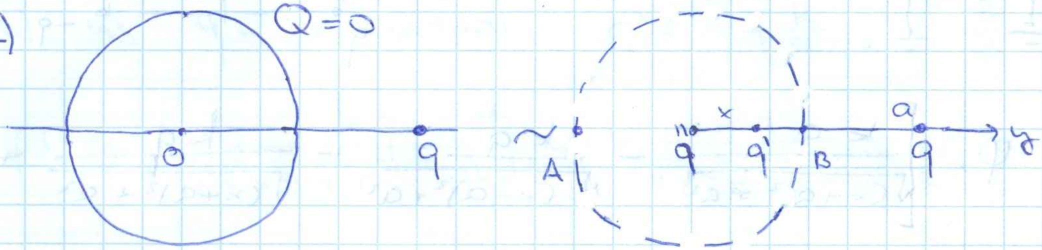
$$x = \frac{R^2}{y}$$

$$E_y = \frac{kq'}{(y-x)^2} = -kq\frac{Ry}{(y^2-R^2)^2}$$

$$A = -\int_a^{+\infty} qE_y\,dy = q\int_{+\infty}^{a} -\frac{kqRy\,dy}{(y^2-R^2)^2} = -\frac{kq^2R}{2}\int_{+\infty}^{a^2}\frac{dy^2}{(y^2-R^2)^2} =$$

$$= -\frac{kq^2R}{2}\int_{+\infty}^{a^2-R^2}\frac{dt}{t^2} = \frac{kq^2R}{2}\frac{1}{(a^2-R^2)} = \frac{q^2R}{8\pi\varepsilon_0(a^2-R^2)}$$

2)



$$\varphi_0 = \frac{kq}{y} + \oint\frac{k\sigma\,dS}{R} = k\left(\frac{q}{y} + \frac{Q}{R}\right) = \frac{kq}{y}$$

из ч.2 известно: $q' = -q\frac{R}{y}$; $x = \frac{R^2}{y}$

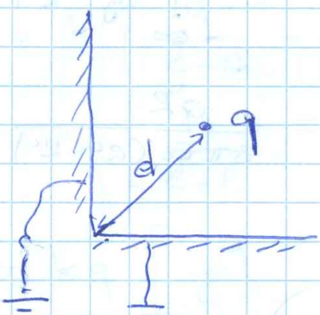
$$\varphi_0 = \frac{kq}{y} = \frac{kq''}{R} \Rightarrow q'' = q\frac{R}{y}$$

$$E_y = \frac{kq''}{y^2} + \frac{kq'}{(y-x)^2} = \frac{kqR}{y^3} - kq\frac{Ry}{(y^2-R^2)^2}$$

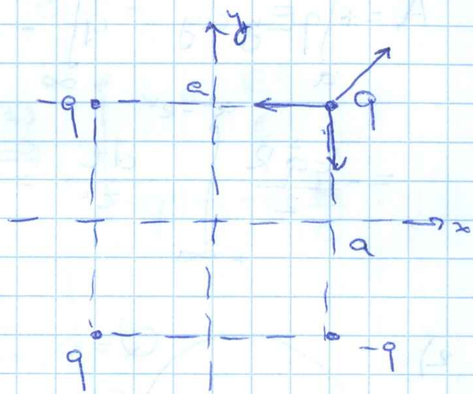
$$A = -\int_a^{+\infty} q E_y \, dy = -kq^2 R \left[-\int_a^{+\infty} \frac{y\,dy}{(y^2-R^2)^2} + \int_a^{+\infty} \frac{dy}{y^3} \right] =$$

$$= kq^2 R \left[\frac{1}{2(a^2-R^2)} - \frac{1}{2a^2} \right] = \frac{1}{8\pi\varepsilon_0} \frac{q^2 R^3}{a^2(a^2-R^2)}$$

$\boxed{4.6}$



$\sim$



$$x: \quad \varphi_x = \frac{kq}{\sqrt{(x+a)^2+a^2}} - \frac{kq}{\sqrt{(x-a)^2+a^2}} - \frac{kq}{\sqrt{(x+a)^2+a^2}} +$$

$$+ \frac{kq}{\sqrt{(x-a)^2+a^2}} = 0 \qquad a = \frac{\sqrt{2}}{2} d$$

в силу симметрии $\varphi_y = 0$.

$$\overline{F} = kq^2 \left[\frac{1}{4d^2} \left(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2} \right) + \frac{1}{2d^2}(-1, 0) + \frac{1}{2d^2}(0, -1) \right]$$

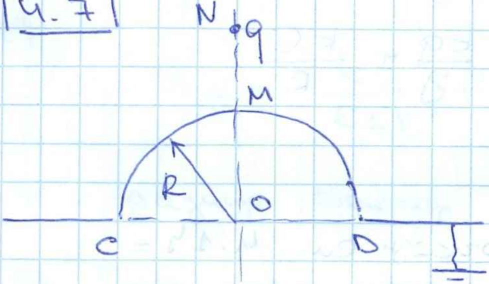
$$x: \quad -\frac{kq^2}{2d^2} + \frac{\sqrt{2}}{2} \frac{kq^2}{4d^2} = \frac{kq^2}{4d^2} \left(\frac{\sqrt{2}}{2} - 2 \right) = F_x$$

$$y: \quad F_y = \frac{kq^2}{4d^2} \left(\frac{\sqrt{2}}{2} - 2 \right)$$

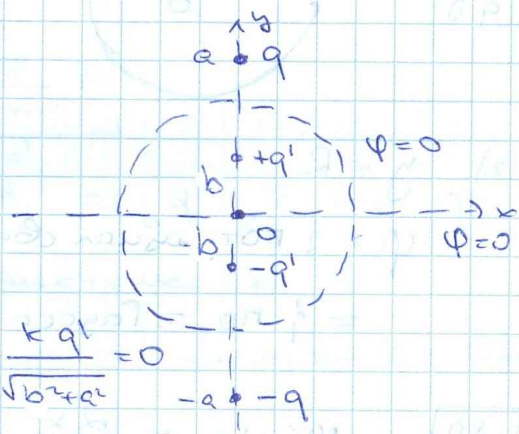
$$F = \sqrt{2}\, F_x = -(2\sqrt{2}-1)\frac{kq^2}{4d^2} = -\frac{q^2}{8\pi\varepsilon_0 d^2}\left(\sqrt{2}-\frac{1}{2}\right)$$

$$\bar{F} = F\left(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right).$$

$\boxed{4.7}$



$\sim$



1) $$\frac{kq}{\sqrt{x^2+a^2}} + \frac{kq'}{\sqrt{b^2+a^2}} - \frac{kq}{\sqrt{x^2+a^2}} + \frac{kq'}{\sqrt{b^2+a^2}} = 0$$

2) из 4.2 известно: $q' = -q\frac{R}{a}$; $b = \frac{R^2}{a}$

$$\varphi(x,y,z) = \frac{kq}{\sqrt{x^2+(y-a)^2+z^2}} - \frac{kq}{\sqrt{x^2+(y+a)^2+z^2}} +$$

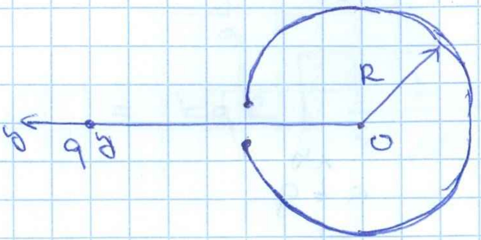
$$+ \frac{kq'}{\sqrt{x^2+(y-b)^2+z^2}} - \frac{kq'}{\sqrt{x^2+(y+b)^2+z^2}}$$

$\boxed{4.8}$ Начальное состояния соотв. случаю 4.5.1.

Конечное состояние соотв. случаю 4.5.2.

Тогда легко видеть, что с Земли перетечёт заряд $q_0 = -q'' = -q\frac{R}{d}$.

4.9



1) $y \geq R$

$$\varphi = \frac{kq}{y} + \oint \frac{k\sigma\, dS}{R} =$$

$$= \frac{kq}{y} + \frac{kQ}{R}$$

2) $y \leq R$

$\varphi = \{$ поле вне сферы однородн. см. 4.14 $\} =$

$= \{$ по т. Гаусса: $\oint E\, dS = \frac{q+Q}{\varepsilon_0} \} = k\frac{q+Q}{R}$

4.10 $\quad \varphi(x) = -\frac{\alpha x^2}{2} + C$

$$\overline{E} = -\operatorname{grad}\varphi = -\frac{\partial \varphi}{\partial x}\,\overline{i} = \alpha x \cdot \overline{i}$$

E параллельно оси Ox.

Воспользуемся ур-нем Пуассона:

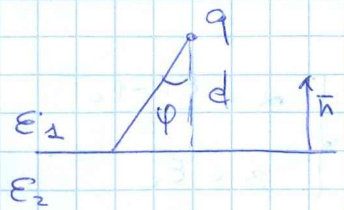
$$\Delta\varphi = \frac{\partial^2}{\partial x^2}\varphi(x) = -\alpha = -\frac{\rho}{\varepsilon_0}$$

$$\Downarrow$$

$$\rho = +\alpha\varepsilon_0$$

Полоса с объёмной плотностью заряда ρ.

4.11



1) Посчитаем поле в диэлектр. 1.

Предположим, что оно создаётся зарядом q и q' (симметр. q).

$$\bar{E}_1 = \frac{kq}{\varepsilon_1 r^3}\bar{r} + \underbrace{\frac{kq'}{\varepsilon_1 r'^3}\bar{r}'}_{\bar{E}_1'}$$

$\bar{r}$ – радиус вектор в т. M от q, $\bar{r}'$ – от q'.

2) Посчитаем поле в диэлектрике 2.

Предположим, что оно создаётся зарядом q'' вместо q

$$\bar{E}_2 = \frac{kq''}{\varepsilon_2 r^3}\bar{r}$$

3) E_1 и E_2 должны удовл. граничным условиям.

$$E_{1\tau} = E_{2\tau}: \quad \frac{q}{\varepsilon_1}\cancel{\sin\varphi} + \frac{q'}{\varepsilon_1}\cancel{\sin\varphi} = \frac{q''}{\varepsilon_2}\cancel{\sin\varphi}$$

$D_{n_1} - D_{n_2} = \sigma$: т.к. против нормали $(-\bar{n})$.

$$[q\cancel{\cos\varphi} - q'\cancel{\cos\varphi}] - q''\cancel{\cos\varphi} = 0.$$

$$\Downarrow$$

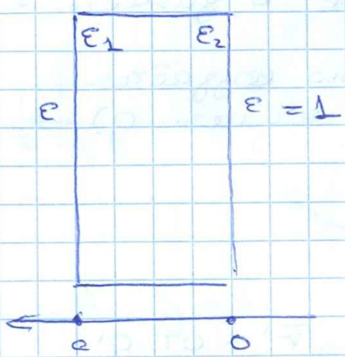
$$q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}q; \quad q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2}q$$

$$F = \bar{E}_1' q = \cancel{\frac{kq}{\varepsilon_1 r^3}\bar{r}} - \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\frac{\bar{r}'}{r'^3}\cdot\frac{kq^2}{\varepsilon_1} = \ldots =$$

$$= -\frac{kq^2}{\varepsilon_1}\cdot\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\cdot\frac{1}{4d^2}\bar{n} \text{ (притягивается)}$$

$\boxed{4.13}$

(1)



$$\begin{cases} \varepsilon(x) = \alpha x + \beta \\ \varepsilon(0) = \varepsilon_2 \\ \varepsilon(a) = \varepsilon_1 \end{cases} \Rightarrow \begin{cases} \alpha = \dfrac{\varepsilon_1 - \varepsilon_2}{a} \\ \beta = \varepsilon_2 \end{cases}$$

$$\varepsilon(x) = \frac{\varepsilon_1 - \varepsilon_2}{a} x + \varepsilon_2 =$$

$$= \frac{(\varepsilon_1 - \varepsilon_2)x + \varepsilon_2 a}{a}$$

$$E(x) = \frac{D}{\varepsilon(x)\varepsilon_0} = \frac{E}{\varepsilon(x)} = \frac{E a}{(\varepsilon_1 - \varepsilon_2)x + \varepsilon_2 a}$$

$$E(x) = -\operatorname{grad}\varphi(x) = -\frac{\partial \varphi}{\partial x} =$$

$$\Delta\varphi = \frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial}{\partial x}\left(-E(x)\right) = Ea \frac{\varepsilon_1 - \varepsilon_2}{[(\varepsilon_1 - \varepsilon_2)x + \varepsilon_2 a]^2} =$$

$$= \frac{E}{\varepsilon(x)^2} \frac{\varepsilon_1 - \varepsilon_2}{a}$$

$$\Delta\varphi = -\frac{\rho}{\varepsilon\varepsilon_0} = -\frac{\rho}{\varepsilon_0} \quad \text{(ур-ие Лапласа)}$$

$$\rho = -\frac{\alpha \varepsilon_0 E}{(\alpha x + \varepsilon_2)^2}, \quad \text{где} \quad \alpha = \frac{\varepsilon_1 - \varepsilon_2}{a}$$

(2) $\varepsilon_1 = 2$

$\varepsilon_2 = 4$

$a = 1\ \text{см} = 10^{-2}\ \text{м}$

$E = 3 \cdot 10^3\ \frac{\text{В}}{\text{м}}$

$$\alpha = \frac{\varepsilon_1 - \varepsilon_2}{a} = -2 \cdot 10^2\ \text{м}^{-1}$$

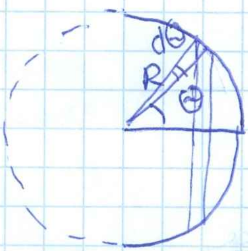
$$P\left(\frac{a}{2}\right) = \frac{2\cdot 10^2\,м^{-1}\cdot 8{,}85\cdot 10^{-12}\,\frac{Кл^2}{Н\cdot м^2}\cdot 3\cdot 10^3\,\frac{В}{М}}{\left(-2\cdot 10^2\,м^{-1}\,\frac{1}{2}\cdot 10^{-2}\,м + 4\right)^2} =$$

$$= 5.9\cdot 10^{-7}\,\frac{Кл^2}{\cancel{Н}\cdot м^3}\cdot\frac{\cancel{Н}}{\cancel{Кл}} = 0.59\,\frac{мкКл}{м^3}$$

$\boxed{4.14.}$ Из задачи 3.21 известно, что поле создаваем на поверхности сферы создают однородное поле внутри сферы.

$$\sigma = \sigma_{max}\cos\Theta = P\cos\Theta \quad (3.20)$$

Легко видеть, что $\sigma_{max} = P = (\varepsilon - 1)\varepsilon_0 E$



$$S_{кольца} = 2\pi R\sin\Theta\; R\,d\Theta$$

$$dS = R^2\sin\Theta\, d\Theta\, d\varphi$$

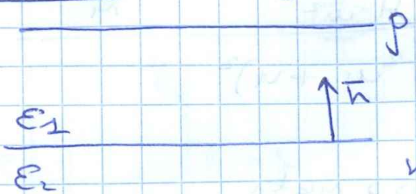
$$dq = \sigma(\Theta)\, dS$$

$$Q = \int_0^{2\pi} d\varphi \int_{-\pi/2}^{\pi/2} \sigma_{max}\cos\Theta\, R^2 \sin\Theta\, d\Theta =$$

$$= 2\pi\cdot\sigma_{max}R^2 \int_{-\pi/2}^{\pi/2} \frac{\sin 2\Theta}{4}\, d(2\Theta) =$$

$$= \sigma_{max}\,\pi R^2\,\frac{1}{2}\left[-\cos 2\Theta\right]\Big|_{-\pi/2}^{\pi/2} = \sigma_{max}\,\pi R^2$$

$\boxed{4.15}$ (задача абсолютно аналогична 4.11)



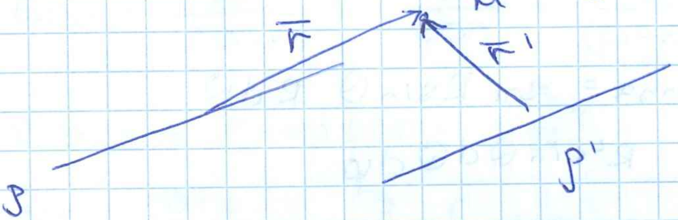
1) Найдём поле в 1 диэл.

Предположим, что оно создаётся нитью ρ и нитью ρ' (симм. ρ)

$$\bar{E}_1 = \frac{1}{2\pi\varepsilon_0\varepsilon_1}\left(\frac{\rho\bar{r}}{r^2} + \frac{\rho'\bar{r}'}{r'^2}\right) \quad (3.12)$$

$\bar{r}$ – вектор перпенд. нити ρ и проходящий (оканчивающийся) в т. М.

$\bar{r}'$ – аналогично для ρ'.

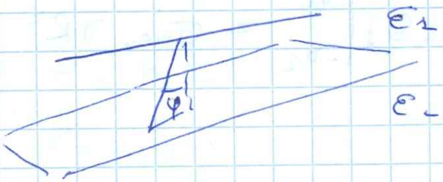


2) найдём поле во 2 м диэлектрике

Предположим, что оно создаётся нить ρ'' вместо ρ

$$\bar{E}_2 = \frac{1}{2\pi\varepsilon_0\varepsilon_2}\,\frac{\rho\bar{r}}{r^2}$$

3) граничные условия.



$E_{\tau 1} = E_{\tau 2}$:

$$\frac{\rho}{\varepsilon_1}\cancel{\sin\varphi} + \frac{\rho'}{\varepsilon_1}\cancel{\sin\varphi} = \frac{\rho''}{\varepsilon_2}\cancel{\sin\varphi}$$

$$D_{n1} - D_{n2} = \sigma: \quad [\rho\cancel{\cos\varphi} - \rho'\cancel{\cos\varphi}] - \rho''\cancel{\cos\varphi} = 0$$

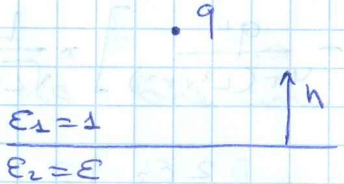
$$p' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} p \qquad p'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} p$$

$\overline{F} = \overline{E}_1 \, dq = \Big\{$ в силу того, что силовые линии $\perp$ нити $\Big\} =$

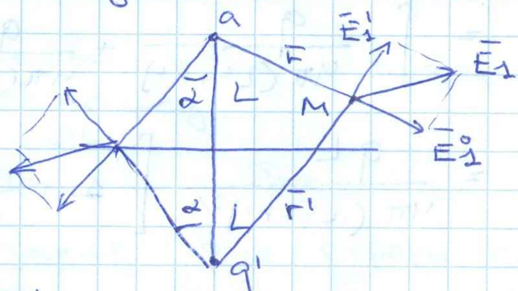
$$= \frac{-dq}{2\pi\varepsilon_0\varepsilon_1} \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} \frac{\overline{r}'}{r'^2} = -\frac{p^2 \, dl}{4\pi\varepsilon_0\varepsilon_1 d} \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} \overline{n}$$

$$\overline{F}_l = \frac{\overline{F}}{dl} = -\frac{p^2}{4\pi\varepsilon_0\varepsilon_1 d} \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} \overline{n} \quad \text{(притяжение)}$$

$\boxed{4.16}$

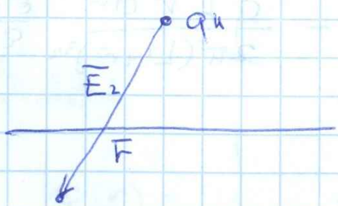


1) найдём поле в (1) области:



$$\overline{E} = \frac{kq}{\varepsilon_1 r^3} \overline{r} + \frac{kq'}{\varepsilon_1 r'^3} \overline{r}'$$

2) найдём поле в (2) области



$$\overline{E}_2 = \frac{kq''}{\varepsilon_2 \overline{r}^3} \overline{r}$$

3) граничные условия.

$$E_{\tau_1} = E_{\tau_2}: \quad \frac{q}{\varepsilon_1} \sin\alpha + \frac{q'}{\varepsilon_1} \cancel{\sin\alpha} = \frac{q''}{\varepsilon_2} \cancel{\sin\alpha}$$

$$D_{n_1} = D_{n_2}: \quad q \cancel{\cos\alpha} - q' \cancel{\cos\alpha} = q'' \cancel{\cos\alpha}$$

$$q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \quad ; \quad q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} q$$

Мы получили систему эквив. исходной.
Теперь ~~границу~~ границу между диэлектриком (ε_2) и вакуумом (ε_1) заменить на металлическую

Тогда,

$$\sigma(r) = D_{n_1} - D_{n_2} = \left[\varepsilon_1 \varepsilon_0 E_1 - \varepsilon_2 \varepsilon_0 E_2\right] \cos\alpha =$$

$$= \left[\frac{q}{4\pi(L^2+r^2)} + \frac{q'}{4\pi(L^2+r^2)} - \frac{q''}{4\pi(L^2+r^2)}\right] \frac{L}{\sqrt{L^2+r^2}}$$

$$= \frac{qL}{4\pi(L^2+r^2)^{3/2}} \left[1 + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} - \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2}\right] =$$

$$= -\frac{qL}{2\pi(L^2+r^2)^{3/2}} \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} = -\frac{qL}{2\pi(L^2+r^2)^{3/2}} \frac{\varepsilon - 1}{\varepsilon + 1}$$

$$Q = \int_0^{2\pi} d\varphi \int_0^{+\infty} \sigma(r)\, dr \cdot r = \cancel{2\pi} \int_0^{+\infty} -\frac{qL\, r\, dr}{\cancel{2\pi}(L^2+r^2)^{3/2}} \frac{\varepsilon - 1}{\varepsilon + 1} =$$

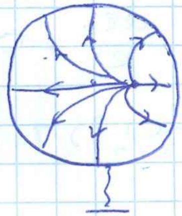
$$= -qL \cdot \frac{\varepsilon - 1}{\varepsilon + 1} \int_0^{+\infty} \frac{r\, dr}{(L^2+r^2)^{3/2}} =$$

$$= q \frac{\varepsilon - 1}{\varepsilon + 1} L \left[\frac{1}{\sqrt{L^2 + x^2}}\right]\Bigg|_0^{+\infty} = -q \frac{\varepsilon - 1}{\varepsilon + 1}$$

$$\bar{F} = \bar{E}_1' q = -\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\cdot\frac{\bar{r}'}{r'^3}\cdot\frac{kq^2}{\varepsilon_1} =$$

$$= -\frac{\varepsilon-1}{\varepsilon+1}\,\frac{kq}{L^2}\,\bar{n} \quad \text{(притягивается)}.$$

$\boxed{4.17}$ 1) Из задачи 3.5 известно, что заряд на внутр. оболочке сферы не взаимодействует с зарядом на внешней оболочке.



Сфера заземлена $\Rightarrow$ $\varphi = 0$.

В случае незаземлённой сферы: $\varphi = \frac{kq}{R}$

Т.о. с Земли перетечёт заряд $-q$ на внешнюю оболочку.

$\Downarrow$

поле вне сферы не будет.

2) На другой заряд вне сферы силы будут действовать, т.к. на внеш. оболочке появится заряд (с Земли) такой, чтобы поле внутри сферы осталось прежним. Этот поверхностный заряд и будет взаимодействовать.

$\boxed{4.18}$

$$\varphi(r) = \begin{cases} a/r, & r \geq R \\ -\dfrac{ar^2}{2R^3} + \dfrac{3a}{2R}, & r < R \end{cases}$$

~~$\varphi(r) = \varphi(0) + \varphi_1(r)$~~

$\varphi(0) = \frac{3a}{R}$ (сфера ?)

$$\cancel{\varphi_1 = \begin{cases} a/r - \dfrac{3a}{R}, & r \geq R \\ -\dfrac{ar^2}{2R^3}, & r < R \end{cases}}$$

$q = 12\pi\varepsilon_0 a$

$$E = -\mathrm{grad}\,\varphi = -\frac{\partial \varphi}{\partial r} = \begin{cases} \dfrac{a}{r^2}, & r \geq R \\ \dfrac{ar}{R^3}, & r < R \end{cases}$$

$$D = \varepsilon_0 E = \begin{cases} \dfrac{a\varepsilon_0}{r^2}, & r \geq R \\ \dfrac{a\varepsilon_0 r}{R^3}, & r < R \end{cases}$$

$$\sigma = \lim_{r \to R+0} D(r) - \lim_{r \to R-0} D(r) = \frac{a\varepsilon_0}{R^3} - \frac{a\varepsilon_0}{R^3} = 0$$

(сферы нет?)

$$-\frac{\rho}{\varepsilon_0} = \Delta\varphi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\varphi_1}{dr}\right) =$$

$$= \frac{1}{r^2} \cdot \begin{cases} 0, & r > R \\ -\dfrac{3ar^2}{R^3}, & r < R \end{cases}$$

$$\rho = \begin{cases} 0, & r > R \\ \dfrac{3a\varepsilon_0}{R^3}, & r < R \end{cases}$$ — равномерно заряженный шар ($r < R$).

$\boxed{4.19}$

$$\varphi(r) = \begin{cases} 0, & r \geq R \\ a\left(\dfrac{1}{r} + \dfrac{r^2}{2R^3} - \dfrac{3}{2R}\right), & r < R \end{cases}$$

$\varphi(r) \sim \dfrac{a}{r}$ ($r \to 0$) — заряд $q = 4\pi\varepsilon_0 a$

$$\varphi(r) = \frac{a}{r} + \underbrace{\begin{cases} -\dfrac{a}{r}, & r \geq R \\ a\left(\dfrac{r^2}{2R^3} - \dfrac{3}{2R}\right), & r < R \end{cases}}_{= \varphi_1(r)}$$

$$E = -\operatorname{grad} \varphi_1 = -\frac{\partial \varphi_1}{\partial r} = \begin{cases} -\dfrac{a}{r^2}, & r \geq R \\ -\dfrac{ar}{R^3}, & r < R \end{cases}$$

$$D(r) = \varepsilon_0 E(r) = \begin{cases} -\dfrac{a\varepsilon_0}{r^2}, & r \geq R \\ -\dfrac{a\varepsilon_0 r}{R^3}, & r < R \end{cases}$$

$$\sigma = \lim_{r \to R+0} D(r) - \lim_{r \to R-0} D(r) = -\frac{a\varepsilon_0}{R^2} + \frac{a\varepsilon_0}{R^2} = 0$$

$$-\frac{\rho}{\varepsilon_0} = \Delta \varphi_1 = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\varphi_1}{dr}\right) = \frac{1}{r^2} \cdot \begin{cases} 0, & r \geq R \\ \dfrac{3ar^2}{R^3}, & r < R \end{cases}$$

$$\rho = \begin{cases} 0, & r \geq R \\ -\dfrac{3\varepsilon_0 a}{R^3}, & r < R \end{cases} \quad (\text{шар})$$

$\boxed{4.20}$

$$\varphi(r) = \begin{cases} 0, & r \geq R \\ a(R^3 - r^3), & r < R \end{cases}$$

$$\lim_{r \to 0+0} \varphi(r) = \varphi(0) = aR^3$$

$$E = -\operatorname{grad}\varphi = -\frac{\partial\varphi}{\partial r} = \begin{cases} 0, & r \geq R \\ 3ar^2, & r < R \end{cases}$$

$$D(r) = \varepsilon_0 E(r) = \begin{cases} 0, & r \geq R \\ 3a\varepsilon_0 r^2, & r < R \end{cases}$$

$$\sigma = \lim_{r\to R+0} D(r) - \lim_{r \to R-0} D(r) = -3a\varepsilon_0 R^2 \quad - \text{сфера}$$

$$-\frac{\rho}{\varepsilon_0} = \Delta\varphi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\varphi}{dr}\right) = \frac{1}{r^2}\cdot\begin{cases} 0, & r \geq R \\ -12ar^3, & r < R \end{cases}$$

$$\rho = \begin{cases} 0, & r > R \\ +12a\varepsilon_0 r, & r < R \end{cases}$$

$\boxed{4.21}$

$$\varphi(r) = \begin{cases} 0, & r \geq R \\ a\left(\frac{1}{r} - \frac{r^2}{R^3}\right), & r < R \end{cases}$$

$$\varphi(r) \sim \frac{a}{r}, \quad r \to 0+0 \quad - \text{заряд } q = 4\pi\varepsilon_0 a$$

$$\varphi(r) = \frac{a}{r} + \varphi_1(r) = \frac{a}{r} + \begin{cases} -\frac{a}{r}, & r \geq R \\ -\frac{ar^2}{R^3}, & r < R \end{cases}$$

$$E = -\operatorname{grad}\varphi_1 = -\frac{\partial\varphi_1}{\partial r} = \begin{cases} -\frac{a}{r^2}, & r \geq R \\ \frac{2ar}{R^3}, & r < R \end{cases}$$

$$D(r) = \varepsilon_0 E(r) = \begin{cases} -\dfrac{a\varepsilon_0}{r^2}, & r \geq R \\ \dfrac{2a\varepsilon_0 r}{R^3}, & r < R \end{cases}$$

$$\sigma = \lim_{r \to R+0} D(r) - \lim_{r \to R-0} D(r) = -\frac{a\varepsilon_0}{R^2} + \frac{2a\varepsilon_0}{R^2} =$$

$$= -\frac{3a\varepsilon_0}{R^2} \quad \text{– сфере}$$

$$-\frac{\rho}{\varepsilon_0} = \Delta\varphi_1 = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\varphi}{dr}\right) =$$

$$= \frac{1}{r^2} \cdot \begin{cases} 0, & r \geq R \\ -\dfrac{6ar^2}{R^3}, & r < R \end{cases}$$

$$\rho = \begin{cases} 0, & r > R \\ \dfrac{6a\varepsilon_0}{R^3}, & r < R \end{cases} \quad \text{– шар}$$

$\boxed{4.22}$

$$\varphi(r) = \frac{b}{r} e^{-ar}$$

$$\varphi(r) \sim \frac{b}{r} \quad (r \to 0+0) \quad \text{– заряд } q = 4\pi\varepsilon_0 b$$

$$\varphi(r) = \frac{b}{r} + \varphi_1(r) = \frac{b}{r} + \frac{b}{r}\left(e^{-ar} - 1\right)$$

$$\frac{\partial\varphi}{\partial r} = \frac{-abe^{-ar}r - be^{-ar} + b}{r^2}$$

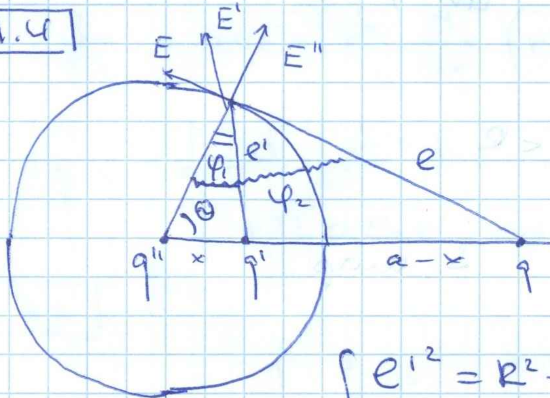
$$-\frac{\rho}{\varepsilon_0} = \Delta\varphi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right) =$$

$$\rho = -\frac{a^2be^{-ar}}{r}\varepsilon_0$$

шар

$$= \frac{1}{r^2}\left[a^2be^{-ar}r - abe^{-ar} + abe^{-ar}\right]$$

$\boxed{4.4}$



из 4.5.2 известно:

$$x = \frac{R^2}{a} \qquad q' = -q\frac{R}{a}$$

$$q'' = q\frac{R}{a}$$

$$\begin{cases} e'^2 = R^2 + x^2 - 2Rx\cos\Theta \\ e^2 = R^2 + a^2 - 2Ra\cos\Theta \\ x^2 = R^2 + e'^2 - 2Re'\cos\varphi_1 \\ a^2 = R^2 + e^2 - 2Re\cos\varphi_2 \end{cases}$$

$$\Downarrow$$

$$\cos\varphi_1 = \frac{R^2 + (R^2 + x^2 - 2Rx\cos\Theta) - x^2}{2Re'} =$$

$$= \frac{R - x\cos\Theta}{e'}$$

$$\cos\varphi_2 = \frac{R - a\cos\Theta}{e}$$

$$E_n = E'' + E' \cos\varphi_1 + E\cos\varphi_2 =$$

$$= \frac{kq''}{R^2} + \frac{kq'}{e'^2}\cos\varphi_1 + \frac{kq}{e^2}\cos\varphi_2 =$$

$$= kq\left[\frac{R}{aR^2} - \frac{R}{a e'^2}\,\frac{R - x\cos\Theta}{e'} + \frac{1}{e^2}\,\frac{R - a\cos\Theta}{e}\right] =$$

$$= kq\left[\frac{1}{aR} - \frac{R}{a}\cdot\frac{aR - R^2\cos\Theta}{a}\left(R^2 + x^2 - 2Rx\cos\Theta\right)^{-3/2} + \right.$$

$$\left. + \frac{R - a\cos\Theta}{\left(R^2 + a^2 - 2Ra\cos\Theta\right)^{3/2}}\right] =$$

$$= kq\left[\frac{1}{aR} - \frac{R^2(a - R\cos\Theta)}{a^2}\left(R^2 + \frac{R^4}{a^2} - 2R\frac{R^2}{a}\cos\Theta\right)^{-3/2} + \right.$$

$$\left. + \frac{R - a\cos\Theta}{\left(R^2 + a^2 - 2Ra\cos\Theta\right)^{3/2}}\right] =$$

$$= kq\left[\frac{1}{aR} - \frac{R^2(a - R\cos\Theta)}{a^2}\left(\frac{R^2\left(a^2 + R^2 - 2Ra\cos\Theta\right)}{a^2}\right)^{-3/2} + \right.$$

$$\left. + \frac{R - a\cos\Theta}{\left(R^2 + a^2 - 2Ra\cos\Theta\right)^{3/2}}\right] =$$

$$= kq\left[\frac{1}{aR} - \frac{R^2(a - R\cos\Theta)}{a^2}\,\frac{a^3}{R^3}\,\frac{1}{\left(R^2 + a^2 - 2Ra\cos\Theta\right)^{3/2}} + \right.$$

$$\left. + \frac{R - a\cos\Theta}{\left(R^2 + a^2 - 2Ra\cos\Theta\right)^{3/2}}\right] =$$

$$= kq\left[\frac{1}{aR} - \frac{a^2 - \cancel{Ra\cos\theta} - R^2 + \cancel{Ra\cos\theta}}{R(R^2+a^2-2Ra\cos\theta)^{3/2}}\right] =$$

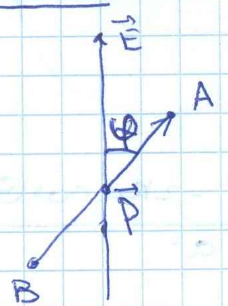
$$= \frac{kq}{Ra}\left[1 - \frac{a(a^2-R^2)}{(R^2+a^2-2Ra\cos\theta)^{3/2}}\right] =$$

$$= \frac{kq}{Ra}\left[1 - \frac{1-\xi^2}{(1-2\xi\cos\theta+\xi^2)^{3/2}}\right]; \quad \xi = \frac{R}{a}$$

$$D_n = \varepsilon_0 E_n = \sigma$$

$$\sigma = \frac{q}{4\pi Ra}\left[1 - \frac{1-\xi^2}{(1-2\xi\cos\theta+\xi^2)^{3/2}}\right]; \quad \xi = \frac{R}{a}.$$

$\boxed{4.12}$



$$\Pi = q(\varphi_A - \varphi_B) = q\int_A^B \vec{E}\,d\vec{l} = -q\vec{E}\vec{l} =$$
$$= -qEl\cos\varphi$$

$$K = \frac{J\omega^2}{2} = \frac{J\dot{\varphi}^2}{2}$$

Известно, что для стержня:

$$J = \frac{ml^2}{12} =$$

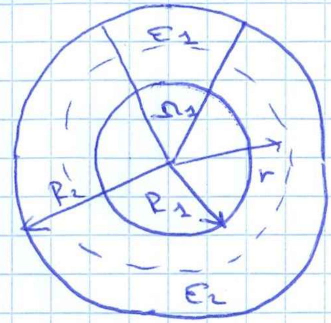
$$L = K - \Pi = \frac{J\dot{\varphi}^2}{2} + qEl\cos\varphi$$

$$\frac{\partial L}{\partial\dot{\varphi}} = J\dot{\varphi} \qquad \frac{\partial L}{\partial\varphi} = -qEl\sin\varphi$$

$$J\ddot{\varphi} + qEl\varphi = 0$$

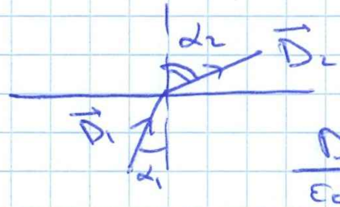
$$\ddot{\varphi} + \frac{qEl}{J}\varphi = 0 \Rightarrow \omega = \sqrt{\frac{pE}{J}} = \frac{2}{l}\sqrt{\frac{pE}{m}}$$

$\boxed{4.23}$



Рассмотрим поле на границе диэлектриков и металла:

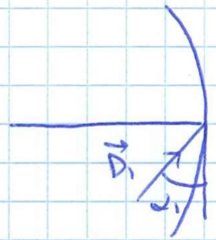
(1)



$$D_1 \cos\alpha_1 = D_2 \cos\alpha_2$$

$$\frac{D_1}{\varepsilon_0 \varepsilon_1} \sin\alpha_1 = \frac{D_2}{\varepsilon_2 \varepsilon_0} \sin\alpha_2$$

(2):



$$D_1 \sin\alpha_1 = \sigma \qquad D_2 \sin\alpha_2 = \sigma$$

$$\frac{D_1}{\varepsilon_1 \varepsilon_0} \cos\alpha_1 = 0 \qquad \frac{D_2}{\varepsilon_2 \varepsilon_0} \cos\alpha_2 = 0$$

$\Rightarrow \alpha_1 = \alpha_2 = \frac{\pi}{2} \Rightarrow$ поле радиально

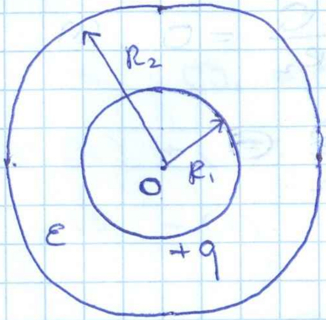
Воспользуемся т. Гаусса:

$$\oint D\, dS = Q \; ; \quad r \in [R_1; R_2]$$

$$\varepsilon_1 \varepsilon_0 E \Omega_1 r^2 + \varepsilon_2 \varepsilon_0 E \Omega_2 r^2 = Q$$

$$E = \frac{Q}{\varepsilon_0 r^2} \frac{1}{\Omega_1 \varepsilon_1 + \Omega_2 \varepsilon_2}$$

5.1.



$$\varphi = \int_{R_1}^{\infty} E\,dr = \int_{R_1}^{R_2} \frac{kq}{\varepsilon r^2}\,dr + \int_{R_2}^{\infty} \frac{kq}{r^2}\,dr = \frac{-q}{4\pi\varepsilon\varepsilon_0}\frac{1}{r}\Big|_{R_1}^{R_2} + \frac{-q}{4\pi\varepsilon_0}\frac{1}{r}\Big|_{R_2}^{+\infty} =$$

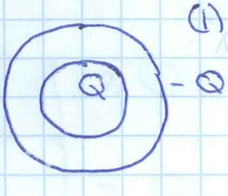
$$= \frac{q}{4\pi\varepsilon\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] + \frac{q}{4\pi\varepsilon_0}\frac{1}{R_2} =$$

$$= \frac{q}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{R_1} + \frac{\varepsilon - 1}{R_2}\right]$$

$$C = \frac{q}{\varphi} = \frac{4\pi\varepsilon\varepsilon_0}{\frac{1}{R_1} + \frac{\varepsilon-1}{R_2}} = \frac{4\pi\varepsilon\varepsilon_0 R_1}{1 + (\varepsilon - 1)\frac{R_1}{R_2}}$$

5.2

(1) до



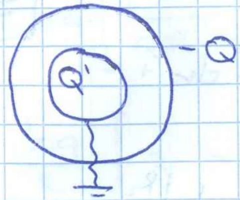
$$\varphi_1 = \frac{kQ}{R_1} - \frac{kQ}{R_2}$$

$$\varphi_2 = \frac{kQ}{R_2} - \frac{kQ}{R_2} = 0$$

$$W_{(1)} = \frac{1}{2}\left[Q\varphi_1 - Q\varphi_2\right] = \frac{Q}{2}\left[\frac{kQ}{R_1} - \frac{kQ}{R_2}\right] =$$

$$= \frac{kQ^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{kQ^2}{2}\left[\frac{R_2 - R_1}{R_1 R_2}\right] = \frac{CU^2}{2}$$

(2) после



$$\varphi_1 = 0 = \frac{kQ'}{R_1} - \frac{kQ}{R_2} = 0 \quad (1)$$

$$\varphi_2 = \frac{kQ'}{R_2} - \frac{kQ}{R_2} =$$

$$(1) \Rightarrow Q' = Q\frac{R_1}{R_2}$$

$$= \frac{kQ}{R_2}\left[\frac{R_1}{R_2} - 1\right]$$

$$W_{(2)} = \frac{1}{2}\left[Q'\varphi_1 - Q\varphi_2\right] = \frac{-kQ^2}{2R_2}\left[\frac{R_1}{R_2} - 1\right] =$$

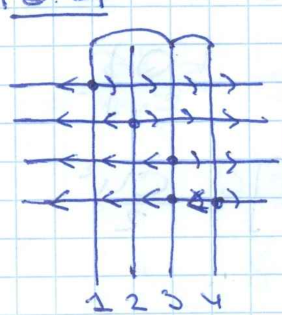
$$= \frac{kQ^2}{2} \cdot \frac{R_2 - R_1}{R_2 R_2}$$

$$\frac{W_{(2)}}{W_{(1)}} = \frac{R_1}{R_2} \Rightarrow W_{(2)} = \frac{R_1}{R_2} W_{(1)} = \frac{R_1}{R_2} \cdot \frac{CU^2}{2}$$

$$\Delta W = W_{(2)} - W_{(1)} = -\frac{CU^2}{2}\left(1 - \frac{R_1}{R_2}\right) =$$

$$= -2\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1} \cdot \frac{R_2 - R_1}{R_2} U^2 = -2\pi\varepsilon_0 R_1 U^2$$

$\boxed{5.3}$



$$Q_2 = Q \quad (1)$$

$$Q_1 + Q_3 + Q_4 = -Q \quad (2)$$

$$\varphi_3 - \varphi_4 = \frac{Q_1 + Q_2 + Q_3 - Q_4}{2\varepsilon_0 S} d_1 = 0 \Rightarrow$$

$$\Rightarrow \quad Q_1 + Q_2 + Q_3 - Q_4 = 0 \quad (3)$$

$$(1); \ (3) + (2): -\cancel{2} Q = \cancel{2}(Q_1 + Q_3) \quad (4)$$

$$(3) - (2): \quad Q_4 = 0$$

$$\varphi_1 - \varphi_3 = \frac{Q_1 - Q_2 - Q_3 - Q_4}{2\varepsilon_0 S} d_1 +$$

$$+ \frac{Q_1 + Q_2 - Q_3 - Q_4}{2\varepsilon_0 S} d = 0$$

$$(Q_1 - Q_3 - Q) d_1 + (Q_1 - Q_3 + Q) d = 0$$

$$(Q_1 - Q_3)(d_1 + d) = Q(d_1 - d)$$

$$\begin{cases} Q_1 - Q_3 = Q \dfrac{d_1 - d}{d_1 + d} \\ Q_1 + Q_3 = -Q \end{cases}$$

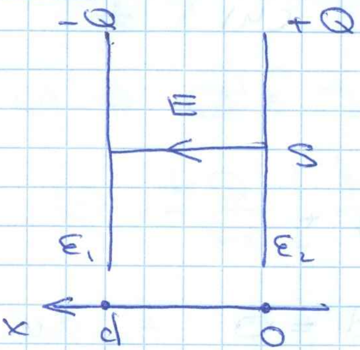
$$\begin{cases} 2Q_1 = -Q\left(\dfrac{d - d_1}{d + d_1} + 1\right) = -Q \dfrac{2d}{d + d_1} \\ 2Q_3 = -Q\left(1 - \dfrac{d - d_1}{d + d_1}\right) = -Q \dfrac{2d_1}{d + d_1} \end{cases}$$

$$U = \varphi_2 - \varphi_3 = \frac{Q_1 + Q_2 - Q_3 - Q_4}{2\varepsilon_0 S} d =$$

$$= \frac{d}{2\varepsilon_0 S} \cdot \left[\frac{-d + d_1}{(d + d_1)} Q + Q\right] = \frac{d}{2\varepsilon_0 S} \cdot \frac{2d_1}{(d + d_1)} Q$$

$$C = \frac{Q}{\varphi} = \frac{2\varepsilon_0 S(d+d_1)}{2dd_1} = \{d = 2d_1\} = \frac{3\varepsilon_0 S}{d} = 3C_0.$$

$\boxed{5.4}$



$$\begin{cases} \varepsilon(x) = \alpha x + \beta \\ \varepsilon(0) = \varepsilon_2 & \beta = \varepsilon_2 \\ \varepsilon(d) = \varepsilon_1 & \alpha = \dfrac{\varepsilon_1 - \varepsilon_2}{d} \end{cases}$$

$$\varepsilon(x) = \frac{\varepsilon_1 - \varepsilon_2}{d} x + \varepsilon_2$$

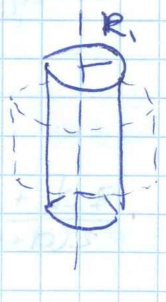
$$U = \int_0^d \frac{Q}{\varepsilon(x)\varepsilon_0 S} dx = \frac{Q}{\varepsilon_0 S} \int_0^d \frac{1}{\alpha x + \beta} dx =$$

$$= \frac{Q}{\varepsilon_0 S} \frac{\ln(\alpha x + \beta)}{\alpha} \Big|_0^d = \frac{Q}{\alpha \varepsilon_0 S} \left[\ln \varepsilon_1 - \ln \varepsilon_2\right] =$$

$$= \frac{Q}{\varepsilon_0 S} \frac{d}{\varepsilon_1 - \varepsilon_2} \ln \frac{\varepsilon_1}{\varepsilon_2}$$

$$C = \frac{Q}{U} = \frac{(\varepsilon_1 - \varepsilon_2)\varepsilon_0 S}{d \ln \frac{\varepsilon_1}{\varepsilon_2}}$$

$\boxed{5.5}$



по т. Гаусса:

$$\oint D\, dS = 2\pi R_1 \cdot l \cdot \sigma$$

$$\varepsilon\varepsilon_0 E \cdot 2\pi r \cdot \cancel{l} = 2\pi R_1 \cdot \cancel{l} \cdot \sigma$$

$$E = \frac{\sigma R_1}{\varepsilon\varepsilon_0 r}$$

$$U = \varphi_1 - \varphi_2 = \int_{R_1}^{R_2} E\,dr = \frac{\sigma R_1}{\varepsilon\varepsilon_0}\int_{R_1}^{R_2}\frac{1}{r}\,dr = \frac{\sigma R_1}{\varepsilon\varepsilon_0}\ln\frac{R_2}{R_1}$$

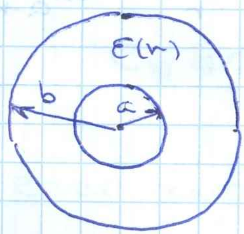
т.о. а) при изменении радиусов $U = \text{const}$

б) ~~двет~~ $E_1 = \frac{\sigma_1 \cancel{R_1}}{\varepsilon\varepsilon_0 \cancel{R_1}}$ $\quad E_2 = \frac{\sigma_2 \cancel{2R_1}}{\varepsilon\varepsilon_0 \cancel{2R_1}}$

$$\sigma_1 = \frac{dQ}{2\pi R_1 \cdot l} \qquad \sigma_2 = \frac{dQ}{4\pi R_1 \cdot l} \Rightarrow \sigma_2 = \frac{\sigma_1}{2}$$

$\Rightarrow E_2 = \frac{E_1}{2}$ (уменьшится вдвое).

$\boxed{5.6}$



$U = \text{const} \qquad \varepsilon = \frac{\varepsilon_\alpha a}{r}$

$$E = \frac{Q}{4\pi\varepsilon_0\,\varepsilon(r)\,r^2}$$

$$U = \int_a^b E(r)\,dr = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{\varepsilon(r)\,r^2} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{\varepsilon_\alpha a r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_\alpha a}\ln\frac{b}{a}$$

$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0\varepsilon_\alpha a}{\ln\left(\frac{b}{a}\right)}$$

$$W = \frac{CU^2}{2} = \frac{2\pi\varepsilon_0\varepsilon_\alpha a}{\ln(b/a)}U^2$$

$\boxed{5.7}$

$$E(x) = \frac{\sigma r}{\varepsilon_0 x} + \frac{\sigma r}{\varepsilon_0 (a-x)} \quad (\text{см. } 5.5)$$

$$U = \varphi_1 - \varphi_2 = \int_r^{a-r} E(x)\,dx = \frac{\sigma r}{\varepsilon_0} \int_r^{a-r} \left[\frac{1}{x} + \frac{1}{a-x}\right] dx$$

$$= \frac{\sigma r}{\varepsilon_0} \left[\ln x - \ln (a-x)\right]\Big|_r^{a-r} =$$

$$= \frac{\sigma r}{\varepsilon_0} \ln \frac{x}{a-x}\Big|_r^{a-r} = \frac{\sigma r}{\varepsilon_0}\left[\ln\frac{a-r}{r} - \ln\frac{r}{a-r}\right] =$$

$$= \frac{2\sigma r}{\varepsilon_0} \ln \frac{a-r}{r}$$

$$C = \frac{2\pi r \cdot h\sigma}{2\sigma r} \cdot \frac{\varepsilon_0}{\ln\frac{a-r}{r}} \approx \frac{\pi \varepsilon_0 h}{\ln \frac{a}{r}}$$

$$C_h = \frac{\pi \varepsilon_0}{\ln \frac{a}{r}}$$

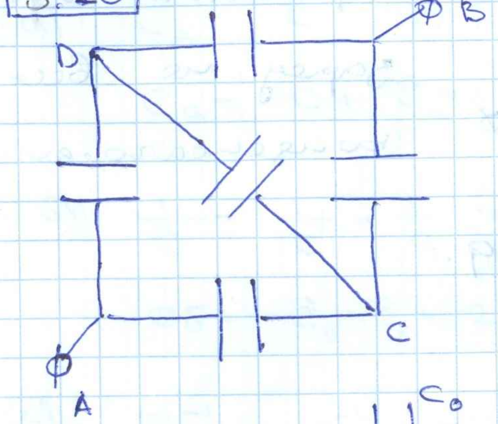
$\boxed{5.9}$ а) не изменилась (физ. хар-ка)

б) 1) $U_0 = \mathcal{E} = \dfrac{nq}{C}$

2) $U = \dfrac{(n-1)q}{C}$ $\Rightarrow$ $\dfrac{U}{U_0} = \dfrac{n-1}{n} < 1$

уменьшилась.

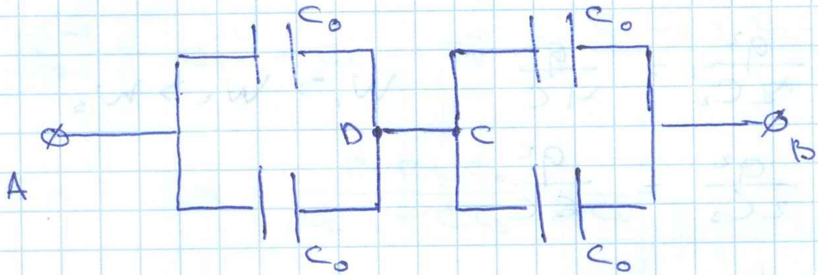
5.10



В силу симметрии задачи

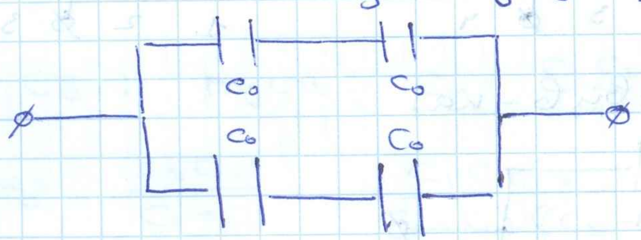
$$\varphi_D = \varphi_C$$

поэтому схема эквив-на схеме:



$\cancel{C_{\parallel} = 2C_0}$; $\quad \frac{1}{C_\Sigma} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_\Sigma = C_0.$

если ключ К не замкнут, то:



$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C_0} = \frac{2}{C_0}$; $\quad C_\Sigma = 2C = C_0.$

$\boxed{5.11}$ В силу закона сохранения заряда

заряд на всех конденсаторах будет одинаковый q.

$$W_1 = \frac{q^2}{2C_1} = \frac{q^2}{2C}$$

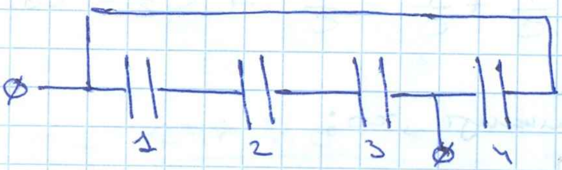
$$W_2 = \frac{q^2}{2C_2} = \frac{q^2}{4C}$$

$$W_1 > W_2 > W_3$$

$$W_3 = \frac{q^2}{2C_3} = \frac{q^2}{6C}$$

$\boxed{5.12}$ 1

а)



б)

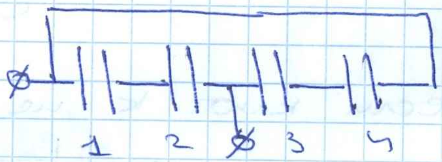


схема а) эквив-на

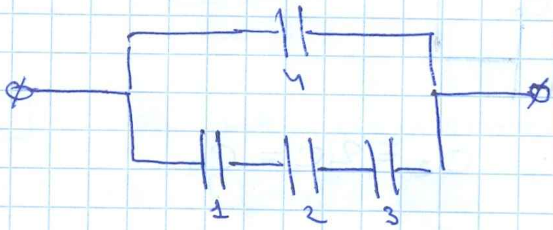
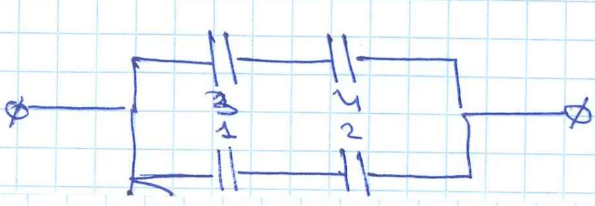


схема б) эквив-на



а) $\frac{1}{\hat{C}_a} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} \Rightarrow \hat{C} = \frac{C}{3}$

$C_a = C + \hat{C}_a = \frac{4C}{3}$

б) $\frac{1}{\hat{C}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow \hat{C} = \frac{C}{2}$

$C_б = \hat{C}_б + \hat{C}_б = 2\hat{C} = C$ $\qquad C_б < C_a$

2 а) $\hat{C}_a^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} = \frac{C_2C_3 + C_1C_3 + C_1C_2}{C_1C_2C_3}$

$C_a = C_4 + \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2}$

б) $\hat{C}_{б_1}^{-1} = C_1^{-1} + C_2^{-1} = \frac{C_1 + C_2}{C_1C_2}$ $\qquad \hat{C}_{б_2}^{-1} = \frac{C_3 + C_4}{C_3C_4}$

$C_б = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$

$C_a = C_б$: пусть $C_4 = \frac{C_1C_2}{C_1 + C_2}$

тогда $\frac{C_3C_4}{C_3 + C_4} = \frac{C_1C_2C_3}{(C_1 + C_2)\left(C_3 + \frac{C_1C_2}{C_1 + C_2}\right)} =$

$= \frac{C_1C_2C_3}{C_1C_3 + C_2C_3 + C_1C_2}$

$\boxed{5.13}$

$$w_{E_1} = \frac{W_1}{V} = \frac{1}{2}\varepsilon\varepsilon_0 E_1^2$$

$$w_{E_2} = \frac{W_2}{V} = \frac{1}{2}\varepsilon_0 E_2^2$$

$$D = E_1\varepsilon\varepsilon_0 = E_2\varepsilon_0 = const$$

$$\frac{W_1}{W_2} = \frac{\varepsilon E_1^2}{E_2^2} = \frac{\varepsilon E_1^2}{\varepsilon^2 E_1^2} = \frac{1}{\varepsilon} \Rightarrow W_2 = \varepsilon W_1$$

$\boxed{5.14}$

$R_1 = 10\text{ см} = 10^{-2}\text{ м}$ $\qquad \varphi_1 = 450\text{ В}$

$R_2 = 20\text{ см} = 2\cdot10^{-2}\text{ м}$ $\qquad \varphi_2 = 300\text{ В}$

$R_3 = 30\text{ см} = 3\cdot10^{-3}\text{ м}$ $\qquad \varphi_3 = 150\text{ В}$

$$\frac{kq_i}{R_i} = \varphi_i \Rightarrow q_i = \frac{\varphi_i R_i}{k}$$

$$W_1 = \frac{1}{2}\sum_{i=1}^{3} q_i\varphi_i = \frac{1}{2k}\sum_{i=1}^{3}\varphi_i^2 R_i$$

$$\varphi_0 = \frac{kq_i'}{R_i} \qquad q_i' = \varphi_0\cdot\frac{R_i}{k} = q_1'\frac{R_i}{R_1}$$

$$W_2 = \frac{1}{2}\sum_{i=1}^{3} q_i'\varphi_0 = \frac{1}{2}\cdot\frac{kq_1'}{R_1}\sum_{i=1}^{3} q_i' = \frac{kq_1'}{2R_1}\sum_{i=1}^{3} q_1'\frac{R_i}{R_1} =$$

$$= \frac{kq_1'^2}{2R_1^2}\sum_{i=1}^{3} R_i \; (=)$$

$$q_1' + q_2' + q_3' = q_1 + q_2 + q_3$$

$$q_1' + q_1' \frac{R_2}{R_1} + q_1' \frac{R_3}{R_1} = \sum_{i=1}^{3} \frac{\varphi_i R_i}{k}$$

$$q_1' \frac{R_1 + R_2 + R_3}{R_1} = \frac{1}{k} \sum_{i=1}^{3} \varphi_i R_i$$

$$q_1' = \frac{R_1}{R_1 + R_2 + R_3} \cdot \frac{1}{k} \sum_{i=1}^{3} \varphi_i R_i = \frac{R_1}{\sum R_i} \frac{1}{k} \sum \varphi_i R_i$$

$$= \frac{k \sum R_i}{2 R_1^2} q_1'^2 = \frac{1}{2k \sum R_i} \left(\sum \varphi_i R_i \right)^2 =$$

$$= \frac{1}{2k} \cdot \frac{1}{R_1 + R_2 + R_3} \cdot \Big[\varphi_1^2 R_1^2 + \varphi_2^2 R_2^2 + \varphi_3^2 R_3^2 +$$

$$+ 2\varphi_1 \varphi_2 R_1 R_2 + 2 \varphi_2 \varphi_3 R_2 R_3 + 2 \varphi_1 \varphi_3 R_1 R_3 \Big]$$

$$W_1 = \frac{1}{2k} \cdot \frac{1}{R_1 + R_2 + R_3} \cdot \Big[\varphi_1^2 R_1^2 + \varphi_1^2 R_1 R_2 + \varphi_1^2 R_1 R_3 +$$

$$+ \varphi_2^2 R_2^2 + \varphi_2^2 R_1 R_2 + \varphi_2^2 R_2 R_3 +$$

$$+ \varphi_3^2 R_3^2 + \varphi_3^2 R_1 R_3 + \varphi_3^2 R_2 R_3 \Big]$$

$$\Delta W = W_2 - W_1 = \frac{1}{2k} \cdot \frac{1}{R_1 + R_2 + R_3} \cdot$$

$$\cdot \Big[(\varphi_1 - \varphi_2)^2 R_1 R_2 + (\varphi_2 - \varphi_3)^2 R_2 R_3 + (\varphi_1 - \varphi_3)^2 R_1 R_3 \Big]$$

$\boxed{5.15}$ (1) $U = const = \mathcal{E}$

$$W_1 = \frac{C_1 U^2}{2}; \quad C_1 = \frac{\varepsilon \varepsilon_0 S}{d}; \quad Q_1 = \frac{\varepsilon \varepsilon_0 S}{d} U$$

$$W_2 = \frac{C_2 U^2}{2}; \quad C_2 = \frac{\varepsilon_0 S}{d}; \quad Q_2 = \frac{\varepsilon_0 S}{d} U$$

$$W_1 + \mathcal{E} \Delta Q + A_{\text{мех}} = W_2$$

$$\Delta W = W_2 - W_1 = (1-\varepsilon) \frac{\varepsilon_0 S}{2d} U^2$$

$$A_{\text{мех}} = \Delta W - U \Delta Q = \Delta W - U (1-\varepsilon) \frac{\varepsilon_0 S}{d} U =$$

$$= \Delta W - 2\Delta W = -\Delta W$$

(2) $Q = const$

$$W_1 = \frac{Q^2}{2C_1}; \quad C_1 = \frac{\varepsilon \varepsilon_0 S}{d} \quad Q = C_1 U$$

$$W_2 = \frac{Q^2}{2C_2}; \quad C_2 = \frac{\varepsilon_0 S}{d}$$

$$W_1 + A_{\text{мех}} = W_2$$

$$A_{\text{мех}} = \Delta W = W_2 - W_1 = \frac{Q^2 d}{2 \varepsilon_0 S} \left(1 - \frac{1}{\varepsilon}\right) =$$

$$= \frac{C_1^2 U^2}{2 C_2} \left(1 - \frac{1}{\varepsilon}\right) = \frac{\varepsilon^2 \varepsilon_0 S}{2d} U^2 \left(1 - \frac{1}{\varepsilon}\right) =$$

$$= \frac{\varepsilon(\varepsilon - 1) \varepsilon_0 S}{2d} U^2$$

$\boxed{5.16}$

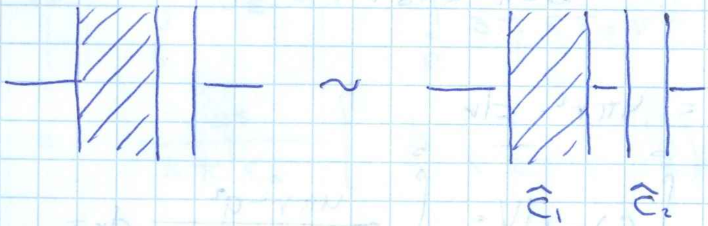
$$W_1 = \frac{q^2}{2C_1} = \frac{q^2 d_0}{2\varepsilon_0 S}$$

$$W_2 = \frac{q^2}{2C_2} = \frac{q^2 d}{2\varepsilon_0 S}$$

$$W_1 + A_{мех} = W_2 \Rightarrow A_{поля} = -A_{мех} = W_1 - W_2 = \frac{q^2}{2\varepsilon_0 S}(d_0 - d)$$

$\boxed{5.17}$ $U = const = \mathcal{E}$

$$W_1 = \frac{C_1 U^2}{2} = \frac{\varepsilon\varepsilon_0 S}{2d} U^2$$



$$\hat{C}_1 = \frac{\varepsilon\varepsilon_0 S}{d} \qquad \hat{C}_2 = \frac{\varepsilon_0 S}{x} \qquad W_2 = \frac{C_2 U^2}{2}$$

$$\frac{1}{C_2} = \frac{1}{\varepsilon_0 S}\left(\frac{d}{\varepsilon} + x\right) = \frac{1}{\hat{C}_1} + \frac{1}{\hat{C}_2} \Rightarrow C_2 = \frac{\varepsilon\varepsilon_0 S}{d + \varepsilon x}$$

$$W_1 + A_{мех} = W_2 \; ; \quad \frac{W_2}{W_1} = \frac{d}{d + \varepsilon x}$$

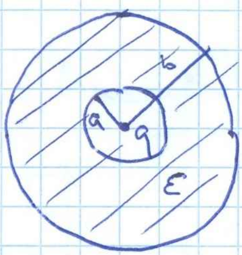
$$A_{мех} = W_2 - W_1 = W_2 - \frac{d + \varepsilon x}{d} W_2 = -\frac{\varepsilon x}{d} W_2$$

$$A = |A_{мех}| = \frac{\varepsilon x}{d} W_2 = \frac{\varepsilon x}{d} \frac{\varepsilon\varepsilon_0 S}{2(d + \varepsilon x)} U^2$$

$$A = \frac{\varepsilon \varepsilon_0 S U^2}{2d\left(\frac{d}{\varepsilon x}+1\right)} \Rightarrow \frac{d}{\varepsilon x} = \frac{\varepsilon \varepsilon_0 S U^2}{2dA} - 1$$

$$x = \frac{d}{\varepsilon} \frac{2Ad}{\varepsilon \varepsilon_0 S U^2 - 2Ad} = \frac{d}{\varepsilon} \cdot \frac{1}{\left(\frac{\varepsilon \varepsilon_0 S U^2}{2Ad} - 1\right)}$$

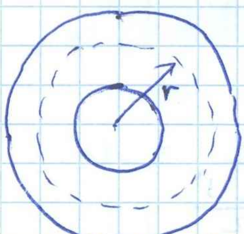
$\boxed{5.18}$



$$E = \frac{kq}{\varepsilon r^2} = \frac{q}{4\pi \varepsilon \varepsilon_0 r^2}$$

$$\omega_E = \frac{1}{2} \varepsilon \varepsilon_0 E^2 = \frac{\varepsilon \varepsilon_0}{2} \cdot \frac{q^2}{16\pi^2 \varepsilon^2 \varepsilon_0^2 r^4} =$$

$$= \frac{q^2}{32\pi^2 \varepsilon \varepsilon_0 r^4}$$



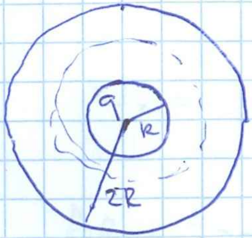
$$dV = 4\pi r^2 \cdot dr$$

$$W = \int_a^b \omega_E \, dV = \int_a^b \frac{4\pi r^2 q^2}{32\pi^2 \varepsilon \varepsilon_0 r^4} dr =$$

$$= \int_a^b \frac{q^2}{8\pi \varepsilon \varepsilon_0 r^2} dr = \left.\frac{-q^2}{8\pi \varepsilon \varepsilon_0 r}\right|_a^b =$$

$$= + \frac{q^2}{8\pi \varepsilon_0 \varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{q^2 (b-a)}{8\pi \varepsilon \varepsilon_0 ab}$$

$\boxed{5.19}$



$$E = \begin{cases} \dfrac{q}{4\pi\varepsilon\varepsilon_0 r^2}, & R < r < 2R \\ \dfrac{q}{4\pi\varepsilon_0 r^2}, & r > 2R \end{cases}$$

$$\omega_E = \begin{cases} \dfrac{\varepsilon\varepsilon_0}{2} \cdot \dfrac{q^2}{16\pi^2\varepsilon^2\varepsilon_0^2 r^4} = \dfrac{q^2}{32\pi^2 \varepsilon\varepsilon_0 r^4}; & R < r < 2R \\ \dfrac{\varepsilon_0}{2} \cdot \dfrac{q^2}{16\pi^2 \varepsilon_0^2 r^4} = \dfrac{q^2}{32\pi^2 \varepsilon_0 r^4}; & r > 2R \end{cases}$$

$$dV = 4\pi r^2 \, dr$$

$$W = \int_R^{+\infty} \omega_E \, dV = \int_R^{2R} \frac{q^2}{8\pi\varepsilon\varepsilon_0 r^2} dr + \int_{2R}^{+\infty} \frac{q^2}{8\pi\varepsilon_0 r^2} dr =$$

$$= \frac{q^2}{8\pi\varepsilon_0} \left[\frac{1}{\varepsilon} \left(\frac{1}{R} - \frac{1}{2R} \right) + \frac{1}{2R} \right] =$$

$$= \frac{q^2}{8\pi\varepsilon_0} \left[\frac{1}{2\varepsilon R} + \frac{1}{2R} \right] = \frac{q^2 (\varepsilon + 1)}{16\pi\varepsilon\varepsilon_0 R}$$

$\boxed{5.20}$ ① $\rho = \dfrac{q}{\frac{4}{3}\pi r^3}$

$$E(x) \cdot 4\pi x^2 = \frac{\rho \cdot \frac{4}{3}\pi x^3}{\varepsilon_0} \quad \text{(т. Гаусса)}, \quad x < r$$

$$E(x) = \begin{cases} \dfrac{\rho x}{3\varepsilon_0} = \dfrac{q x}{4\pi r^3 \varepsilon_0}, & x < r \\ \dfrac{q}{4\pi\varepsilon_0 x^2}, & x \geq r \end{cases}$$

$$w_E = \frac{\varepsilon_0 E^2}{2} = \begin{cases} \frac{q^2 x^2}{32\pi^2 r^6 \varepsilon_0}, & x < r \\ \frac{q^2}{32\pi^2 \varepsilon_0 x^4}, & x \geq r \end{cases}$$

$$dV = 4\pi x^2 \, dx$$

$$W_1 = \int_0^{+\infty} w_E \, dV = \int_0^r \frac{q^2 x^4}{8\pi \varepsilon_0 r^6} dx + \int_r^{+\infty} \frac{q^2}{8\pi \varepsilon_0 x^2} dx =$$

$$= \frac{q^2}{40\pi \varepsilon_0 r^6} \cdot r^5 + \frac{q^2}{8\pi \varepsilon_0 r}$$

② $$W_2 = \frac{1}{2} q \frac{q}{4\pi \varepsilon_0 r} = \frac{q^2}{8\pi \varepsilon_0 r}$$

$$W_1 + A_{вн} = W_2 \Rightarrow A_{поле} = -A_{вн} = W_1 - W_2 =$$

$$= \frac{q^2}{40\pi \varepsilon_0 r}$$

$\boxed{5.21}$ $$C_1 = \frac{\varepsilon_0 \hat{S}}{d} ; \quad C_2 = \frac{\varepsilon_0 \hat{S}}{d-s}$$

① $$W_1 = \frac{\varepsilon_0 \hat{S}}{2d} U^2 \qquad Q_1 = \frac{\varepsilon_0 \hat{S}}{d} U$$

$$W_2 = \frac{\varepsilon_0 \hat{S}}{2(d-s)} U^2 \qquad Q_2 = \frac{\varepsilon_0 \hat{S}}{d-s} U$$

$$A_{вн_1} = W_2 - W_1 - \mathcal{E} \Delta Q = U^2 \frac{\varepsilon_0 \hat{S}}{2} \left[\frac{1}{d-s} - \frac{1}{d} - \right.$$

$$\left. - \left[\frac{2}{d-s} - \frac{2}{d} \right] \right] = \frac{\varepsilon_0 \hat{S}}{2} U^2 \left[\frac{1}{d} - \frac{1}{d-s} \right] = - \frac{\varepsilon_0 \hat{S} s}{2(d-s) d} U^2$$

② $W_1 = \frac{\varepsilon_0 \hat{S}}{2d} U^2 \qquad Q = \frac{\varepsilon_0 \hat{S}}{d} U$

$$W_2 = \frac{Q^2}{2C_2} = \frac{\varepsilon_0^2 \hat{S}^2 U^2}{2d^2} \cdot \frac{d-s}{\cancel{\varepsilon_0 \hat{S}}} = \frac{\varepsilon_0 \hat{S}(d-s)}{2d^2} U^2$$

$$A_{вн2} = W_2 - W_1 = \frac{\varepsilon_0 \hat{S}}{2d} U^2 \left(\frac{d-s}{d} - 1 \right) = -\frac{\varepsilon_0 \hat{S} s}{2d^2} U^2$$

$$\frac{A_1}{A_2} = \frac{-A_{вн1}}{-A_{вн2}} = \frac{d}{d-s}$$

$\boxed{5.22}$ из 5.5 известно:

$$E(r) = \frac{\sigma R}{\varepsilon \varepsilon_0 r}, \quad r \in [R, 4R]$$

$$w_E(r) = \frac{\varepsilon \varepsilon_0 E^2}{2} = \frac{\sigma^2 R^2}{2\varepsilon\varepsilon_0 r^2}, \quad R < r < 4R$$

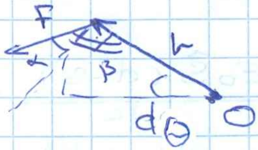
$$dV = 2\pi r \cdot \ell \cdot dr$$

$$W_1 = \int_R^{2R} \frac{\sigma^2 R^2}{\cancel{2}\varepsilon\varepsilon_0 r^{\cancel{2}}} \cdot \cancel{2}\pi \cancel{r} \ell \, dr = \frac{\sigma^2 R^2 \pi \ell}{\varepsilon\varepsilon_0} \ln \frac{2R}{R}$$

$$W_2 = \int_{2R}^{4R} \frac{\sigma R^2}{2\varepsilon\varepsilon_0 r^2} \cdot 2\pi r \ell \, dr = \frac{\sigma^2 R^2 \pi \ell}{\varepsilon\varepsilon_0} \ln \frac{4R}{2R}$$

$$\frac{W_1}{W_2} = \frac{1}{1}$$

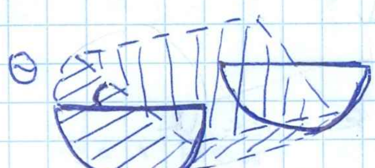
$\boxed{5.24}$ 1) Установим соотношение между работой и моментом силы.



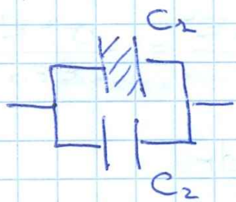
$$dA = F\cos\alpha \cdot r\,d\theta =$$
$$= F \sin\left(\beta + \frac{\pi}{2}\right) r\,d\theta =$$
$$= F r \sin\beta\, d\theta$$

$A = \int M\,d\theta$, тогда можно записать следующее

2) $$M = \frac{\partial W}{\partial \theta} = \frac{1}{2} U^2 \frac{\partial C}{\partial \theta}$$



конденсатор можно разбить на два



$$C_1 = \frac{\varepsilon\varepsilon_0 S_1}{d} = \frac{\varepsilon\varepsilon_0 (\pi - \theta) R^2}{2d}$$

$$C_2 = \frac{\varepsilon_0 S_2}{d} = \frac{\varepsilon_0 \theta R^2}{2d}$$

$$C = C_1 + C_2 = \frac{\varepsilon_0 \theta R^2}{2d} + \frac{\varepsilon\varepsilon_0 (\pi - \theta) R^2}{2d}$$

$$\frac{\partial C}{\partial \theta} = \frac{\varepsilon_0 R^2}{2d} - \frac{\varepsilon\varepsilon_0 R^2}{2d} = (1 - \varepsilon)\frac{\varepsilon_0 R^2}{2d}$$

$$M = \frac{1}{4}\frac{\varepsilon_0 R^2}{d} U^2 (1 - \varepsilon) < 0$$ (поле пытается втянуть диэлектр.)

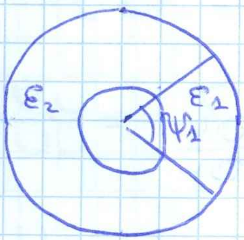
5.11 (продолжение)

3 конденсатора можно заменить одним

$$\frac{1}{C_0} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} = \frac{11}{6C}$$

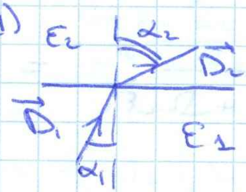
$$q = UC_0 = \frac{6}{11} UC$$

5.8



Рассмотрим поле на границе диэлектриков и металла:

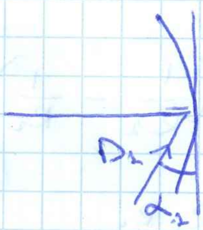
(1)



$$D_1 \cos\alpha_1 = D_2 \cos\alpha_2$$

$$\frac{D_1}{\varepsilon_0 \varepsilon_1} \sin\alpha_1 = \frac{D_2}{\varepsilon_0 \varepsilon_2} \sin\alpha_2$$

(2):



$$D_1 \sin\alpha_1 = \sigma \qquad D_2 \sin\alpha_2 = \sigma$$

$$\frac{D_1}{\varepsilon_1 \varepsilon_0} \cos\alpha_1 = 0 \qquad \frac{D_2}{\varepsilon_2 \varepsilon_0} \cos\alpha_2 = 0$$

$$\Rightarrow \alpha_1 = \alpha_2 = \pi/2$$

$\Rightarrow$ поле радиально. Воспользуемся т. Гаусса:

$$\oint D\, dS = Q \ ; \ r \in [R_1 ; R_2]$$

$$\varepsilon_1 \varepsilon_0 E \psi_1 \cdot r \cdot l + \varepsilon_2 \varepsilon_0 E \psi_2 r \cdot l = Q$$

$$E = \frac{Q}{\varepsilon_0 l r (\varepsilon_1 \psi_1 + \varepsilon_2 \psi_2)}$$

$$E = -\frac{\partial\varphi}{\partial r} \Rightarrow \varphi = \frac{-Q}{\varepsilon_0 l\,(\varepsilon_1\psi_1 + \varepsilon_2\psi_2)} \ln r$$

$$U = \varphi_1 - \varphi_2 = \frac{Q}{\varepsilon_0 l\,(\varepsilon_1\psi_1 + \varepsilon_2\psi_2)} \ln \frac{R_2}{R_1}$$

$$C = \frac{Q}{U} = \frac{\varepsilon_0 l\,(\varepsilon_1\psi_1 + \varepsilon_2\psi_2)}{\ln R_2/R_1}$$

$\boxed{5.23}$ из 4.23 известно, что

$$E = \frac{q}{\varepsilon_0 r^2} \frac{1}{\Omega_1\varepsilon_1 + \Omega_2\varepsilon_2} = -\frac{\partial\varphi}{\partial r}$$

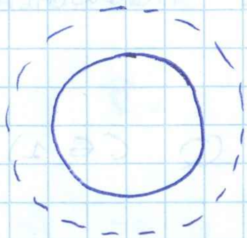
$$\varphi = \frac{q}{\varepsilon_0 r} \frac{1}{\Omega_1\varepsilon_1 + \Omega_2\varepsilon_2}$$

$$U = \varphi_1 - \varphi_2 = \frac{q}{\varepsilon_0(\Omega_1\varepsilon_1 + \Omega_2\varepsilon_2)} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{q}{U} = \frac{\varepsilon_0(\Omega_1\varepsilon_1 + \Omega_2\varepsilon_2) R_1 R_2}{R_2 - R_1}$$

6.1

6



$$I = \oint_S \vec{j}\, d\vec{S} = \oint_S \lambda \frac{\vec{D}}{\varepsilon\varepsilon_0} d\vec{S} =$$

$$= \frac{\lambda}{\varepsilon\varepsilon_0} Q_1 = \frac{\lambda}{\varepsilon\varepsilon_0} C_1 U_1$$

$$I = -\frac{\lambda}{\varepsilon\varepsilon_0} Q_2 = -\frac{\lambda}{\varepsilon\varepsilon_0} C_2 U_2$$

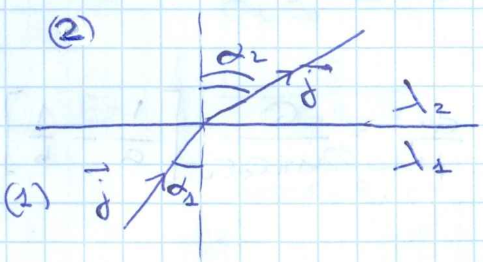
$$I\varepsilon\varepsilon_0 = \lambda C_1 U_1 = -\lambda C_2 U_2$$

$$\frac{U_1}{U_2} = -\frac{C_2}{C_1} = -\frac{4\pi\varepsilon_0 R_2}{4\pi\varepsilon_0 R_1} = -\frac{R_2}{R_1}$$

$$U = U_1 - U_2 = -U_2\left(1 + \frac{R_2}{R_1}\right)$$

$$U_2 = -\frac{UR_1}{R_1 + R_2} \qquad U_1 = \frac{UR_2}{R_1 + R_2}$$

6.2



1) $E_{\tau 1} = E_{\tau 2}$

$E_1 \sin\alpha_1 = E_2 \sin\alpha_2$

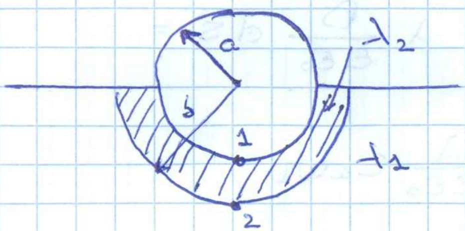
2) из непрерывности тока следует:

$$j_1 \cos\alpha_1 = j_2 \cos\alpha_2$$

$$\lambda_1 E_1 \cos\alpha_1 = \lambda_2 E_2 \cos\alpha_2$$

$$\frac{\operatorname{tg}\alpha_1}{\lambda_1} = \frac{\operatorname{tg}\alpha_2}{\lambda_2} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\operatorname{tg}\alpha_2}{\operatorname{tg}\alpha_1}$$

$\boxed{6.3}$



1) Погрузим шар полностью в землю:

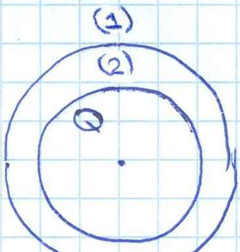
$$I = \oint_S \bar{j}\, d\bar{S} = \frac{\lambda}{\varepsilon \varepsilon_0} Q \quad (6.1)$$

$$I = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} Q = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} Q$$

$$U = \overbrace{(\varphi_1 - \varphi_2)}^{U_2} + \overbrace{(\varphi_2 - \varphi_3)}^{U_1} = I\hat{R} = I(R_1 + R_2) =$$

$$= IR_1 + IR_2 = U_1 + U_2$$

$$U_1 = IR_1 = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} Q R_1 \qquad U_2 = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} Q R_2$$



$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 \varepsilon_1 r^2}, & \text{область (1)} \\ \dfrac{Q}{4\pi\varepsilon_0 \varepsilon_2 r^2}, & \text{область (2)} \end{cases}$$

$$U_2 = \int_a^b \bar{E}\, d\bar{r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2} \int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$U_1 = \int_b^\infty \bar{E}\, d\bar{r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1} \frac{1}{b}$$

$$\frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{\lambda_2}{\varepsilon_2\varepsilon_0} Q R_2 \Rightarrow R_2 = \frac{1}{4\pi\lambda_2}\left[\frac{1}{a} - \frac{1}{b}\right]$$

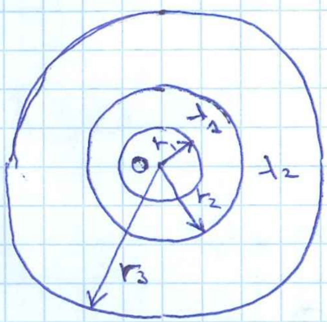
$$\frac{Q}{4\pi\varepsilon_0\varepsilon_1}\frac{1}{b} = \frac{\lambda_1}{\varepsilon_0\varepsilon_1} Q R_1 \Rightarrow R_1 = \frac{1}{4\pi\lambda_1}\cdot\frac{1}{b}$$

$$\hat{R} = R_1 + R_2 = \frac{1}{4\pi\lambda_2}\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{4\pi\lambda_1 b}$$

г) тогда для шара погружённого наполовину:

$$R = 2\hat{R} = \frac{1}{2\pi\lambda_2}\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{2\pi\lambda_1 b}$$

$\boxed{6.10}$ аналогично 6.3



$$I = \frac{\lambda_1}{\varepsilon_1\varepsilon_0} Q = \frac{\lambda_2}{\varepsilon_2\varepsilon_0} Q$$

$$U = \underbrace{(\varphi_1 - \varphi_2)}_{U_1} + \underbrace{(\varphi_2 - \varphi_3)}_{U_2} =$$

$$= I R_1 + I R_2$$

$$U_1 = \frac{\lambda_1}{\varepsilon_1\varepsilon_0} Q R_1 \qquad U_2 = \frac{\lambda_2}{\varepsilon_2\varepsilon_0} Q R_2$$

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0\varepsilon_1 r^2}, & r \in (r_1, r_2) \\ \dfrac{Q}{4\pi\varepsilon_0\varepsilon_2 r^2}, & r \in (r_2, r_3) \end{cases}$$

$$U_1 = \int_{r_1}^{r_2} \bar{E}\, d\bar{r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$U_2 = \int_{r_2}^{r_3} \bar{E}\, d\bar{r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

$$R_1 = \frac{1}{4\pi\lambda_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$R_2 = \frac{1}{4\pi\lambda_2}\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

$$\cancel{N_1 = \frac{U^2}{R_1} = \frac{4\pi\lambda_1 U^2}{r_2 - r_1} \cdot r_2 r_1}$$

$$\cancel{N_2 = \frac{U^2}{R_2} = \frac{4\pi\lambda_2 U^2}{r_3 - r_2} \cdot r_3 r_2}$$

$$N_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1 + R_2)^2}$$

$$N_2 = I^2 R_2 = \frac{U^2 R_2}{(R_1 + R_2)^2}$$

$\boxed{6.11}$ аналогично 6.10.

из 5.5 известно:

$$E = \begin{cases} \dfrac{\sigma r_1}{\varepsilon_1 \varepsilon_0 r}, & r_1 < r < r_2 \\ \dfrac{\sigma r_1}{\varepsilon_2 \varepsilon_0 r}, & r_2 < r < r_3 \end{cases}$$

$$U_1 = \int_{r_1}^{r_2} \bar{E}\, d\bar{r} = \frac{\sigma r_1}{\varepsilon_1 \varepsilon_0} \ln \frac{r_2}{r_1}$$

$$U_2 = \int_{r_2}^{r_3} \bar{E}\, d\bar{r} = \frac{\sigma r_1}{\varepsilon_2 \varepsilon_0} \ln \frac{r_3}{r_1}$$

$$U_1 = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} Q R_1 = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} \sigma \cdot 2\pi r_1 \ell \cdot R_1$$

$$U_2 = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} \sigma \cdot 2\pi r_2 \ell \cdot R_2$$

$$\frac{\lambda_1}{\cancel{\varepsilon_1 \varepsilon_0}} \cancel{\sigma}\, 2\pi \cancel{r_1} \ell R_1 = \cancel{\sigma r_1} \frac{1}{\cancel{\varepsilon_0 \varepsilon_1}} \ln \frac{r_2}{r_1}$$

$$R_1 = \frac{1}{2\pi \lambda_1 \ell} \ln \frac{r_2}{r_1} \quad ; \quad R_2 = \frac{1}{2\pi \lambda_2 \ell} \ln \frac{r_3}{r_2}$$

$$N_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1 + R_2)^2} \qquad N_2 = \frac{U^2 R_2}{(R_1 + R_2)^2}$$

$\boxed{6.3}$ $$I = \oint_S \vec{j}\, d\vec{S} = \frac{\lambda}{\varepsilon \varepsilon_0} Q_1 = -\frac{\lambda}{\varepsilon \varepsilon_0} Q_2 \quad (6.1)$$

$$U = \varphi_1 - \varphi_2 = \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = \frac{Q_1}{C_1} + \frac{Q_1}{C_2} = $$
$$= Q_1 \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$U = IR = \frac{\lambda}{\varepsilon \varepsilon_0} Q_1 R$$

$$R = \frac{\varepsilon \varepsilon_0}{\lambda} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$\boxed{6.4}$ из задачи 6.3 известно:

$$\hat{R} = \frac{\varepsilon \varepsilon_0}{\lambda} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

тогда для шаров полностью утоплен. в земле

$$\hat{R} = \frac{\varepsilon \varepsilon_0}{\lambda} \left(\frac{1}{4\pi \varepsilon \varepsilon_0 a_1} + \frac{1}{4\pi \varepsilon \varepsilon_0 a_2} \right) = \frac{1}{4\pi \lambda} \left(\frac{1}{a_1} + \frac{1}{a_2} \right)$$

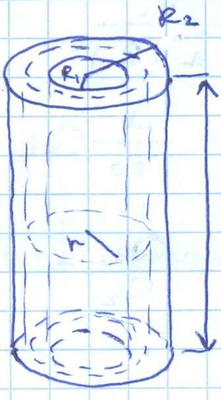
тогда для шаров погружённых наполовину:

$$R = 2\bar{R} = \frac{1}{2\pi\lambda}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$

$\boxed{6.8}$ из задачи 6.5 известно:

$$R = \frac{\varepsilon\varepsilon_0}{\lambda}\left(\frac{1}{4\pi\varepsilon\varepsilon_0 r} + \frac{1}{4\pi\varepsilon\varepsilon_0 r}\right) = \frac{1}{2\pi\lambda r}$$

$\boxed{6.5}$



т. Гаусса:

$$\oint_S D_1\, ds = Q$$

$$D_1 \cdot 2\pi r \cdot \cancel{\ell} = \sigma \cdot \cancel{2\pi} R_1 \cdot \cancel{\ell}$$

$$E_1 = \frac{\sigma R_1}{\varepsilon_1 \varepsilon_0 r}$$

$$U_1 = \varphi_1 - \varphi_2 = \int_{R_1}^{R_2} E\, dr = \frac{\sigma R_1}{\varepsilon_1\varepsilon_0} \ln\frac{R_2}{R_1}$$

$$C_1 = \frac{Q}{U} = 2\pi \cancel{R_1} \cdot h\, \cancel{\sigma} \cdot \frac{\varepsilon_1\varepsilon_0}{\cancel{\sigma R_1} \ln\frac{R_2}{R_1}} =$$

$$= \frac{2\pi\varepsilon_1\varepsilon_0 h}{\ln\frac{R_2}{R_1}}$$

$$N_1 = \frac{U^2}{R_1} = \left\{ R_1 = \frac{\varepsilon_1\varepsilon_0}{\lambda_1 C_1}\ (*) \right\} = \frac{\lambda_1 C_1}{\varepsilon_1\varepsilon_0} U^2 =$$

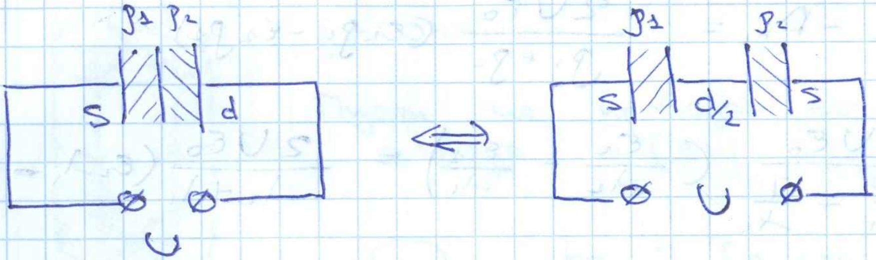
$$N_1 = \frac{2\pi \lambda_1 h}{\ln \frac{R_2}{R_1}} U^2 \qquad N_2 = \frac{2\pi \lambda_2 h}{\ln \frac{R_2}{R_1}} U^2$$

$$\frac{N_1}{N_2} = \frac{\lambda_1}{\lambda_2}$$

$$(*): \quad I = \oint_S \vec{j}\, d\vec{S} = \frac{\lambda}{\varepsilon\varepsilon_0} Q = \frac{\lambda}{\varepsilon\varepsilon_0} CU$$

$$U = IR \Rightarrow R = \frac{\varepsilon\varepsilon_0}{\lambda C}$$

$\boxed{6.6}$

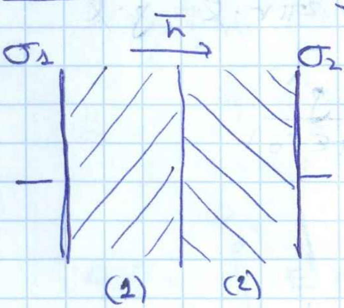


$U \equiv \mathcal{E}_0$

$$R_1 = \rho_1 \frac{d}{2S} \ ; \qquad R_2 = \rho_2 \frac{d}{2S}$$

$$N_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1 + R_2)^2} \qquad N_2 = \frac{U^2 R_2}{(R_1 + R_2)^2}$$

$\boxed{6.7}$ $\quad \lambda_i = \frac{1}{\rho_i}$



Граничное условие для границы:

$D_{n2} - D_{n1} = \sigma$ (убрали диэлектр)

$D_{n1} = \varepsilon_0 \varepsilon_1 E_1$

$D_{n2} = \varepsilon_0 \varepsilon_2 E_2$

Граничное условие для пластин:

$$\sigma_1 = D_1 = D_{n1} \qquad \sigma_2 = -D_2 = -D_{n2}$$

$\bar{j} = const \qquad \vec{j} = \lambda \vec{E} \qquad j = \frac{I}{S} = \frac{U}{R_1+R_2}\cdot\frac{1}{S}$

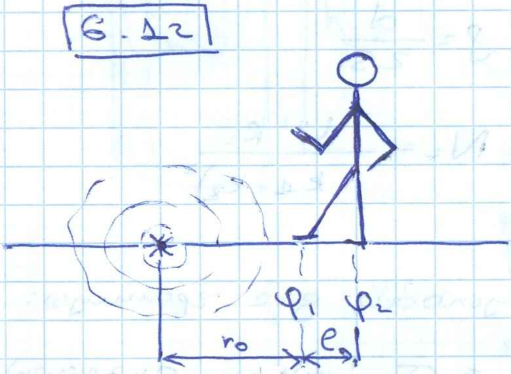
$$E_1 = \frac{j}{\lambda_1} = \frac{I}{\lambda_1 S} = \frac{U}{R_1+R_2}\,\frac{1}{\lambda_1 S} = \frac{2U\rho_1}{(\rho_1+\rho_2)d}$$

$$E_2 = \frac{j}{\lambda_2} = \frac{2U\rho_2}{(\rho_1+\rho_2)d}$$

$$\sigma = D_{n_2} - D_{n_1} = \frac{2U\varepsilon_0}{\rho_1+\rho_2}(\varepsilon_2\rho_2 - \varepsilon_1\rho_1) =$$

$$= \frac{2U\varepsilon_0}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left(\frac{\varepsilon_2}{\lambda_2}-\frac{\varepsilon_1}{\lambda_1}\right) = \frac{2U\varepsilon_0}{\lambda_2+\lambda_1}(\varepsilon_2\lambda_1 - \varepsilon_1\lambda_2)$$

$\boxed{6.12}$



$$U_{max} = \varphi_1 - \varphi_2$$

$$\oint D\,ds = Q$$

$$\varepsilon\varepsilon_0 E \cdot 2\pi r\cdot \ell = \rho\cdot\ell$$

$$E = \frac{\rho}{2\pi\varepsilon\varepsilon_0 r}$$

$$j = \lambda E = \frac{\lambda\rho}{2\pi\varepsilon\varepsilon_0 r}$$

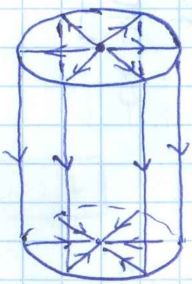
$$I = \oint_S \bar{j}\,d\bar{s} = \underbrace{\frac{\lambda\rho}{2\pi\varepsilon\varepsilon_0 r}}_{j}\cdot\underbrace{\frac{2\pi r\cdot L}{2}}_{S} = \frac{\lambda\rho L}{2\varepsilon\varepsilon_0} \Rightarrow$$

$$\Rightarrow \rho = \frac{2I\varepsilon\varepsilon_0}{\lambda L}$$

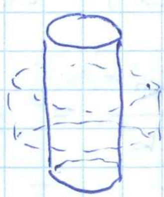
$$E = \frac{1}{2\pi\varepsilon\varepsilon_0 r} \cdot \frac{2I\varepsilon\varepsilon_0}{\lambda L} = \frac{I}{\pi\lambda L \cdot r}$$

$$U_{max} = \varphi_1 - \varphi_2 = \int_{r_0}^{r_0+l} E\,dr = \frac{I}{\pi\lambda L} \ln\left(1+\frac{l}{r_0}\right)$$

$\boxed{6.93}$



Пусть на проводах есть плотность заряда $\rho\ [\frac{Кл}{м}]$, тогда



по т. Гаусса:

$$E = \frac{\rho}{2\pi\varepsilon\varepsilon_0 r}$$

$$j = \lambda E = \frac{\rho\lambda}{2\pi\varepsilon\varepsilon_0 r}$$

$$I = \oint \vec{j}\,d\vec{S} = \frac{\rho\lambda}{\cancel{2\pi}\varepsilon\varepsilon_0 \cancel{r}} \cdot \cancel{2\pi r} \cdot \delta = \frac{\rho\lambda\delta}{\varepsilon\varepsilon_0}$$

$$E = \frac{I}{2\pi\cancel{\varepsilon\varepsilon_0} r} \cdot \frac{\cancel{\varepsilon\varepsilon_0}}{\lambda\delta} = \frac{I}{2\pi\lambda\delta r}$$

$$\Delta\varphi = \int_{d/2}^{D/2} E\,dr = \frac{I}{2\pi\lambda\delta} \ln\frac{D}{d}$$

$$R_{торц.} = \frac{\Delta\varphi}{I} = \frac{1}{2\pi\lambda\delta} \ln\frac{D}{d}$$

из 6.5 известно:

$$R_{бок} = \frac{\varepsilon\varepsilon_0}{\lambda C_{бок}} = \left\{ C_{бок} = \frac{\varepsilon\varepsilon_0 S}{d} = \frac{\varepsilon\varepsilon_0 \pi D \delta}{\ell} \right\} =$$

$$= \frac{\ell}{\lambda \pi D \delta}$$

$$R = 2R_{торц} + R_{бок} = \frac{2}{2\pi\lambda\delta} \ln\frac{D}{d} + \frac{\ell}{\lambda\pi D\delta} =$$

$$= \frac{1}{\pi\lambda\delta}\left[\frac{\ell}{D} + \ln\frac{D}{d}\right]$$

$\boxed{6.16}$ $\lambda(x) = \frac{k}{x^2}$

$$I = const = \oint_S \bar{j}\, d\bar{s} = j(x) \cdot 2\pi x \cdot \ell$$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{2\pi x \cdot \ell} \cdot \frac{x^2}{k} = \frac{I}{2\pi\ell k} x$$

$$U_0 = \int_{R_1}^{R_2} E(x)\, d\bar{x} = \frac{I}{4\pi\ell k} \cdot \left[R_2^2 - R_1^2\right]$$

$$E(x) = \frac{I}{2\pi\ell \cdot k} x = \frac{x}{\cancel{2\pi\ell \cdot k}} \cdot \frac{\overset{2}{\cancel{4\pi\ell k}}}{R_2^2 - R_1^2} U_0 =$$

$$= \frac{2U_0 x}{R_2^2 - R_1^2}$$

$$D(x) = \varepsilon\varepsilon_0 E(x) = \varepsilon\varepsilon_0 \frac{2U_0 x}{R_2^2 - R_1^2}$$

$$\rho(x) = \operatorname{div} D \overset{(x)}{=} \frac{6 U_0 \varepsilon\varepsilon_0}{R_2^2 - R_1^2}$$ (свободные заряды)

$$\operatorname{div} D = \frac{1}{x^2}\frac{\partial}{\partial x}\left(D(x)x^2\right) = \frac{1}{x^2}\frac{\partial}{\partial x}\left(\frac{2U_0\varepsilon\varepsilon_0}{R_2^2-R_1^2}x^3\right) =$$

$$= \frac{1}{x^2}\cdot\frac{6U\varepsilon\varepsilon_0}{R_2^2-R_1^2}x^2 = \frac{6U\varepsilon\varepsilon_0}{R_2^2-R_1^2}$$

Другой способ посчитать $\rho(x)$ –
– уравнение Пуассона

$$\Delta\varphi = -\frac{\rho}{\varepsilon\varepsilon_0} = \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial\varphi}{\partial x}\right) = -\frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2 E(x)\right)$$

$$\rho = \frac{6U\varepsilon\varepsilon_0}{R_2^2-R_1^2}$$

$\boxed{6.17}$ $\quad \mathcal{D} = \lambda(x)\cdot E^2(x) = \text{const}$

$$I = \oint_S \bar{j}\,d\bar{S} = j(x)\cdot 4\pi x^2 = \text{const}$$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{4\pi x^2\lambda(x)}$$

$$\mathcal{D} = \frac{I^2}{16\pi^2 x^4\lambda(x)} = \text{const} \Rightarrow \lambda(x) = \frac{k}{x^4}$$

$\boxed{6.18}$ $\quad I = \oint_S \bar{j}\,d\bar{S} = j(x)\cdot 2\pi x\cdot l = \text{const}$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{2\pi x l}\cdot\frac{1}{\lambda(x)} = \text{const} \Rightarrow$$

$$\Rightarrow \lambda(x) = c_1 j(x) = c_2\cdot\frac{1}{x}$$

$\boxed{6.19}$

$$\begin{cases} \lambda(x) = \alpha x + \beta \\ \lambda(0) = \lambda_1 \\ \lambda(l) = \lambda_2 \end{cases} \Rightarrow \quad \beta = \lambda_1, \quad \alpha = \frac{\lambda_2 - \lambda_1}{l}$$

$$E(x) = \frac{j}{\lambda(x)} = \frac{j}{\alpha x + \beta}$$

$$-\frac{\rho}{\varepsilon\varepsilon_0} = \Delta\varphi = \frac{\partial}{\partial^2 x}\varphi(x) = -\frac{\partial E(x)}{\partial x} = \frac{\alpha j}{(\alpha x + \beta)^2}$$

$$\rho = -\frac{\alpha\varepsilon\varepsilon_0 j}{(\alpha x+\beta)^2} = -\frac{\lambda_2-\lambda_1}{l}\cdot\frac{\varepsilon\varepsilon_0 j}{\left(\frac{\lambda_2-\lambda_1}{l}+\lambda_1\right)^2} =$$

$$= \frac{\varepsilon\varepsilon_0 j(\lambda_1-\lambda_2)l}{((\lambda_2-\lambda_1)x + l\lambda_1)^2}$$

$\boxed{6.20}$ $$I_i = \frac{\lambda}{\varepsilon\varepsilon_0} q_i \quad (6.1)$$

$$N_i = I_i U_i = I_i \varphi_i = \frac{\lambda}{\varepsilon\varepsilon_0} q_i \varphi_i$$

$$N = \sum_{i=1}^{n} \frac{\lambda}{\varepsilon\varepsilon_0} q_i\varphi_i = \frac{\lambda}{\varepsilon\varepsilon_0}\sum_{i=1}^{n} q_i\varphi_i$$

$\boxed{6.21}$ $$R = \frac{\varepsilon\varepsilon_0}{\lambda C} \qquad N = \frac{U_0^2}{R} = \frac{U_0^2\lambda C}{\varepsilon\varepsilon_0} =$$

$$= \frac{U_0^2\lambda}{\cancel{\varepsilon\varepsilon_0}}\cdot 4\pi\cancel{\varepsilon\varepsilon_0}\frac{R_1R_2}{R_2-R_1} = \frac{4\pi U_0^2\lambda}{\frac{1}{R_1}-\frac{1}{R_2}}$$

$\boxed{6.22}$

Ток разрядки: $I = -\frac{dq}{dt}$

с другой стороны: $I = \frac{U}{R} = \frac{q}{RC} = \frac{\lambda q}{\varepsilon\varepsilon_0}$

$$\frac{dq}{dt} + \frac{\lambda q}{\varepsilon\varepsilon_0} = 0$$

$$\frac{dq}{q} = -\frac{\lambda}{\varepsilon\varepsilon_0} dt$$

$$q(t) = q_0 \exp\left(-\frac{\lambda}{\varepsilon\varepsilon_0} t\right) \quad ; \quad q_0 = U_0 C$$

$$I(t) = \frac{\lambda q(t)}{\varepsilon\varepsilon_0} = \lambda \cdot U_0 \cdot \frac{2\pi\cancel{\varepsilon\varepsilon_0}}{\cancel{\varepsilon\varepsilon_0}} \frac{\ell}{\ln R_2/R_1} \exp\left(-\frac{\lambda}{\varepsilon\varepsilon_0} t\right) =$$

$$= \frac{2\pi\lambda\ell U_0}{\ln R_2/R_1} \exp\left(-\frac{\lambda}{\varepsilon\varepsilon_0} t\right)$$

(*): ёмкость такого конденсатора (6.5)

$\boxed{6.23}$ $C(t) = \frac{\varepsilon\varepsilon_0 S}{vt + d_0}$; $q(t) = U_0 C(t)$ $\varepsilon = 1$

$$I_0 = \frac{dq}{dt} = \frac{d}{dt}\left(U_0 \frac{\varepsilon\varepsilon_0 S}{vt + d_0}\right) = -\frac{v\varepsilon\varepsilon_0 S U_0}{(vt + d_0)^2}$$

$$\Delta q = q_2 - q_1 = C_2 U_0 - C_1 U_0 = \varepsilon\varepsilon_0 S U_0 \left(\frac{1}{vt + d_0} - \frac{1}{d_0}\right) < 0$$

$\Rightarrow$ отрицат. заряд набегает на пластину конденсатора $\Rightarrow$

$\Rightarrow$ $I = -I_0$.

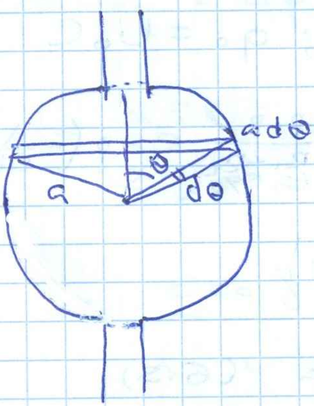
$\boxed{6.24}$ $C(t) = \frac{\varepsilon\varepsilon_0 \alpha t^2}{d}$; $q = U_0 C(t)$

$$I_0 = \frac{dq}{dt} = U_0 \frac{2\varepsilon\varepsilon_0 \alpha t}{d}$$

$$\Delta q = q_2 - q_1 = U_0 \cdot (C_2 - C_1) = \frac{\varepsilon\varepsilon_0\alpha}{d}(t_2^2 - t_1^2) > 0$$

$$I = I_0.$$

$\boxed{6.15}$



т.к. $\delta \ll a$, то сферу можно представить в виде цилиндров.

из задачи 6.13 известно, что

$$R_{бок} = R = \frac{\ell}{4\pi d\delta}$$

тогда $$dR = \frac{\cancel{a}\, d\Theta}{\lambda \cdot 2\pi \cancel{a} \sin\Theta\, \delta}$$

$$R_0 = \int\limits_{\Theta_0}^{\pi-\Theta_0} \frac{d\Theta}{\lambda \cdot 2\pi \sin\Theta\, \delta} = \frac{1}{2\pi\lambda\delta} \ln \operatorname{tg}\frac{\Theta}{2}\Big|_{\Theta_0}^{\pi-\Theta_0} =$$

$$= \frac{1}{2\pi\lambda\delta}\left[\ln \operatorname{tg}\left(\frac{\pi}{2} - \frac{\Theta_0}{2}\right) - \ln \operatorname{tg}\frac{\Theta_0}{2}\right] =$$

$$= -\frac{1}{\pi\lambda\delta} \ln \operatorname{tg}\frac{\Theta_0}{2} \;;$$

$\operatorname{tg}\frac{\Theta_0}{2} < 1$

$\Theta_0 < \frac{\pi}{2}$

$\ln\left(\operatorname{tg}\frac{\Theta_0}{2}\right) < 0.$

$\boxed{6.14}$

$$I = const = \frac{U}{R_0} = \frac{U\pi\lambda\delta}{\ln ctg\frac{\Theta_0}{2}} \quad (6.15)$$

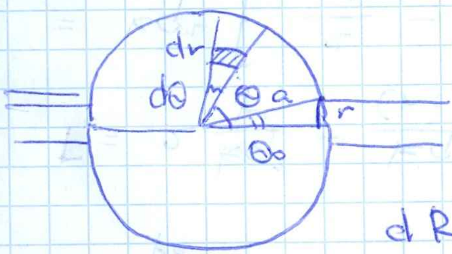
пусть $\varphi(A) = 0$, тогда:

$$\varphi(\Theta) - \varphi(A) = I R(\Theta) = I\int_{\Theta_0}^{\Theta} dR =$$

$$= \frac{U\cancel{\pi\lambda\delta}}{\ln ctg\frac{\Theta_0}{2}} \cdot \frac{1}{2\cancel{\pi\lambda\delta}} \ln tg\frac{\Theta}{2}\Big|_{\Theta_0}^{\Theta} =$$

$$= U\,\frac{\ln tg\frac{\Theta}{2} - \ln tg\frac{\Theta_0}{2}}{-2\ln tg\frac{\Theta_0}{2}}$$

$\boxed{6.25}$



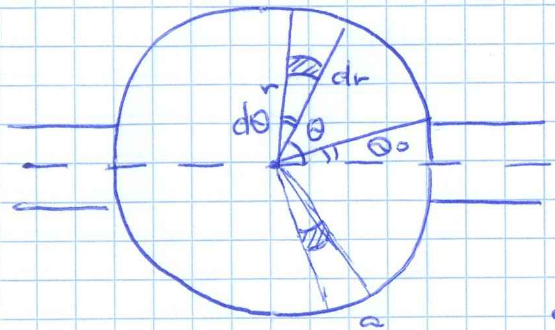
Шар можно представить в виде мн-ва концентр сфер толщины dr.

$$dR = -\frac{1}{\pi\lambda dr}\ln tg\frac{\Theta_0}{2} \quad (6.15).$$

$$A = \int_0^a \frac{1}{dR} = -\frac{\pi\lambda}{\ln tg\frac{\Theta_0}{2}}\int_0^a dr = -\frac{\pi\lambda a}{\ln tg\frac{\Theta_0}{2}} \Rightarrow$$

$$\Rightarrow R = \frac{1}{A} = -\frac{\ln tg\frac{\Theta_0}{2}}{\pi\lambda a} \approx -\frac{\rho}{\pi a}\ln\frac{r}{2a}$$

$\boxed{6.25}$



$$dR = \rho \frac{de}{dS} =$$

$$= \rho \frac{r\,d\theta}{2\pi r \sin\theta\, dr} = \frac{\rho\, d\theta}{2\pi \sin\theta\, dr}$$

$$d\Lambda = \frac{1}{dR}$$

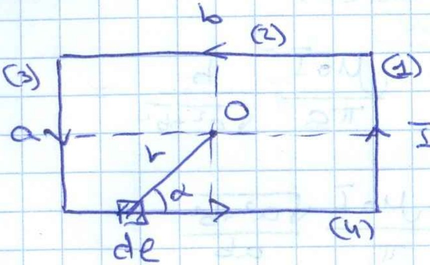
$$d\Lambda_\theta = \int_0^a \frac{1}{dR} = \int_0^a 2\pi\lambda\, dr\, \frac{\sin\theta}{d\theta} = 2\pi a \lambda \frac{\sin\theta}{d\theta}$$

$$dR_\theta = \frac{1}{d\Lambda_\theta} \qquad R = \int_{\theta_0}^{\pi-\theta_0} dR_\theta = \frac{\rho}{2\pi a} \int_{\theta_0}^{\pi-\theta_0} \frac{d\theta}{\sin\theta} =$$

$$= \frac{\rho}{2\pi a} \ln \left| tg \frac{\theta_0}{2} \right| \Big|_{\theta_0}^{\pi-\theta_0} = \frac{-\rho}{\pi a} \ln \frac{r}{2a}$$

$\ln \frac{r}{2a} < 0$, т.к. $\frac{r}{2a} < 1$.

$\boxed{7.1}$ $\boxed{7}$



$$dB = \frac{\mu_0 I}{4\pi}\,\frac{de \cdot r \cdot \sin\alpha}{r^3} = \left\{\begin{array}{l} r\sin\alpha = h_1 \\ de\,\sin\alpha = r\,d\alpha = \dfrac{h\,d\alpha}{\sin\alpha} \end{array}\right\} =$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{\sin^2\alpha}{h^2}\cdot\frac{h\,d\alpha}{\sin\alpha} = \frac{\mu_0 I}{4\pi h}\sin\alpha\,d\alpha$$

$$B_1(O) = B_3(O) = \left\{ tg\,\alpha_1 = \frac{b}{a};\ h = \frac{b}{2}\right\} =$$

$$= \int_{\alpha_1}^{\pi-\alpha_1} \frac{\mu_0 I}{4\pi h}\sin\alpha\,d\alpha = 2\cdot\frac{\mu_0 I}{4\pi}\cdot\frac{2}{b}\cdot\cos\alpha_1 =$$

$$= \left\{ tg^2\alpha_1 + 1 = \frac{1}{\cos^2\alpha_1} \Rightarrow \cos\alpha_1 = \frac{a}{\sqrt{a^2+b^2}}\right\} =$$

$$= \frac{\mu_0 I}{\pi b}\cdot\frac{a}{\sqrt{a^2+b^2}}$$

$$B_2(O) = B_4(O) = \left\{ tg\,\alpha_2 = \frac{a}{b};\ h = \frac{a}{2}\right\} =$$

$$= \int_{\alpha_2}^{\pi-\alpha_2} \frac{\mu_0 I}{4\pi h}\sin\alpha\,d\alpha = 2\cdot\frac{\mu_0 I}{4\pi}\cdot\frac{2}{a}\cdot\cos\alpha_2 =$$

$$= \left\{ tg^2\alpha_2 + 1 = \frac{1}{\cos^2\alpha_2} \Rightarrow \cos\alpha_2 = \frac{b}{\sqrt{a^2+b^2}}\right\} =$$

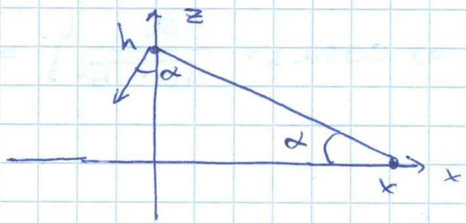
$$= \frac{\mu_0 I}{a}\cdot\frac{b}{\sqrt{a^2+b^2}}$$

$$B(O) = B_1(O) + B_2(O) + B_3(O) + B_4(O) =$$

$$= 2 \cdot \frac{\mu_0 I}{\pi b} \frac{a}{\sqrt{a^2+b^2}} + 2 \cdot \frac{\mu_0 I}{\pi a} \frac{b}{\sqrt{a^2+b^2}} =$$

$$= \frac{2\mu_0 I}{\pi\sqrt{a^2+b^2}} \left(\frac{a}{b} + \frac{b}{a} \right) = \frac{2\mu_0 I}{\pi} \frac{\sqrt{a^2+b^2}}{ab}$$

$\boxed{7.2}$ Проведём на обтекаемой током плоскости линию, перпенд. к направл. тока. Ток, приходящ. на един. длины такой линии, называется линейной плотностью тока, и рассматривается как вектор $\vec{i}$ направленный вдоль тока.



Разобьём плоскость на тонкие нити, тогда

$$dB = \frac{\mu_0 i \, dx}{2\pi\sqrt{x^2+h^2}}$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+h^2}} \qquad \sin\alpha = \frac{h}{\sqrt{x^2+h^2}}$$

$$B_x = \int_{-\infty}^{+\infty} dB \sin\alpha = \frac{\mu_0 i}{2\pi} h \int_{-\infty}^{+\infty} \frac{dx}{x^2+h^2} =$$

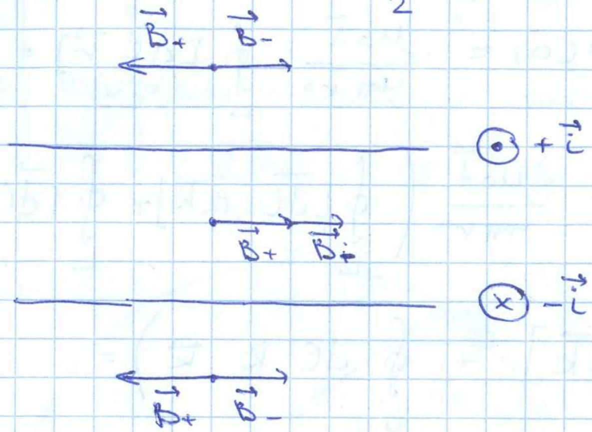
$$= \frac{\mu_0 i}{2\pi} \cdot h \cdot \left[\frac{1}{h} \operatorname{arctg} \frac{x}{h} \right] \Bigg|_{-\infty}^{+\infty} = \frac{\mu_0 i}{2}$$

$$B_y = \int_{-\infty}^{+\infty} dB \cos\alpha = \frac{\mu_0 i}{2\pi} \int_{-\infty}^{+\infty} \frac{x\,dx}{x^2+h^2} = \frac{\mu_0 i}{4\pi} \ln(a^2+x^2) \Big|_{-\infty}^{+\infty}$$

$$= 0$$

$\boxed{7.3}$ из зад. 7.2 известно:

$$B = \frac{\mu_0 i}{2}; \quad \vec{B} = \frac{\mu_0 [\vec{i}, \vec{n}]}{2}$$



поле вне пластин:

$$\vec{B} = \vec{B}_+ + \vec{B}_- = \frac{\mu_0 [+\vec{i}, \vec{n}]}{2} + \frac{\mu_0 [-\vec{i}, \vec{n}]}{2} = 0$$

поле внутри пластин:

$$\vec{B} = \vec{B}_+ + \vec{B}_- = \frac{\mu_0 [\vec{i}, \vec{n}]}{2} + \frac{\mu_0 [-\vec{i}, -\vec{n}]}{2} =$$

$$= \mu_0 [\vec{i}, \vec{n}].$$

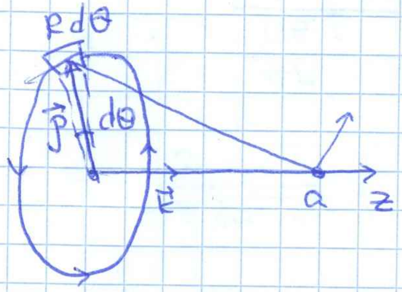
$\boxed{7.7}$ аналогично задаче 7.2

$$dB = \frac{\mu_0 i \, dx}{2\pi \sqrt{x^2+h^2}}; \quad \cos\alpha = \frac{x}{\sqrt{x^2+h^2}}; \quad \sin\alpha = \frac{h}{\sqrt{x^2+h^2}}$$

$$B_x = \int_{-l/2}^{l/2} dB \sin\alpha = \frac{\mu_0 i}{2\pi} \operatorname{arctg} \frac{x}{h} \Big|_{-l/2}^{l/2} = \frac{\mu_0 i}{\pi} \operatorname{arctg} \frac{l}{2h}$$

$$B_y = \int_{-l/2}^{l/2} dB \cos\alpha = \frac{\mu_0 i}{4\pi} \ln(h^2+x^2) \Big|_{-l/2}^{l/2} = 0.$$

7.6 1) Вычислим магнитную индукцию $\vec{B}$ кольца:



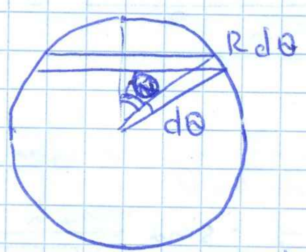
$$\vec{B}(a) = \frac{\mu_0 I}{4\pi r^3} \oint_L [\vec{dl}, \vec{r}] =$$

$$= \frac{\mu_0 I}{4\pi r^3} \left(\oint_L [\vec{dl}, a\vec{k}] + \oint_L [\vec{dl}, \vec{\rho}] \right) =$$

$$= \frac{\mu_0 I}{4\pi r^3} \left(\Big[\underbrace{\oint_L \vec{dl}}_{=0}, a\vec{k} \Big] + \oint_L dl \cdot R \cdot \vec{k} \right) =$$

$$= \frac{\mu_0 I}{4\pi r^3} \vec{k} \oint_0^{2\pi} R \cdot R\,d\theta = \frac{\mu_0 I R^2}{2 r^3} = \frac{\mu_0 I R^2}{2 (R^2 + a^2)^{3/2}}$$

2) Разобьём сферу на кольца:



$$dI = \frac{dq}{T} = \frac{\sigma\, dS}{T} = \frac{\sigma \cdot 2\pi R \sin\theta \cdot R\,d\theta}{T} =$$

$$= \sigma \omega R^2 \sin\theta\, d\theta$$

Можно найти $\vec{B}$ в центре сферы от кольца:

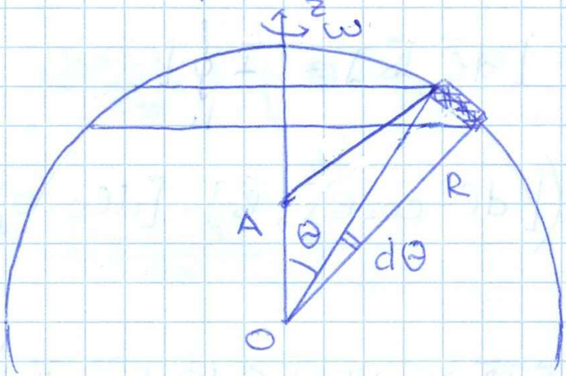
$$dB_0 = \frac{\mu_0\, dI}{2} \frac{R^2 \sin^2\theta}{(R^2 \sin^2\theta + R^2 \cos^2\theta)^{3/2}} =$$

$$= \frac{\mu_0 \sigma \omega\, d\theta}{2} \cdot \frac{R^4 \sin^3\theta}{R^3} = \frac{\mu_0 \omega \sigma R}{2} \sin^3\theta\, d\theta$$

$$\vec{B}_0 = \int_0^{\pi} dB = \frac{\mu_0 \omega \sigma R}{2} \int_0^{\pi} \sin^3\Theta \, d\Theta = \frac{2}{3} \mu_0 \vec{\omega} \sigma R$$

3) Найдём линейную плотность токов:

$$i = \frac{dI}{R\,d\Theta} = \sigma \omega R \sin\Theta = i_0 \sin\Theta$$



по т. косинусов:

$$r^2 = R^2 + z^2 - 2Rz\cos\Theta$$

$$dB = \frac{\mu_0 \, dI \cdot R^2 \sin^2\Theta}{2r^3} = \frac{\mu_0 i_0 R^3 \sin^3\Theta \, d\Theta}{2r^3} =$$

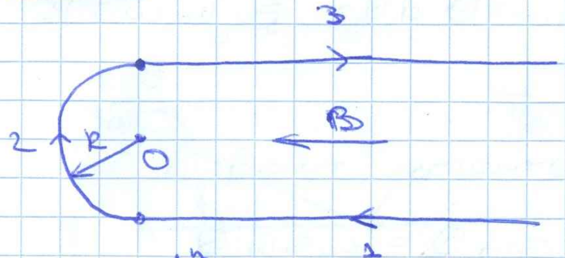
$$= \frac{\mu_0 i_0 R^3 \sin^3\Theta}{2} \left(R^2 + z^2 - 2Rz\cos\Theta\right)^{-3/2} d\Theta =$$

$$= \frac{\mu_0 i_0}{2} \frac{\sin^3\Theta \, d\Theta}{\left(1 + \zeta^2 - 2\zeta\cos\Theta\right)^{3/2}}, \quad \zeta = \frac{z}{R}$$

$$\vec{B} = \frac{\mu_0 i_0}{2} \int_0^{\pi} \frac{\sin^3\Theta \, d\Theta}{\left(1 + \zeta^2 - 2\zeta\cos\Theta\right)^{3/2}} = \frac{2\mu_0 i_0}{3} =$$

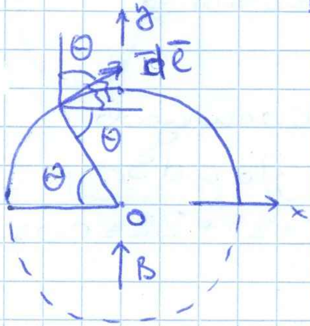
$$= \frac{2}{3} \mu_0 \sigma R \vec{\omega}$$

7.10



$F_1 = I\,[\vec{l}, \vec{B}] = 0$

$F_3 = I\,[-\vec{l}, \vec{B}] = 0$



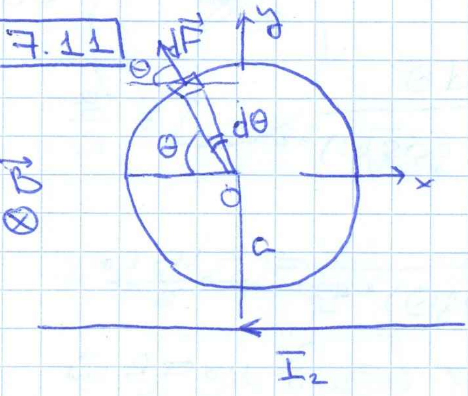
$$\vec{F} = I \oint_L [d\vec{l}, \vec{B}] =$$

$$= I \oint_L \left([dl \cdot \cos\theta\, \vec{j}, \vec{B}] + [dl \sin\theta\, \vec{i}, \vec{B}]\right) =$$

$$= I \oint_L \left(R\,d\theta \cos\theta \underbrace{[\vec{j}, \vec{B}]}_{=0} + R \sin\theta\, d\theta\, [\vec{i}, \vec{B}]\right) =$$

$$= I \oint_0^{\pi} BR \sin\theta\, d\theta \cdot \vec{i} = 2BRI \cdot \vec{i}$$

7.11



$$B = \frac{\mu_0 I_2}{2\pi (a + R \sin\theta)}$$

$$dF = I_1 [d\vec{l}, \vec{B}]$$

$$F_y = I_1 \oint_L [d\vec{l} \cdot \sin\theta, \vec{B}] = I_1 \int_0^{2\pi} B\, R\, d\theta \cdot \sin\theta =$$

$$= I_1 \int_0^{2\pi} \frac{\mu_0 I_2}{2\pi}\cdot\frac{R\sin\theta}{a+R\sin\theta}\,d\theta =$$

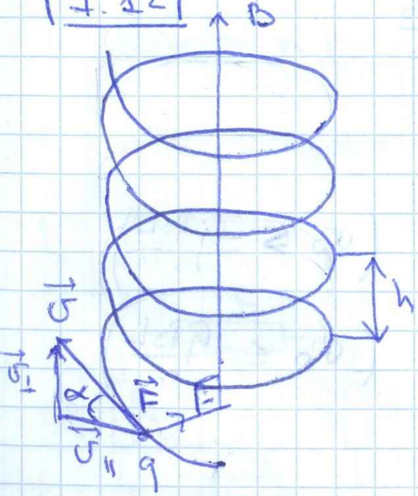
$$= \frac{\mu_0 I_1 I_2}{2\pi}\int_0^{2\pi}\frac{R\sin\theta}{a+R\sin\theta}\,d\theta =$$

$$= \mu_0 I_1 I_2\left(-1+\sqrt{\frac{a^2}{a^2-R^2}}\right)$$

$$F_x = I_2 \oint_L [\vec{dl}\cos\theta, \vec{B}] = \frac{\mu_0 I_1 I_2}{2\pi}\int_0^{2\pi}\frac{R\cos\theta}{a+R\sin\theta}\,d\theta =$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\ln(a+R\sin\theta)\Big|_0^{2\pi} = 0$$

$\boxed{7.12}$



Частицы двигаются по спирали с радиусом витка R

$$\begin{cases} \vec{F} = q[\vec{v},\vec{B}] = qv\sin\alpha B = ma \\ a = \dfrac{v_\parallel^2}{R} = \dfrac{v^2\sin^2\alpha}{R} \end{cases}$$

$$\Rightarrow R = \frac{mv\sin\alpha}{Bq}$$

и шагом спирали h:

$$h = v_\perp T = v_\perp \cdot \frac{2\pi R}{v_\parallel} = \frac{2\pi m \cancel{v\sin\alpha}}{Bq\,\cancel{v\sin\alpha}} v\cos\alpha =$$

$$= \frac{2\pi m v\cos\alpha}{Bq}$$

Точки максимально удаляется друг от друга на расстояние

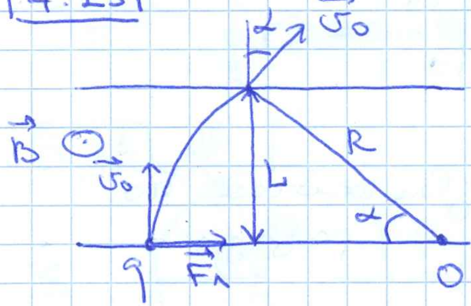
$$d = 2R + 2R = 4R$$

И первый раз встрется через время

$$T = \frac{2\pi R}{v_\parallel} = \frac{2\pi m\, v\sin\alpha}{Bq\,\cancel{v\sin\alpha}} = \frac{2\pi m}{Bq}$$

на расстоянии h от точки старта

$\boxed{7.13}$



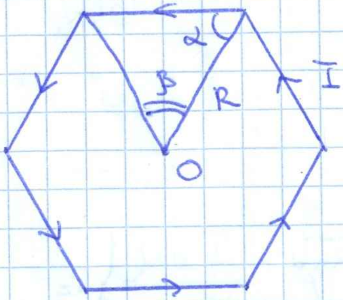
$$\frac{m v_0^2}{R} = q v_0 B \Rightarrow$$

$$\Rightarrow R = \frac{m v_0}{qB}$$

$R > L$: $\sin\alpha = \frac{L}{R} = \frac{LqB}{mv_0}$ $\left(v_0 > \frac{qBL}{m}\right)$

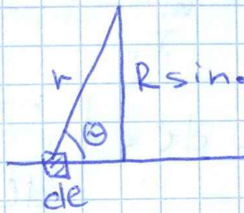
$R \leq L$: $\alpha = \pi$. $\left(v_0 \leq \frac{qBL}{m}\right)$

$\boxed{7.14}$



$$2\alpha = \pi - \beta = \pi - \frac{2\pi}{n} \Rightarrow$$

$$\Rightarrow \alpha = \frac{n-2}{2n}\pi$$



$$dB = \frac{\mu_0 I}{4\pi r^2} d\ell \sin\theta = \left\{ \begin{array}{l} r = \dfrac{R\sin\alpha}{\sin\theta} \\ d\ell\sin\theta = r\,d\theta = \dfrac{R\sin\alpha}{\sin\theta} d\theta \end{array} \right\}$$

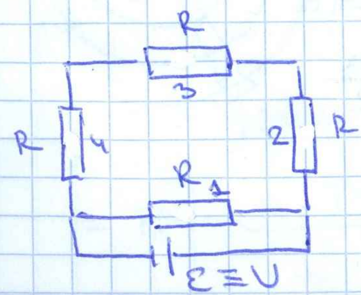
$$= \frac{\mu_0 I \sin\theta}{4\pi R \sin\alpha} d\theta$$

$$B = \frac{\mu_0 I}{4\pi R\sin\alpha} \int_{\alpha}^{\pi-\alpha} \sin\theta\, d\theta = \frac{\mu_0 I}{2\pi R} \operatorname{ctg}\alpha$$

$$B_\Sigma = nB = \frac{\mu_0 n I}{2\pi R} \operatorname{ctg}\alpha, \quad \alpha = \frac{n-2}{2n}\pi$$

$$\operatorname{ctg}\alpha = \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{tg}\left(\frac{\pi}{2} - \frac{n-2}{2n}\pi\right) = \operatorname{tg}\frac{\pi}{n}$$

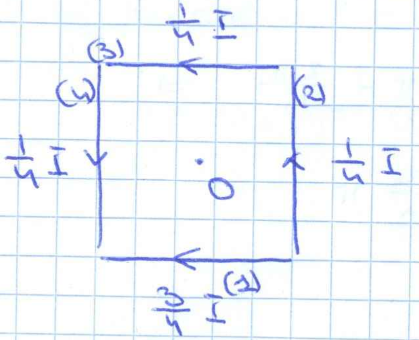
$\boxed{7.15}$

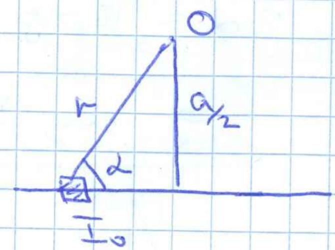


$$U = I_1 \cdot R = I_2 \cdot (R+R+R)$$

пусть через ЭДС течёт ток I:

$$I_1 = \frac{3I}{4} \qquad I_2 = \frac{I}{4}$$





$$dB = \frac{\mu_0 I_0}{4\pi r^2}\sin\alpha \, dl = \left\{ \begin{array}{l} r = \frac{a}{2\sin\alpha} \\ dl \cdot \sin\alpha = r\,d\alpha \end{array} \right\} =$$

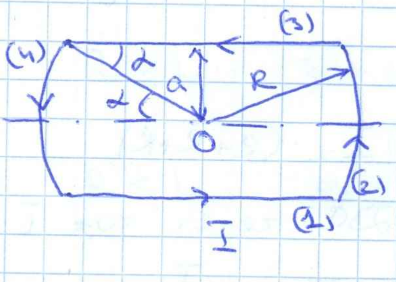
$$= 2\frac{\mu_0 I_0}{4\pi a}\sin\alpha \, d\alpha$$

$$B = 2\frac{\mu_0 I_0}{4\pi a}\int\limits_{\pi/4}^{3\pi/4}\sin\alpha\, d\alpha = \frac{\mu_0 I_0}{\pi a}\cos\frac{\pi}{4}$$

$$B_\Sigma = -B_1 + B_2 + B_3 + B_4 =$$

$$= -\frac{3}{4}\frac{\mu_0 I}{\pi a}\cos\frac{\pi}{4} + 3\cdot\frac{\mu_0 I}{4\pi a}\cos\frac{\pi}{4} = 0.$$

$\boxed{7.16}$

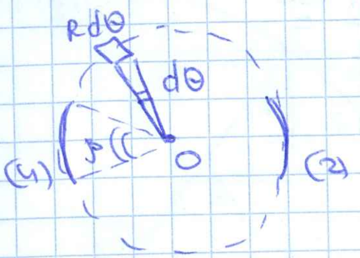


известно, что (7.15)

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi a}\cos\alpha,$$

$$\sin\alpha = \frac{a}{R}$$

$$\otimes B_\Sigma$$



$$\vec{B}_0(O) = \frac{\mu_0 I}{4\pi R^3} \oint_L [\vec{dl}, \vec{R}] =$$

$$= \frac{\mu_0 I}{4\pi R^3} \oint_L dl \cdot R \cdot \vec{k} =$$

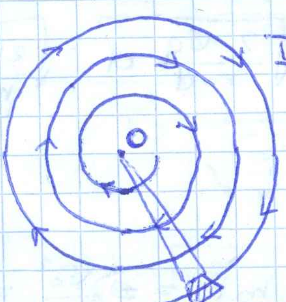
$$= 2\frac{\mu_0 I}{4\pi R^3} \oint_0^{\beta} R^2 \, d\rho \; \vec{k} = \frac{\mu_0 I}{4\pi R} \cdot 2\beta \; ; \; \beta = 2\alpha$$

$$B_{\Sigma} = B_1 + B_2 + B_0 = 2\frac{\mu_0 I}{2\pi a} \sqrt{1 - \frac{a^2}{R^2}} + \frac{\mu_0 I \alpha}{\pi R} =$$

$$= \frac{\mu_0 I}{R\pi} \left(\frac{\sqrt{R^2 - a^2}}{a} + \arcsin \frac{a}{R} \right)$$

$\boxed{7.17}$

$$\begin{cases} \rho(\theta) = \alpha\theta + \beta \\ \rho(0) = R_2 & \Rightarrow \quad \beta = R_2 \\ \rho(2\pi N) = R_1 & \Rightarrow \quad \alpha = \dfrac{R_1 - R_2}{2\pi N} \end{cases}$$



$$\vec{B}(O) = \frac{\mu_0 I}{4\pi} \int_L \frac{[\vec{dl}, \vec{\rho}(\theta)]}{\rho^3(\theta)} =$$

$\otimes \; \vec{n}$

$$= \frac{\mu_0 I}{4\pi} \int_0^{2\pi N} \frac{\rho(\theta)\, d\theta \cdot \rho(\theta)}{\rho^3(\theta)} \vec{n} =$$

$$= \frac{\mu_0 I}{4\pi} \vec{n} \int_0^{2\pi N} \frac{d\theta}{\alpha\theta + \beta} = \frac{\mu_0 I}{4\pi\alpha} \vec{n} \ln(\alpha\theta + \beta) \Big|_0^{2\pi N} =$$

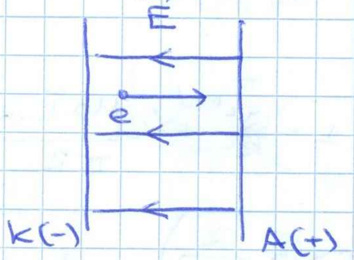
$$= \vec{n} \cdot \frac{\mu_0 I N}{2} \frac{\ln R_1/R_2}{R_1 - R_2}$$

$$\lim_{R_1\to R_2} B(0) = \lim_{R_1\to R_2} \frac{\mu_0 I N}{2}\,\frac{\ln R_1/R_2}{R_1 - R_2} \overset{Л}{=}$$

$$\overset{Л}{=} \frac{\mu_0 I N}{2} \lim_{R_1\to R_2} \frac{\frac{R_2}{R_1}\cdot\frac{1}{R_2}}{1} = \frac{\mu_0 I N}{2R_1}$$

$\boxed{7.18}$ Разобьём задачу на 3 этапа:

1) Электрическое поле



$$\begin{cases} U = Ed \\ F = Eq = ma \end{cases} \qquad |e| \equiv q$$

$$\Rightarrow a = \frac{Eq}{m} = \frac{Uq}{md}$$

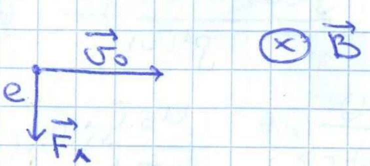
$$\upsilon_0 = at$$

$$d = \frac{at^2}{2} \Rightarrow t^2 = \frac{2d}{a} = \frac{2md^2}{Uq}$$

$$\Rightarrow t = d\sqrt{\frac{2m}{Uq}} = \sqrt{\frac{2d}{a}}$$

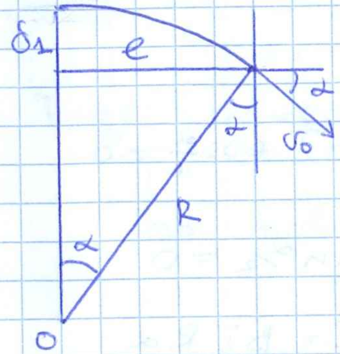
$$\upsilon_0 = at = a\sqrt{\frac{2d}{a}} = \sqrt{2da} = \sqrt{\frac{2Uq}{md}\,d} = \sqrt{\frac{2Uq}{m}}$$

2) Магнитное поле



$$F_л = Bq\upsilon_0 = ma = \frac{m\upsilon_0^2}{R} \Rightarrow$$

$$\Rightarrow R = \left(\frac{Bq}{m\upsilon_0}\right)^{-1} = \frac{m\upsilon_0}{Bq}$$



$\delta_1 = R - R\cos\alpha;$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{e^2}{R^2}}$

$\delta_1 = R\left(1-\sqrt{1-\frac{e^2}{R^2}}\right) =$

$= \frac{m v_0}{Bq}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2v_0^2}}\right)$

$v_1 = v_0\cos\alpha \qquad v_2 = v_0\sin\alpha$

3) Свободный полёт:

$D = v_0\cos\alpha\, t \Rightarrow t = \frac{D}{v_0\cos\alpha}$

$\delta_2 = v_0\sin\alpha\, t = D\, tg\alpha = D\cdot\frac{e}{\sqrt{R^2-e^2}}$

$\delta = \delta_1+\delta_2 = \frac{m}{Bq}\cdot\sqrt{\frac{2Uq}{m}}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2v_0^2}}\right) + D\frac{e}{\sqrt{R^2-e^2}}$

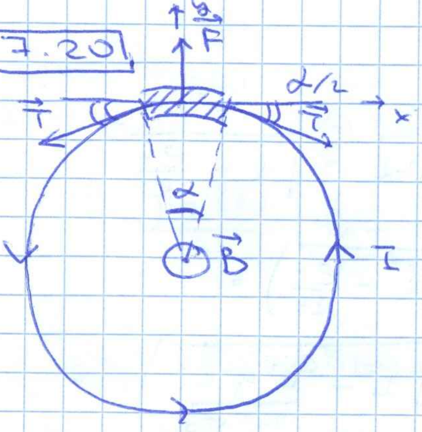
$= \frac{1}{B}\sqrt{\frac{2Um}{q}}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2v_0^2}}\right) +$

$+ D\frac{1}{\sqrt{\frac{m^2v_0^2}{B^2q^2e^2}-1}} =$

$= \frac{1}{B}\sqrt{\frac{2Um}{q}}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2}\cdot\frac{m}{2Uq}}\right) +$

$+ D\frac{1}{\sqrt{\frac{m^2}{B^2q^2e^2}\cdot\frac{2Uq}{m}-1}}$

7.20



$\alpha \ll 1$

$x:\quad T\cos\frac{\alpha}{2} - T\cos\frac{\alpha}{2} = 0$

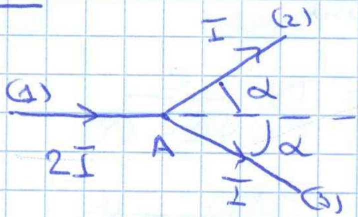
$y:\quad F - 2T\sin\frac{\alpha}{2} = 0$

$F = BI\,dl = BIR\alpha$

$BIR\alpha = 2T\sin\frac{\alpha}{2} \approx T\alpha$

$\Rightarrow \quad I = \dfrac{T}{BR}$

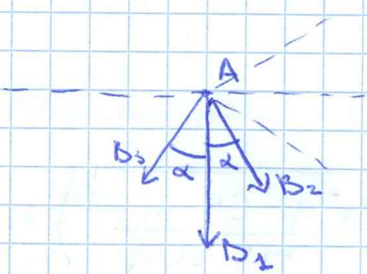
7.8



из 7.14 известно, что:

$$B_{(1)} = 2B_{(2)} = 2B_{(3)} = \frac{\mu_0 2I}{4\pi a}\cdot\int_0^{\pi/2}\sin\theta\, d\theta = \frac{\mu_0 2I}{4\pi a}.$$

Легко видеть, что:



$$B_\Sigma = B_1 + 2B_2\cos\alpha = \frac{\mu_0 I}{2\pi a}(1+\cos\alpha)$$

7.5 Из зад. 7.4 известно, что магн. инд. внутри провода равна 0.

Тогда исходная система равносильна такой системе:

$$\underbrace{\text{по поверхности провода течёт ток}}_{\text{с учётом мощности щели}}$$

$$\hat{I} = I + I_d$$

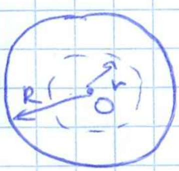
по поверхности щели течёт ток $-I_d$.

$$I_d = \frac{I}{2\pi R} \cdot d = i d$$

Тогда магнитная индукция будет создаваться только щелью (проводом)

$$B = \frac{\mu_0 I_d}{2\pi r} = \frac{\mu_0 I}{4\pi^2 R r} d$$

7.4 Воспользуемся т. о циркуляции магнитного поля.



$\odot I$ 1) $r < R$:

$$B \cdot 2\pi r = 0 \Rightarrow B = 0$$

2) $r \geq R$:

$$B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}.$$

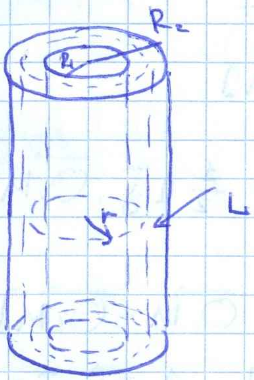
7.8 По т. о циркуляции магн. поля:

1) $r < R$: $B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$

2) $R \leq r$: $B \cdot 2\pi r = 0 \Rightarrow B = 0.$

8.1

8



Воспользуемся теор. о циркуляции:

$$\oint_L \vec{H}\, d\vec{l} = H\cdot 2\pi r = \tilde{I}$$

1) $r \le R_1$: $H\cdot 2\pi r = I\cdot \dfrac{\pi r^2}{\pi R_1^2} \Rightarrow$

$$\Rightarrow H = \frac{I r}{2\pi R_1^2}$$

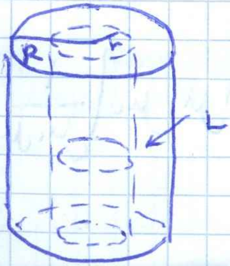
2) $R_1 \le r \le R_2$: $H\cdot 2\pi r = I \Rightarrow H = \dfrac{I}{2\pi r}$

3) $R_2 \le r$: $H\cdot 2\pi r = 0 \Rightarrow H = 0$

$$H = \begin{cases} \dfrac{I r}{2\pi R_1^2} & ; \; r \le R_1 \\ \dfrac{I}{2\pi r} & ; \; R_1 \le r \le R_2 \\ 0 & ; \; R_2 \le r \end{cases}$$

8.2 Рассмотрим систему из сплошных цилиндров с токами плотности j, текущ. в противоположных направлениях.

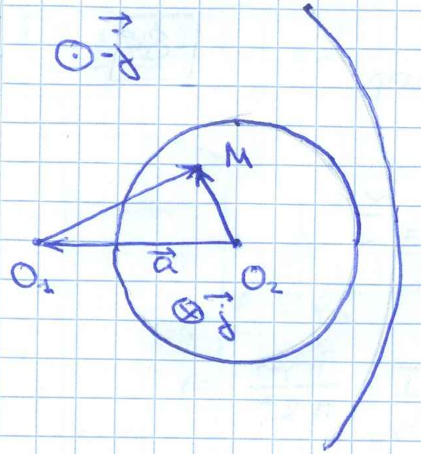
Эта система будет эквивалентна данной.



Найдём H внутри цилиндра

$$\oint_L \vec{H}\, d\vec{l} = H\cdot 2\pi r = j\pi r^2 \Rightarrow$$

$$\Rightarrow H = \frac{jr}{2}$$

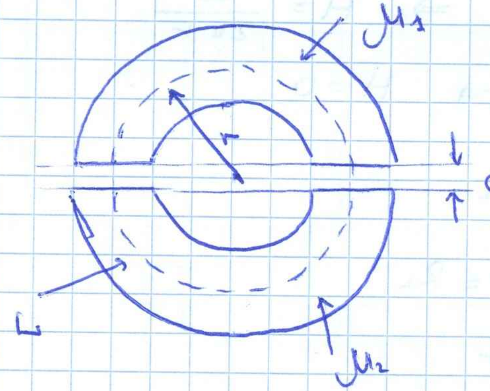


$$\vec{H}_1 = \frac{1}{2}[\vec{j}_1, \vec{r}_1] = -\frac{1}{2}[\vec{j}, \overrightarrow{O_1M}]$$

$$\vec{H}_2 = \frac{1}{2}[\vec{j}_2, \vec{r}_2] = \frac{1}{2}[\vec{j}, \overrightarrow{O_2M}]$$

$$\vec{H}_M = \vec{H}_1 + \vec{H}_2 = \frac{1}{2}[\vec{j}, \overrightarrow{O_2M} - \overrightarrow{O_1M}] = \frac{1}{2}[\vec{j}, \overrightarrow{O_2O_1}] = \frac{1}{2}[\vec{j}, \vec{a}]$$

$\boxed{8.3}$



$$\vec{H}_1 = \frac{\vec{B}(r)}{\mu_0\mu_1}$$

$$\vec{H}_2 = \frac{\vec{B}(r)}{\mu_0\mu_2}$$

$$\vec{H}_0 = \frac{\vec{B}(r)}{\mu_0}$$

$$\oint_L \vec{H}\,d\vec{l} = H_1\left(\frac{L}{2} - d\right) + H_2\left(\frac{L}{2} - d\right) + H_0 \cdot 2d =$$

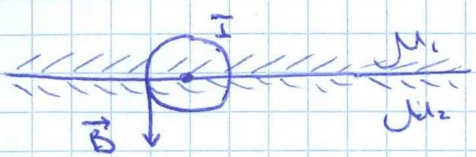
$$= \frac{B(r)}{\mu_0}\left[\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\left(\frac{L}{2} - d\right) + 2d\right] =$$

$$= \frac{B(r)}{\mu_0}\left[(\mu_2 + \mu_1)\left(\frac{L}{2} - d\right) + 2d\mu_1\mu_2\right]\frac{1}{\mu_1\mu_2} =$$

$$= IN$$

$$B(r) = \frac{\mu_0 \mu_1 \mu_2 I N}{(\mu_1+\mu_2)\left(\frac{L}{2}-d\right) + 2d\,\mu_1\mu_2}$$

$\boxed{8.4}$



$$B_{1n} = B_{2n} = B$$

$$\mu_1\mu_0 H_1 = \mu_2\mu_0 H_2 = B$$

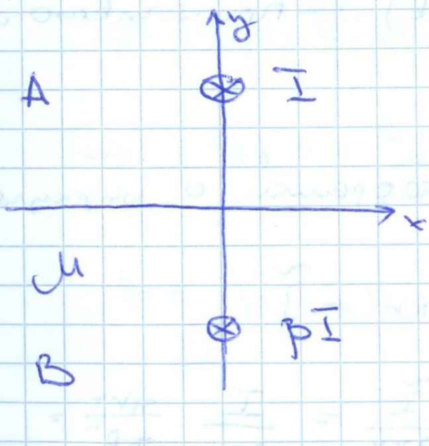
$$\mu_1 H_1 = \mu_2 H_2$$

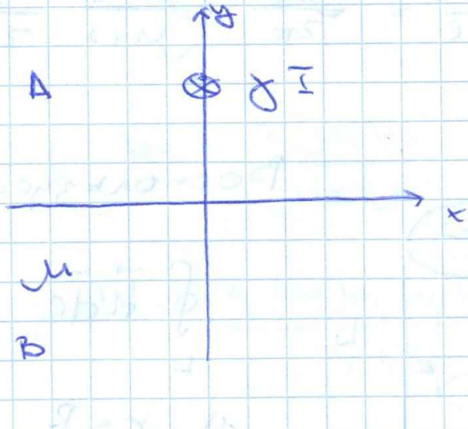
$$I = \oint H\,dl = \pi r\cdot H_1 + \pi r\cdot H_2 = \pi r(H_1+H_2)$$

$$I = \pi r\left(\frac{\mu_2 H_2}{\mu_1} + H_2\right) \Rightarrow \frac{\pi r}{\mu_1}(\mu_2 H_2 + \mu_1 H_2)$$

$$H_2 = \frac{I\mu_1}{\pi r(\mu_1+\mu_2)} \Rightarrow B = \frac{\mu_1\mu_2 I \mu_0}{\pi r(\mu_1+\mu_2)}$$

$\boxed{8.7}$ Воспользуемся методом изображений





для определения поля в A

для определения поля в B

$$H = \frac{I}{2\pi r} \sim I$$

$$\begin{cases} H_{\tau_1} = H_{\tau_2} \\ H_{n_1} = \mu H_{n_2} \end{cases} \Rightarrow \begin{cases} \beta - 1 = -\gamma \\ \beta + 1 = \mu\gamma \end{cases}$$

$$1 - \gamma + 1 = \mu\gamma \Rightarrow \gamma = \frac{2}{\mu+1}$$

$$\beta = 1 - \gamma \Rightarrow \quad \Rightarrow \beta = \frac{\mu - 1}{\mu + 1}$$

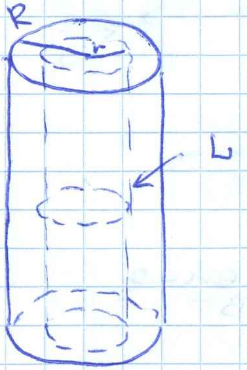
Т. о.

$$\vec{dF} = I\,[\vec{dl}, \vec{B}_\beta] = I \cdot dl \cdot \frac{\beta I \mu_0}{2\pi \cdot 2h}(-\vec{k}) =$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I^2}{2h} \cdot \frac{\mu - 1}{\mu + 1}\, dl\, (-\vec{k})$$

$$\frac{\vec{dF}}{dl} = \frac{\mu_0}{2\pi} \frac{\mu-1}{\mu+1} \frac{I^2}{2h} (-\vec{k})$$ – притягиваются.

$\boxed{8.8}$



Воспользуемся теоремой о циркуляции

$$\oint_L \vec{H}\vec{dl} = H \cdot 2\pi r = \tilde{I}$$

1) $r \le R$ $\quad H = \frac{\tilde{I}}{2\pi r} = \frac{I}{2\pi r} \cdot \frac{\pi r^2}{\pi R^2} = \frac{I r}{2\pi R^2}$

2) $r \ge R$ $\quad H = \frac{I}{2\pi r}$

$$B = \mu_0 \tilde{\mu} H = \begin{cases} \mu\mu_0 \dfrac{Ir}{2\pi R^2}, & r \le R \\ \mu_0 \dfrac{I}{2\pi r}; & r \ge R \end{cases}$$

$\boxed{8.5}$ Очевидно, что для контура $H_{\tau_1} = H_{\tau_2} = 0$.

$$H_1 \mu_1 = H_2 \mu_2 \Rightarrow \quad H_2 = \frac{\mu_1}{\mu_2} H_1$$

Поле контура соленоидально и симм. относительно границы раздела:

$$\oint_L H_0\, dl = I \quad (1)$$

$$\oint_L \tilde{H}\, dl = \oint_{L_1} H_1\, dl + \oint_{L_2} H_2\, dl = I \quad (2)$$

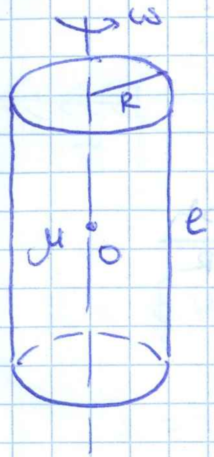
$$L_1 = L_2; \; L_1 \cup L_2 = L$$

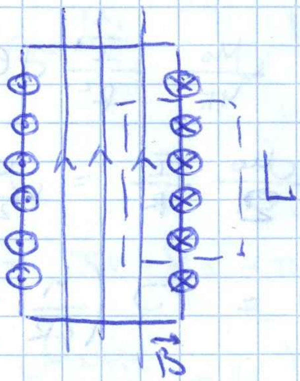
$$\Rightarrow \; 2H_0 = H_1 + H_2$$

$$\Rightarrow \quad H_1 = \frac{2\mu_2 H_0}{\mu_2 + \mu_1}; \quad H_2 = \frac{2\mu_1 H_0}{\mu_2 + \mu_1}$$

$$B = \mu_1 \mu_0 H_1 = \mu_2 \mu_0 H_2 = \frac{2\mu_0 \mu_1 \mu_2 H_0}{\mu_1 + \mu_2}$$

$\boxed{8.13}$





Цилиндр тонкий и длинный

$\Downarrow$

$\ell \ll R$

$$I = \frac{Q}{2\pi}\omega$$

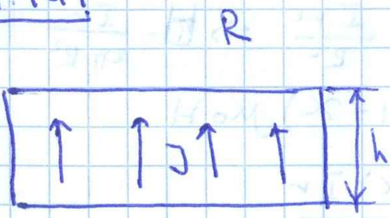
Поле внутри соленоида почти однородно.

Поле вне соленоида (вблизи его) почти отсутствует.

$$\oint \vec{H}\, d\vec{l} = \frac{B}{\mu\mu_0} L = \frac{Q}{2\pi}\omega$$

$$B(O) = \frac{\mu\mu_0 Q\omega}{2\pi L} \; ; \quad H = \frac{B}{\mu\mu_0} = \frac{Q\omega}{2\pi L}$$

$\boxed{8.14}$



Т.к. намагниченность однородна, то внутри магнита токов нет

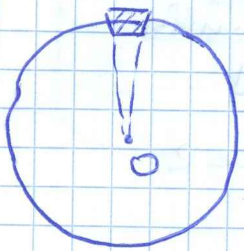
$\Downarrow$

только на поверхности

из гранич. условий $\underbrace{J_{2\tau}}_{J} - \underbrace{J_{1\tau}}_{0} = i'$

$\Rightarrow \; i' = J$

Диск тонкий $\Rightarrow$ можно считать кольцом радиуса R с током $I' = h i'$.

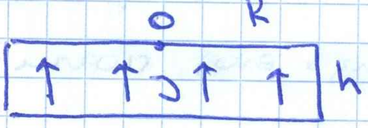


$$dB = \frac{\mu_0}{4\pi} \frac{I'}{R^2} dl$$

$$B = \frac{\mu_0}{2} \frac{I'}{R} = \frac{\mu_0}{2} \frac{Jh}{R}$$

$$H = \frac{B}{\mu_0} - J = J\left(\frac{h}{2R} - 1\right)$$

$\boxed{8.11}$

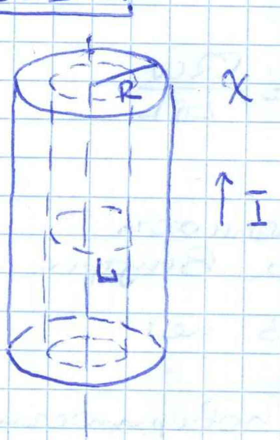


Диск тонкий $\Rightarrow$

$\Rightarrow$ можно считать кольцом.

$$(O): \quad B_{n_1} = B_{n_2} = B_1 = B_2 = \frac{\mu_0}{2} \frac{Jh}{R} \quad (8.14)$$

$\boxed{8.15}$



1) $0 \le r \le R$

$$\oint_L H dl = \frac{I}{\pi R^2} \pi r^2 = \frac{I r^2}{R^2}$$

$$2\pi r \cdot H = \frac{I r^2}{R^2} \Rightarrow H = \frac{I r}{2\pi R^2}$$

$$B = \mu \mu_0 H = (1+\chi) \mu_0 H$$

$$B = \frac{\mu_0 (1+\chi) I r}{2\pi R^2}$$

2) $R \le r$ $\quad \oint_L H dl = I \Rightarrow 2\pi r \cdot H = I \Rightarrow H = \frac{I}{2\pi r}$

$$B = \mu_0 H = \mu_0 \frac{I}{2\pi r}$$

$\boxed{8.12}$

$$\begin{cases} I = \alpha z + \beta \\ I(0) = I_m \\ I(\ell) = 0 \end{cases} \Rightarrow \quad I(z) = \frac{I_m}{\ell}(\ell - z)$$

$$I(z) = i(z) \quad \Rightarrow \quad dI = i(z)\,dz$$

$$dB = \frac{\mu_0 dI}{2}\,\frac{R^2}{(R^2+z^2)^{3/2}} = \frac{\mu_0 I(z)}{2}\,\frac{R^2\,dz}{(R^2+z^2)^{3/2}} =$$

$$B = \int_0^\ell dB = \underbrace{\int_0^\ell \frac{\mu_0 I_m}{2}\,\frac{R^2}{(R^2+z^2)^{3/2}}\,dz}_{I_1} - \underbrace{\int_0^\ell \frac{\mu_0 I_m}{2\ell}\,\frac{R^2 z\,dz}{(R^2+z^2)^{3/2}}}_{I_2}$$

$$I_1 = \frac{\mu_0 I_m}{2}\int_0^\ell \frac{R^2}{(R^2+z^2)^{3/2}}\,dz = \frac{\mu_0 I_m}{2}\,\frac{\ell}{(R^2+\ell^2)^{3/2}}$$

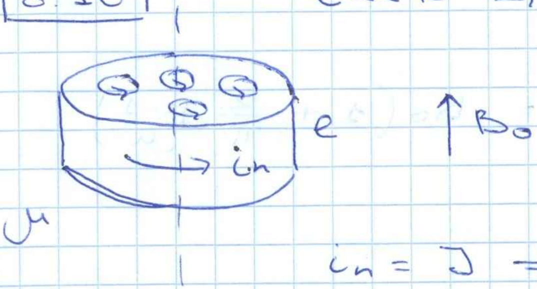
$$I_2 = \frac{\mu_0 I_m R^2}{2\ell}\int_0^\ell \frac{z\,dz}{(R^2+z^2)^{3/2}} = \frac{\mu_0 I_m R^2}{2\ell}\left[\frac{-1}{\sqrt{R^2+z^2}}\right]\Bigg|_0^\ell =$$

$$= \frac{\mu_0 I_m R^2}{2\ell}\left(\frac{1}{R} - \frac{1}{(R^2+\ell^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I_m}{2}\,\frac{\ell}{(R^2+\ell^2)^{3/2}} + \frac{\mu_0 I_m}{2}\left(\frac{R}{\ell} - \frac{R^2}{\ell(R^2+\ell^2)^{1/2}}\right) =$$

$$= \frac{\mu_0 I_m}{2\ell}\left(\sqrt{R^2+\ell^2} - R\right) \approx \frac{\mu_0 I_m}{2}\left(1 - R/\ell\right)$$

$\boxed{8.10}$ $\ell \ll R \Rightarrow$ считаем кольцом



$$B = \mu\mu_0 H = (1+\chi)\mu_0 H = B_0 + \underbrace{\mu_0 \chi H}_{B'}$$

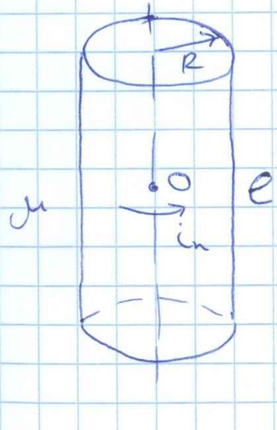
$$i_n = J \Rightarrow I = i_n \ell = J\ell$$

$$B' = \frac{\mu_0 I}{2R} = \frac{\mu_0 J \ell}{2R} =$$

$$J = \chi H = (\mu - 1)\frac{B_0}{\mu\mu_0}$$

$$= \frac{\mu-1}{\mu}\frac{B_0 \ell}{2R} \Rightarrow B = B_0\left(1 + \frac{\mu-1}{\mu}\frac{\ell}{2R}\right)$$

$\boxed{8.9}$ $R \ll \ell$



$$B = \mu\mu_0 H = (1+\chi)\mu_0 H = B_0 + \underbrace{\mu_0 \chi H}_{B'}$$

$$i_n = J \Rightarrow I = i_n \ell = J\ell$$

$$dB' = \frac{\mu_0}{2\pi}\frac{dI \cdot R^2}{(R^2+z^2)^{3/2}} = \frac{\mu_0}{2\pi}\frac{R^2 J\, dz}{(R^2+z^2)^{3/2}}$$

$$B' = \int_{-\ell/2}^{\ell/2} \frac{\mu_0}{2\pi}\frac{J R^2}{(R^2+z^2)^{3/2}}\,dz =$$

$$= \frac{\mu_0 J}{2\pi}\left[\frac{z}{(z^2+R^2)^{1/2}}\right]\Bigg|_{-\ell/2}^{\ell/2} = \frac{\mu_0 J}{2\pi}\frac{\ell}{(\ell^2/4+R^2)^{1/2}} =$$

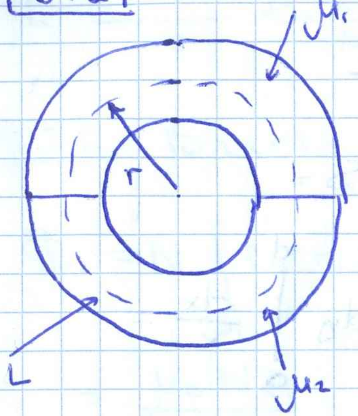
$$= \frac{\mu_0 J}{2\pi}4 = \frac{2\mu_0 J}{\pi}$$

$$J = \chi H = (\mu - 1) \frac{B_0}{\mu \mu_0}$$

$$B' = \frac{2}{\pi} \frac{\mu - 1}{\mu} B_0 \Rightarrow B = B_0 \left(1 + \frac{2}{\pi} \frac{\mu - 1}{\mu}\right)$$

3.1

9



$$H_1 = \frac{B(r)}{\mu_0 \mu_1}$$

$$H_2 = \frac{B(r)}{\mu_0 \mu_2}$$

$$\oint H dl = H_1 \pi r + H_2 \pi r = NI =$$

$$= \frac{B(r)}{\mu_0} \pi r \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right) = \frac{B(r)}{\mu_0} \frac{\mu_1 + \mu_2}{\mu_1 \mu_2} \pi r$$

$$B(r) = \frac{\mu_0 \mu_1 \mu_2 N I}{B(r) \pi r (\mu_1 + \mu_2)}$$

$$\Phi = N \int_{S_1} B dS = \frac{\mu_0 \mu_1 \mu_2 N^2 I}{\pi (\mu_1 + \mu_2)} \int_S \frac{dS}{r} =$$

$$= \frac{\mu_0 \mu_1 \mu_2 N^2 I}{\pi (\mu_1 + \mu_2)} \int_0^h dz \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\mu_0 \mu_1 \mu_2 N^2 I}{\pi (\mu_1 + \mu_2)} h \ln \frac{R_2}{R_1}$$

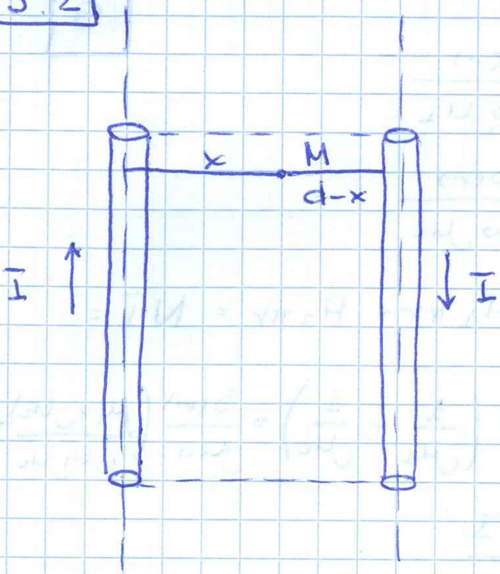
$$\Phi = LI \Rightarrow L = \frac{\mu_0 \mu_1 \mu_2 N^2 h}{\pi (\mu_1 + \mu_2)} \ln \frac{R_2}{R_1} =$$

$$= \frac{4\pi \cdot 10^{-7}\ \text{Н}/\text{А}^2 \cdot 400 \cdot 100 \cdot 400^2}{\pi (400 + 100)} 6 \cdot 10^{-2}\ \text{м} \ln \frac{10{,}8}{4} =$$

$$= 0{,}3\ \text{Гн}$$

Решение не согласуется с действительностью :С

$\boxed{9.2}$



$$B_1 = \frac{\mu_0 I}{2\pi x}$$

$$\Phi_1 = \int_S B_1 \, dS =$$

$$= \frac{\mu_0 I}{2\pi} \int_0^l dy \int_a^d \frac{dx}{x} =$$

$$= \frac{\mu_0 I l}{2\pi} \ln \frac{d}{a} = \Phi_2$$

$$\Phi = \Phi_1 + \Phi_2 = \frac{\mu_0 I l}{\pi} \ln \frac{d}{a}$$

$$\Phi = IL \Rightarrow L = \frac{\mu_0 l}{\pi} \ln \frac{d}{a} =$$

$$= \frac{4\pi \cdot 10^{-7}\,Н/А^2 \cdot 1\,м}{\pi} \ln \frac{100}{0.5} = 1{,}2 \cdot 10^{-6}\,Гн$$

$\boxed{9.3}$

$$\oint_L H \, dl = 2\pi r \frac{B}{\mu \mu_0} = I \qquad \mu = 1$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi = \int_S B \, dS = \frac{\mu_0 I}{2\pi} \int_0^l dz \int_{R_1}^{R_2} \frac{dr}{r} =$$

$$= \frac{\mu_0 I}{2\pi} l \cdot \ln \frac{R_2}{R_1} \Rightarrow L = \frac{\mu_0 l}{2\pi} \ln \frac{R_2}{R_1}$$

$$L = \frac{\mu_0 \ell}{2\pi} \ln \frac{R_2}{R_1} = \frac{4\pi \cdot 10^{-7}\, H/A^2 \cdot 1\text{м}}{2\pi} \ln \frac{100}{0.5} = 0{,}6\,\text{мкГн}$$

$\boxed{9.5}$ $$B_1(r) = \frac{\mu_0 I_1}{2\pi r} \quad \text{(провод)}$$

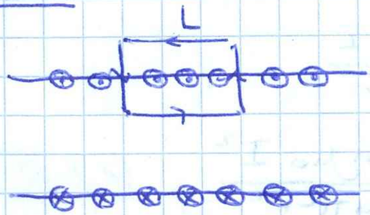
$$\Phi_2 = N \int_{S_1} B_1(r)\, dS = N \cdot \frac{\mu_0 I_1}{2\pi} \int_0^h dz \int_{R_1}^{R_2} \frac{dr}{r} =$$

$$= \frac{\mu_0 N I_1}{2\pi} h \ln \frac{R_2}{R_1}$$

$$L_{12} = \frac{\Phi_2}{I_1} = \frac{\mu_0 N h}{2\pi} \ln \frac{R_2}{R_1} = \frac{4\pi \cdot 10^{-7}\, H/A^2 \cdot 500 \cdot 6 \cdot 10^{-2}\,\text{м}}{2\pi} \ln \frac{10{,}8}{4}$$

$$= 6\,\text{мГн}$$

$\boxed{9.6}$



$$\oint_L H d\ell = \frac{B_1}{\mu\mu_0} \ell = n \cdot \ell \cdot I_1$$

$$\Rightarrow B_1 = \mu\mu_0 n I_1 \quad \text{(внутр. катушка)}$$

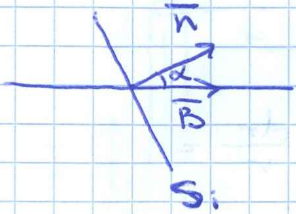
$$\Phi_2 = N \int_{S_1} B_1\, dS = \{\text{поток через (2)} = \text{поток через (1)}\} =$$

$$= N \mu\mu_0 n I_1 S$$

$$L_{12} = \frac{\Phi_2}{I_1} = \mu\mu_0 n N S = 4\pi \cdot 10^{-7}\, H/A^2 \cdot 10^3 \tfrac{1}{м}\, 20 \cdot 10 \cdot 10^{-4}\, м^2 =$$

$$= 25\,\text{мкГн}.$$

$\boxed{9.8}$ $B_1 = \mu\mu_0 n I_1$ (для соленоида)



$$\Phi_2 = N \int_{S_1} B_1 \, dS =$$

$$= N \cdot \mu\mu_0 n I_1 \cdot S \cdot \cos\alpha \quad (\mu = 1)$$

$$L_{12} = \frac{\Phi_2}{I_1} = \mu\mu_0 n N S \cos\alpha \quad (\mu = 1)$$

$\boxed{9.10}$

$$\oint_L H \, dl = H \cdot 2\pi r = I \Rightarrow H = \frac{I}{2\pi r}$$

$$r \in [R_1, R_2]$$

$$B = \mu\mu_0 H = \frac{\mu\mu_0 I}{2\pi r}$$

$$w_M = \frac{1}{2} BH = \frac{1}{2} \cdot \frac{\mu\mu_0 I^2}{4\pi^2 r^2}$$

$$W = \int_V w_M \, dV = \int_0^h dz \int_0^{2\pi} d\varphi \int_{R_1}^{R_2} r \, dr \left[\frac{1}{2} \frac{\mu\mu_0 I^2}{4\pi^2 r^2} \right] =$$

$$= h \cdot 2\pi \cdot \frac{1}{2} \mu\mu_0 I^2 \frac{1}{4\pi^2} \ln \frac{R_2}{R_1} =$$

$$= \frac{\mu\mu_0 I^2 h}{4\pi} \ln \frac{R_2}{R_1} = \Bigg| = 0{,}62*$$

$$= \frac{1000 \cdot 4\pi \cdot 10^{-7} \, H/A^2 \cdot 100^2 A^2 \cdot 0{,}2 m}{4\pi} \ln \frac{60}{3} =$$

$\boxed{9.11}$ $\oint H\,dl = H \cdot 2\pi r = NI \Rightarrow H = \dfrac{NI}{2\pi r}$

$$B = \mu\mu_0 H = \frac{\mu\mu_0 NI}{2\pi r}$$

$$w_m = \frac{1}{2}BH = \frac{1}{2}\frac{\mu\mu_0 N^2 I^2}{4\pi^2 r^2}$$

$$W = \int_V w_m\,dV = \int_0^h dz \int_0^{2\pi} d\varphi \int_{R_1}^{R_2} r\,dr \cdot \frac{1}{2}\frac{\mu\mu_0 N^2 I^2}{4\pi^2 r^2} =$$

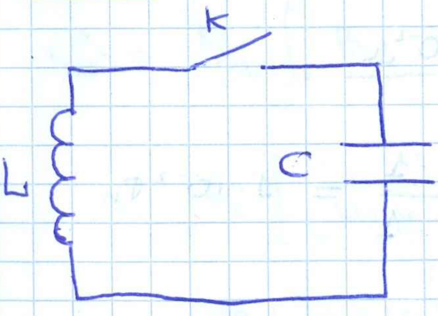
$$= h \cdot 2\pi \cdot \frac{1}{2}\frac{\mu\mu_0 N^2 I^2}{4\pi^2}\ln\frac{R_2}{R_1} =$$

$$= \frac{\mu\mu_0 N^2 I^2 h}{4\pi}\ln\frac{R_2}{R_1} =$$

$$= \frac{500 \cdot 4\pi \cdot 10^{-7}\,\frac{H}{A^2} \cdot 100^2 \cdot 10^2 A^2 \cdot 3 \cdot 10^{-2}\,м}{4\pi} \cdot \ln\frac{5{,}4}{2} =$$

$$= 0{,}15\ Дж$$

$\boxed{9.13}$



U_0

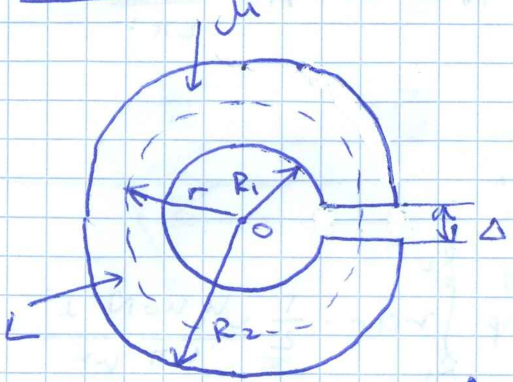
Вся энергия конденсатора должна перейти в энергию катушки.

$$\frac{CU_0^2}{2} = \frac{LI_{max}^2}{2}$$

$$I_{max} = U_0\sqrt{\frac{C}{L}} =$$

$$= 12\,В \cdot \sqrt{\frac{1 \cdot 10^{-6}\,Ф}{10 \cdot 10^{-3}\,Гн}} = 0.12\ A$$

$\boxed{9.7}$



$$\oint_L H\,dl = H_1(2\pi r - \Delta) + H_2\Delta =$$

$$= \frac{B(r)}{\mu\mu_0}(2\pi r - \Delta) + \frac{B(r)}{\mu_0}\Delta =$$

$$= NI$$

$$B(r) = \frac{\mu\mu_0 NI}{2\pi r - \Delta + \mu\Delta}$$

$$\Phi = N\int_{S_1} B(r)\,dS = N\cdot\mu\mu_0 NI\cdot\int_0^h dz\int_{R_1}^{R_2}\frac{dr}{2\pi r - \Delta + \mu\Delta} =$$

$$= \frac{\mu\mu_0 N^2 I h}{2\pi}\ln\frac{2\pi R_2 + (\mu-1)\Delta}{2\pi R_1 + (\mu-1)\Delta}$$

$$\Phi = LI \Rightarrow L = \frac{\mu\mu_0 N^2 h}{2\pi}\ln\frac{2\pi R_2 + (\mu-1)\Delta}{2\pi R_1 + (\mu-1)\Delta} =$$

$$= \frac{200\cdot 4\pi\cdot 10^{-7}\,\text{Н}/\text{А}^2\cdot 200^2\cdot 3\cdot 10^{-2}\,\text{м}}{2\pi}\cdot$$

$$\cdot\ln\frac{2\pi\cdot 54 + (200-1)\cdot 1}{2\pi\cdot 20 + (200-1)\cdot 1} = 1\cdot 10^{-2}\,\text{Гн}.$$

$\boxed{3.4}$

$$\begin{cases} \Phi = LI = \Phi_1 + \Phi_2 \\ \Phi_1 = L_1 I \\ \Phi_2 = L_2 I \end{cases} \Rightarrow L = L_1 + L_2$$

Пусть L_0 – индуктивность одного витка.

$$\begin{cases} L_1 = N_1 L_0 \\ L_2 = N_2 L_0 \\ L = N L_0 = (N_1 + N_2) L_0 \end{cases} \Rightarrow N = N_1 \left(1 + \frac{L_2}{L_1}\right)$$

$\boxed{3.1}$ (видимо, правильное решение)

$$\oint H_1 dl = H_1 \cdot \pi r = \frac{N}{2} I \Rightarrow B_1 = \mu_1 \mu_0 \frac{NI}{2\pi r}$$

аналогично для $B_2 = \mu_2 \mu_0 \frac{NI}{2\pi r}$

$$\Phi = N/2 \int_{S_1} B_1 dS + N/2 \int_{S_1} B_2 dS =$$

$$= N/2 \left[\mu_1 \mu_0 \frac{NI}{2\pi} \int_0^h dz \int_{R_1}^{R_2} \frac{dr}{r} + \mu_2 \mu_0 \frac{NI}{2\pi} \int_0^h dz \int_{R_1}^{R_2} \frac{dr}{r} \right] =$$

$$= \frac{\mu_0 (\mu_1 + \mu_2) N^2 I}{4\pi} h \ln \frac{R_2}{R_1}$$

$$\Phi = LI \Rightarrow L = \frac{\mu_0 (\mu_1 + \mu_2) N^2 h}{4\pi} \ln \frac{R_2}{R_1} = 0{,}47 \text{ Гн}$$

$\boxed{3.12}$ 1) $B_2 = \mu_0 n I \qquad H_2 = nI$

$$\Phi_2 = \int_S B_2 \, dS = n\ell \int_{S_i} B_2 \, dS = \mu_0 n^2 I \, \ell S_2$$

$$W_2 = \frac{\Phi_2 I}{2} = \frac{\mu_0 n^2 I^2 \ell}{2} S_2$$

2) $B_1 = \frac{\mu_0 I}{2\pi r} \qquad H_1 = \frac{I}{2\pi r}$

$$w_m = \frac{1}{2} B_1 H_1 = \frac{1}{2} \frac{\mu_0 I^2}{4\pi^2 r^2}$$

$$W_1 = \int_{V_0} w_m \, dV = \int_0^{\ell} dz \int_0^{2\pi} d\varphi \int_{R_1}^{R_2} \frac{r\,dr}{r^2} \frac{1}{2} \frac{\mu_0 I^2}{4\pi^2} =$$

$$= \frac{\mu_0 I^2}{4\pi^2} \cdot \frac{1}{2} 2\pi \ell \ln \frac{R_2}{R_1} =$$

$$= \frac{1}{2} \cdot \frac{\mu_0 I^2}{4\pi} \ell \ln \frac{S_2}{S_1}$$

3) $$\frac{W_2}{W_1} = \frac{4\pi S_2 n^2}{\ln S_2/S_1} \approx 2700$$

$\boxed{3.15}$ 1) $B = \mu_0 n I \qquad \Phi = N \int_{S_i} B \, dS = \mu_0 n N I S$

$$IL = \Phi \Rightarrow L = \mu_0 n N S = \mu_0 \frac{N^2}{\ell} S$$

$$L_1 = L_0 = \mu_0 \frac{N^2}{\ell_0} S_0 \Rightarrow L_2 = \mu_0 \frac{N^2}{\ell_0} \beta \frac{S_0}{\beta^2} = \frac{L_0}{\beta}$$

2) $B_1 = \mu_0 n I_1 \qquad n = \frac{N}{\ell_0}$

$$\Phi_2 = N \int\limits_{S_i} B_1 \, dS = N \mu_0 n I_1 S_2 = N \mu_0 n I_1 \frac{S_0}{\beta^2}$$

$$L_{12} = \frac{\Phi_2}{I_1} = \mu_0 n N \frac{S_0}{\beta^2} = \mu_0 \frac{N^2}{\ell_0} \frac{S_0}{\beta^2} = \frac{L_0}{\beta^2}$$

3) $W = \frac{1}{2} L I^2 = \frac{1}{2} L_1 I^2 + L_{12} I^2 + \frac{1}{2} L_2 I^2$

$$L = L_1 + 2L_{12} + L_2 =$$
$$= L_0 \left[1 + \frac{1}{\beta} + \frac{2}{\beta^2} \right]$$

$\boxed{9.14}$ Вдвинем соленоид (2) в соленоид (1) на Δx.

Тогда поток изменится на $\Delta \Phi$.

$$\begin{cases} \Delta \Phi = (B_2 - B_1) \cdot S \cdot \underbrace{n_1 \Delta x}_{\Delta N} \\ B_2 = \mu_0 n_1 I_1 + \mu_0 n_2 I_2 \text{ — внутри (2) и (1)} \\ B_1 = \mu_0 n_1 I_1 \text{ — внутри (2) вдали от (1)} \end{cases}$$

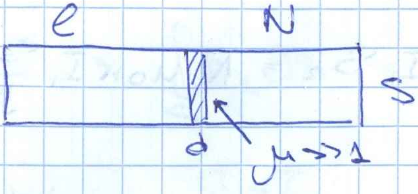
$$\Rightarrow \Delta \Phi = \mu_0 n_2 I_2 \cdot S \cdot n_1 \Delta x$$

$$F = \frac{\Delta A}{\Delta x} = \frac{\Delta \Phi I_1}{\Delta x} = \mu_0 n_1 n_2 I_1 I_2 S = \mu_0 n_1 n_2 I_1 I_2 \pi r^2 =$$

$$= 4\pi \cdot 10^{-7} \, Н/А^2 \cdot 10^3 \frac{1}{м} \cdot 10^3 \frac{1}{м} \cdot 2 А \cdot 5 А \cdot \pi \cdot 1^2 \cdot 10^{-4} \, м^2 =$$

$$= 40 \ Н$$

9.9



$n = \frac{N}{l} \qquad d \ll l$

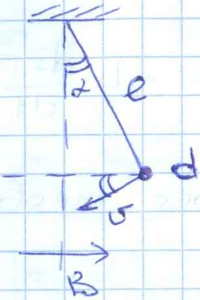
$$\begin{cases} B_1 = \mu_0 n J \\ B_2 = \mu \mu_0 n J \end{cases} \Rightarrow \begin{cases} \Phi_1 = \mu_0 n J \cdot S \, \frac{l-d}{l} N \\ \Phi_2 = \mu \mu_0 n J \cdot S \cdot \frac{d}{l} N \end{cases}$$

$$L = \frac{\Phi_1 + \Phi_2}{J} = \frac{\mu_0 n \cancel{J} S N}{\cancel{J} l} (l - d + \mu d) =$$

$$= \mu_0 S \cdot \frac{N^2}{l^2} (l - d + \mu d) \approx$$

$$\approx \frac{\mu_0 S N^2}{l}$$

$\boxed{10.1}$ $\boxed{10}$



Известно, что уравнение колебаний матем. маят. выглядит следующ. образом.

$$\ddot{\alpha} + \hat{\omega}^2 \alpha = 0, \quad \hat{\omega} = \sqrt{\frac{g}{l}}$$

Решение данного дифф. ур.:

$$\alpha = \alpha_0 \cos(\hat{\omega} t)$$

$$\Delta U(t) = d \cdot [\bar{B} \times \bar{v}] = d B v(t) \sin\alpha(t) =$$

$$= \left\{ \begin{array}{l} v(t) = \omega(t) l = \frac{d\alpha(t)}{dt} l = \frac{d\alpha}{dt} l \\ \sin\alpha(t) \approx \alpha(t), \text{ т.к. } \alpha(t) \leq \alpha_0 \end{array} \right\} =$$

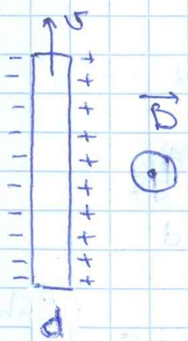
$$= dB \alpha_0 \cos\hat{\omega} t \cdot \frac{d\alpha}{dt} l =$$

$$= -d \cdot B \cdot \alpha_0 \cos\hat{\omega} t \cdot l \alpha_0 \sin\hat{\omega} t \cdot \hat{\omega} =$$

$$= -\frac{1}{2} B \alpha_0^2 d l \hat{\omega} \sin 2\hat{\omega} t =$$

$$= -\frac{1}{2} \alpha_0^2 B \sqrt{gl}\, d \cdot \sin 2\sqrt{\frac{g}{l}}\, t$$

$\boxed{10.2}$



d — толщина пластины.

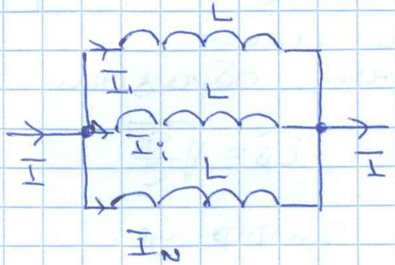
$$\Delta U = d[\bar{B} \times \bar{v}] = B v d$$

$$\left\{ \begin{array}{l} E = \frac{\Delta U}{d} = B v \\ E = \frac{\sigma}{\varepsilon_0} \end{array} \right. \Rightarrow \sigma = \varepsilon_0 B v$$

$$\sigma = \varepsilon_0 B v = 8{,}85 \cdot 10^{-12} \frac{\text{Кл}^2}{\text{Н} \cdot \text{м}^2} \cdot 10 \cdot 10^{-3} \text{Тл} \cdot 10 \text{ м/с} =$$

$$= 8{,}85 \cdot 10^{-13} \frac{\text{Кл}}{\text{м}^2}$$

10.3



$$\Phi_i = L \cdot I_i$$

$$\varepsilon_i = U_i = -\frac{d\Phi_i}{dt} = -L\frac{dI_i}{dt}$$

В силу паралл. соед. проводн.

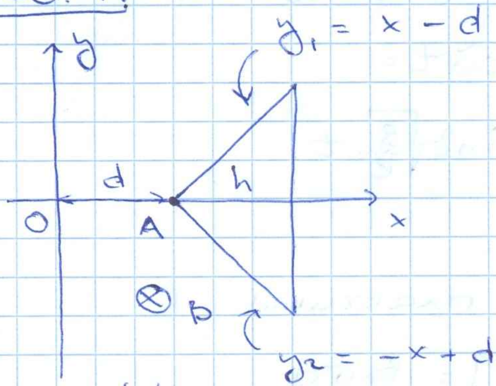
$$U_1 = \ldots = U_N = U_0$$

$\Rightarrow$ $I_1 = \ldots = I_N = I_0$, с другой стороны

$$\begin{cases} U_0 = -\dfrac{d\Phi}{dt} = -L_{эф}\dfrac{dI}{dt} = -L_{эф} N \dfrac{dI_0}{dt} \\ U_0 = -L\dfrac{dI_0}{dt} \end{cases} \Rightarrow L_{эф} = \frac{L}{N}.$$

10.4

$$I_1(t) = I_0 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$$



$$B = \frac{\mu_0 I_1(t)}{2\pi x}$$

$$\Phi = \int_S B\,dS =$$

$$= \int_d^{d+h} dx \int_{-x+d}^{x-d} \frac{\mu_0 I_1(t)}{2\pi x} dy = \frac{\mu_0 I_1(t)}{2\pi} \int_d^{d+h} \frac{dx}{x} \int_{-x+d}^{x-d} dy =$$

$$= \frac{\mu_0 I_1(t)}{\pi} \int_d^{d+h} \left(1 - \frac{d}{x}\right) dx = \frac{\mu_0 I_1(t)}{\pi} \Big[x - d\ln x\Big]_d^{d+h}$$

$$= \frac{\mu_0 I_1(t)}{\pi} \left[h - d \ln \frac{d+h}{d} \right]$$

$$\mathcal{E}_i = - \frac{\partial \Phi}{\partial t} = \frac{\mu_0}{\pi} \left[d \ln\left(1 + \frac{h}{d}\right) - h \right] \frac{\partial I_1}{\partial t} =$$

$$= \frac{\mu_0}{\pi} \left[d \ln\left(1 + \frac{h}{d}\right) - h \right] \cdot \left[I_0 \exp\left(-\frac{t^2}{\tau^2}\right) \left(-\frac{2t}{\tau^2}\right) \right] =$$

$$= \frac{\mu_0 I_0}{\pi \tau^2} \left(h - d \ln\left(1 + \frac{h}{d}\right)\right) \left[\exp\left(-\frac{t^2}{\tau^2}\right) 2t \right]$$

$$f(t) = \exp\left(-\frac{t^2}{\tau^2}\right) t$$

$$f'(t) = \exp\left(-\frac{t^2}{\tau^2}\right) + t \exp\left(-\frac{t^2}{\tau^2}\right) \cdot \left(-\frac{2t}{\tau^2}\right) =$$

$$= \exp\left(-\frac{t^2}{\tau^2}\right) \left[1 - \frac{2t^2}{\tau^2} \right] = 0$$

$- \quad -\frac{\tau}{\sqrt{2}} \quad + \quad \frac{\tau}{\sqrt{2}} \quad - \quad t \quad , \quad t > 0$

$$I_2(t) = \frac{\mathcal{E}_i}{R} = \frac{2 \mu_0 I_0}{\tau^2 \pi R} \left(h - d \ln\left(1 + \frac{h}{d}\right)\right) \left[\exp\left(-\frac{t^2}{\tau^2}\right) \cdot t \right]$$

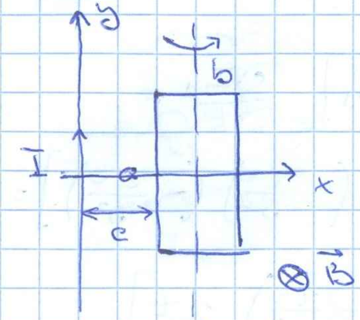
$$I_{max} = I_2\left(\frac{\tau}{\sqrt{2}}\right) = \frac{2 \mu_0 I_0}{\tau^2 \pi R} \left(h - d \ln\left(1 + \frac{h}{d}\right)\right) \frac{\tau}{\sqrt{2}} \cdot (\sqrt{e})^{-1} =$$

$$= \sqrt{\frac{2}{e}} \cdot \frac{\mu_0 I_0}{\pi R \tau} \left(h - d \cdot \ln\left(1 + \frac{h}{d}\right)\right) =$$

$$= \sqrt{\frac{2}{e}} \cdot \frac{4\pi \cdot 10^{-7}\, H/A^2 \cdot 100\, A}{\pi \cdot 0{,}70\, \Omega \cdot 10^{-3}\, c} \cdot \left(0{,}17\, m - 0{,}1\, m \cdot \right.$$

$$\left. \cdot \ln\left(1 + \frac{0{,}17}{0{,}1}\right)\right) = 5{,}3\, mA$$

$\boxed{10.5}$



$$I_i = \frac{\mathcal{E}_i}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = \frac{dq}{dt}$$

$$q(t_2) - \underbrace{q(t_1)}_{=0} = \frac{1}{R}\left(\Phi(t_1) - \Phi(t_2)\right)$$

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right)$$

$$\Phi_н = \int\limits_{-a/2}^{a/2} dy \int\limits_{c}^{c+b} \underbrace{\frac{\mu_0 I}{2\pi x}}_{B(x)} dx = \frac{a\mu_0 I}{2\pi}\ln\left(1+\frac{b}{c}\right)$$

Легко видеть, что абсолютное значение потока через рамку не поменялось при повороте на 180°.

Однако, нормаль поменялась, она тоже повернулась на 180°.

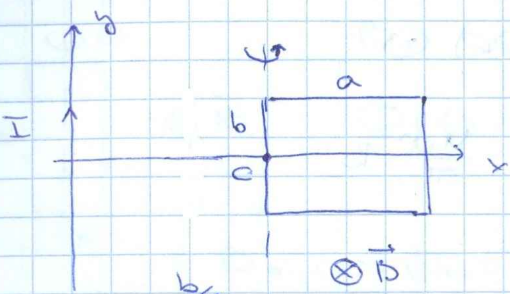
Т.о. $\Phi_к = -\Phi_н$

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right) = \frac{2}{R}\Phi_н = \frac{\mu_0 I a}{\pi R}\ln\left(1+\frac{b}{c}\right) =$$

$$= \frac{4\pi\cdot 10^{-7}\,Н/А^2 \cdot 10\,А \cdot 20\cdot 10^{-2}\,м}{\pi\cdot 0,7\,Ом}\cdot\ln\left(1+\frac{17}{10}\right) =$$

$$= 1\,мкКл.$$

$\boxed{10.6}$



$$l = 2(a+b)$$

$$R = \rho \frac{l}{S} \qquad B(x) = \frac{\mu_0 I}{2\pi x}$$

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right) \quad (10.5)$$

$$\Phi_н = \int\limits_{-b/2}^{b/2} dy \int\limits_{c}^{c+a} \frac{\mu_0 I}{2\pi x}\, dx = \frac{b \mu_0 I}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$

$$\Phi_к = \int\limits_{-b/2}^{b/2} dy \int\limits_{c-a}^{c} \frac{\mu_0 I}{2\pi x}\, dx = \frac{b \mu_0 I}{2\pi} \ln\left(\frac{c}{c+a}\right)$$

По тем же соображениям, что и в 10.5 знаки у $\Phi_н$ и $\Phi_к$ противоположны.

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right) = \frac{1}{R} \cdot \frac{b \mu_0 I}{2\pi}\left[\ln \frac{a+c}{c} + \ln \frac{c}{c-a}\right] =$$

$$= \frac{\mu_0 I S b}{4\pi(a+b)\rho} \ln \frac{a+c}{c-a} = \frac{\mu_0 I S b \lambda}{4\pi(a+b)} \ln \frac{c+a}{c-a} =$$

$$= \frac{\cancel{4\pi} \cdot 10^{-7}\, Н/А^2 \cdot 10 А \cdot 1 \cdot 10^{-6} м^2 \cdot 11 \cdot 10^{-2} м \cdot 6 \cdot 10^{7} \frac{Ом}{м}}{\cancel{4\pi}\,(19 + 11) \cdot 10^{-2} м} \cdot$$

$$\cdot \ln \frac{21+19}{21-19} = 6{,}6 \text{ мкКл}$$

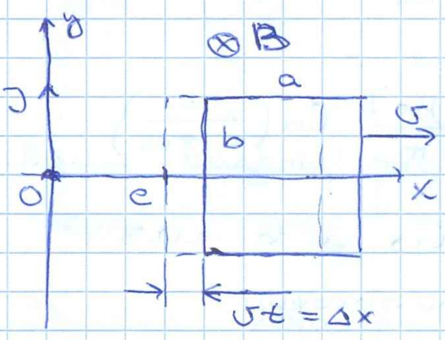
$\boxed{10.7}$ $\quad 4a = 2\pi r \quad \Rightarrow \quad r = \frac{2a}{\pi}$

iz 10.5: $\quad Q = \frac{1}{R}(\Phi_u - \Phi_k) =$

$\Phi_u = B \cdot a^2 \qquad \Phi_k = B \cdot \pi r^2 = \frac{4a^2}{\pi} B$

$= \frac{Ba^2}{R}\left(1 - \frac{4}{\pi}\right)$

$\boxed{10.8}$



$$\Phi(t) = \int_{-b/2}^{b/2} dy \int_{c+\Delta x}^{c+\Delta x+a} \frac{\mu_0 I}{2\pi x}\, dx =$$

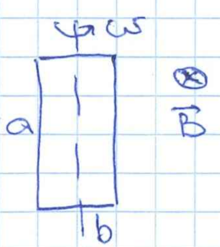
$$= \frac{b\mu_0 I}{2\pi} \ln\left(1 + \frac{a}{c+\Delta x}\right) =$$

$$= \frac{b\mu_0 I}{2\pi} \ln\left(1 + \frac{a}{c+vt}\right)$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = +\frac{b\mu_0 I}{2\pi} \cdot \frac{c+vt}{c+vt+a} \cdot \frac{av}{(c+vt)^2} =$$

$$= \frac{\mu_0 I abv}{(c+vt+a)(c+vt)} \cdot \frac{1}{2\pi}$$

$\boxed{10.9}$

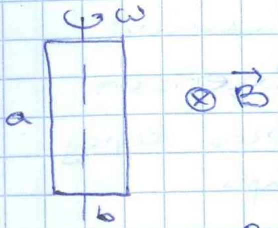


$$\Phi(t) = \int_{S(t)} B\, dS = BS\cos\alpha(t) =$$

$$= Bab\cos\omega t$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = Bab\omega\sin\omega t$$

$\boxed{10.10}$



$$B = B_0 \cos(\Omega t)$$

$$\Phi(t) = \int_{S(t)} B(t)\, dS = B(t)\, S \cos\alpha(t) =$$

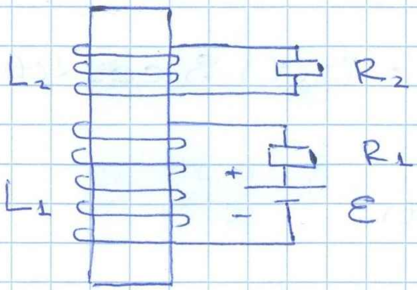
$$= B_0 \cos(\Omega t)\, ab \cos(\omega t)$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = -B_0 ab \left[-\Omega \sin\Omega t \cos\omega t - \right.$$

$$\left. - \omega \sin\omega t \cos\Omega t\right] =$$

$$= B_0\, ab \left[\Omega \sin\Omega t \cos\omega t + \omega \sin\omega t \cos\Omega t\right]$$

10.13



Коэффициент взаимной индукции L_{12} показывает как изменяется поток через вторую катушку при изменении тока в первой.

$$\mathcal{E}_{21} = -L_{12}\frac{dI_1}{dt}$$

с другой стороны $\mathcal{E}_{21} = I_2 R_2 = \frac{dQ}{dt} R_2$

$$-L_{12}\int_{t_1}^{t_2}\frac{dI_1}{\cancel{dt}} = R_2\int_{t_1}^{t_2}\frac{dQ}{\cancel{dt}} \xRightarrow{\text{интегрир.}} Q = -L_{12} I_1 \frac{1}{R_2}$$

$Q = -\mathcal{E}\frac{L_{12}}{R_1 R_2}$, где $L_{12} = \sqrt{L_1 L_2}$ (пример 9.3)

10.12 Аналогично соображениям 10.13

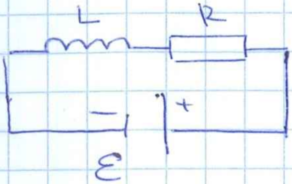
$$\mathcal{E}_{21} = -L_{12}\frac{dI_1}{dt}$$

с другой стороны $\mathcal{E}_{21} = I_2 R_2 = \frac{dQ}{dt} R_2$

$$-L_{12}\int_{t_1}^{t_2}\frac{dI_1}{\cancel{dt}} = R_2\int_{t_1}^{t_2}\frac{dQ}{\cancel{dt}} \xRightarrow{\text{интегр.}} Q = L_{12} I_1 \frac{1}{R_2}$$

$$Q = L_{12} I_1 \frac{1}{R_2} = \frac{\mu_0 n S I}{R}$$ (см. 9.6)

$\boxed{10.14}$ (исходное условие)



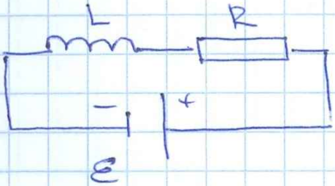
$$I(t) = I_0 e^{-t/\tau}$$

$$\mathcal{E}_i = -L\frac{dI}{dt} = \frac{LI_0}{\tau}e^{-t/\tau}$$

$$\mathcal{E}(t) + \mathcal{E}_i = I(t)R$$

$$\mathcal{E}(t) = RI_0 e^{-t/\tau} - \frac{LI_0}{\tau}e^{-t/\tau} = I_0 e^{-t/\tau}\left(R + \frac{L}{\tau}\right)$$

$\boxed{10.14}$ (немного другое условие)



$$I(t) = I_0\left(1 - e^{-t/\tau}\right)$$

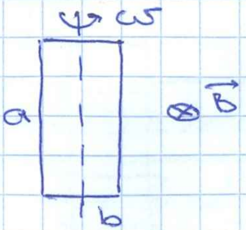
$$\mathcal{E}_i = -L\frac{dI}{dt} = -\frac{LI_0}{\tau}e^{-t/\tau}$$

$$\mathcal{E}(t) + \mathcal{E}_i = I(t)R$$

$$\mathcal{E}(t) = RI_0 - Re^{-t/\tau}I_0 + \frac{LI_0}{\tau}e^{-t/\tau} =$$

$$= RI_0 - I_0 e^{-t/\tau}\left(R - \frac{L}{\tau}\right)$$

$\boxed{10.11}$



$$\omega(t) = \omega_0\left(1 - e^{-\delta t}\right) = \frac{d\alpha(t)}{dt}$$

$$\alpha(t) = \omega_0 t + \omega_0 \frac{e^{-\delta t}}{\delta} = \omega_0\left(t + \frac{e^{-\delta t}}{\delta}\right)$$

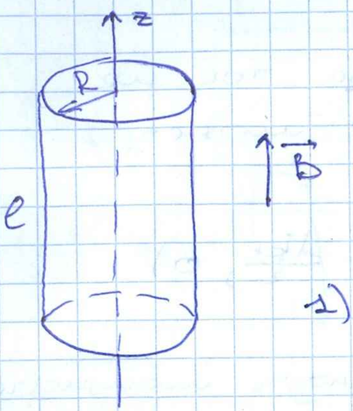
$$\Phi(t) = \int_{S(t)} B\,dS = BS\cos\alpha(t) = Bab\cos\left[\omega_0\left(t + \frac{e^{-\delta t}}{\delta}\right)\right]$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = Bab\sin\left[\omega_0\left(t + \frac{e^{-\delta t}}{\delta}\right)\right] \cdot \left[\omega_0\left(1 - e^{-\delta t}\right)\right] = \mathcal{E}_i(t)$$

$$\omega_0\left(1 - e^{-\delta t_0}\right) = \omega_0/2 \Rightarrow t_0 = \frac{\ln 2}{\delta}$$

$$\tilde{\mathcal{E}}_i = \mathcal{E}_i(t_0) = Bab\sin\left[\omega_0\left(\frac{\ln 2}{\delta} - \frac{1}{2\delta}\right)\right]\omega_0 \cdot \frac{1}{2} = \frac{1}{2}abB\omega_0\sin\left[\omega_0\left(\frac{\ln 2}{\delta} - \frac{1}{2\delta}\right)\right]$$

$\boxed{11.1}$ $\boxed{11}$



$$B(t) = \begin{cases} kt, & 0 \le r \le R_1 \\ 0, & r > R_1 \quad (R_1 > R) \end{cases}$$

В цилиндр. координатах (r, φ, z):

$$\vec{B} = (0, 0, kt)$$

1) $\operatorname{rot} \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$:

В силу симметрии: $\dfrac{\partial E}{\partial \varphi} = \dfrac{\partial E}{\partial z} = 0$

$$\begin{cases} \dfrac{1}{r}\dfrac{\partial E_z}{\partial \varphi} - \dfrac{\partial E_\varphi}{\partial z} = 0 \\ \dfrac{\partial E_r}{\partial z} - \dfrac{\partial E_z}{\partial r} = 0 \\ \dfrac{1}{r}\dfrac{\partial (r E_\varphi)}{\partial r} - \dfrac{\partial E_r}{\partial \varphi} = -k \end{cases} \Longrightarrow \begin{cases} 0 = 0 \\ \dfrac{\partial E_z}{\partial r} = 0 \\ \dfrac{1}{r}\dfrac{\partial (r E_\varphi)}{\partial r} = -k \end{cases}$$

$$\Rightarrow E_z = C_1 \; ; \; E_\varphi = -\frac{kr}{2} + \frac{C_2}{r}$$

В силу ограниченности E при $r = 0 \Rightarrow C_2 = 0$

2) $\operatorname{div} D = \rho \Rightarrow \operatorname{div} \varepsilon_0 E = 0 \quad (\rho = 0)$

$$\frac{1}{r}\frac{\partial (r E_r)}{\partial r} + \frac{1}{r}\frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z} = 0$$

В силу симметрии, имеем: $\left(\dfrac{\partial E}{\partial \varphi} = \dfrac{\partial E}{\partial z} = 0\right)$

$$\frac{\partial (r E_r)}{\partial r} = 0 \Rightarrow E_r = \frac{C_3}{r}$$

В силу ограниченности E при $r = 0 \Rightarrow C_3 = 0$

т.о. $\vec{E} = \left(0, -\dfrac{kr}{2}, C_1\right)$

$\vec{j} = \lambda\vec{E} = (0, -\frac{\lambda k r}{2}, \lambda C_2)$

В силу ограниченности кольца ток не является замкнутым $\Rightarrow$ не может поддерживаться $\Rightarrow C_1 = 0$

$\vec{E} = (0, -\frac{kr}{2}, 0) \quad \vec{j} = (0, -\frac{\lambda k r}{2}, 0)$

3) $\mathcal{D} = \frac{j^2}{\lambda} = \vec{j}\vec{E} = \frac{\lambda k^2 r^2}{4}$ (плотность мощности)

$$N = \int\limits_V \mathcal{D} dV = \int\limits_0^{2\pi} d\varphi \int\limits_0^R r dr \int\limits_0^l \mathcal{D} dz = \int\limits_0^{2\pi} d\varphi \int\limits_0^l dz \int\limits_0^R \frac{\lambda k^2 r^3}{4} dr =$$

$$= 2\pi \cdot l \cdot \frac{\lambda k^2 R^4}{16} = \frac{\pi}{8} \lambda k^2 R^4 l =$$

$$= \frac{\pi}{8} \cdot 6 \cdot 10^7 \frac{Ом}{м} \, 10^2 \frac{Тл^2}{с^2} \, (10^{-2})^4 м^4 \cdot 4 \cdot 10^{-2} м =$$

$$= 0{,}94 \, Вт$$

$\boxed{11.3}$ $j(r,t) = kre^{-t/\tau}$ и линии тока — окружности с центром на Oz

$\overline{E} = \frac{\overline{j}}{\lambda} = \frac{kr}{\lambda} e^{-t/\tau}$

$\mathrm{rot}\,\overline{E} = -\frac{\partial \overline{B}}{\partial t} \Rightarrow$

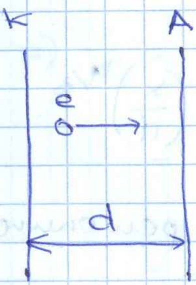
$$\mathrm{rot}\,\overline{E} = \begin{vmatrix} 1/r\,\overline{e}_r & \overline{e}_\varphi & 1/r\,\overline{e}_z \\ \partial/\partial r & \partial/\partial\varphi & \partial/\partial z \\ 0 & \frac{kr}{\lambda} e^{-t/\tau} & 0 \end{vmatrix} =$$

$$= \bar{e}_r \cdot \frac{1}{r}\underbrace{\left[-\frac{\partial}{\partial z}\left(\frac{kr^2}{\lambda}e^{-t/\tau}\right)\right]}_{=0} + \bar{e}_z \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(\frac{kr^2}{\lambda}e^{-t/\tau}\right) =$$

$$= \bar{e}_z \cdot \frac{1}{r}\cdot\frac{2kr}{\lambda}e^{-t/\tau} = \frac{2k}{\lambda}e^{-t/\tau}\bar{e}_z$$

$$\Rightarrow \bar{B} = \bar{e}_z \int_0^t \frac{2k}{\lambda}e^{-\tilde{t}/\tau}\,d\tilde{t} = \frac{2k\tau}{\lambda}\left(e^{-t/\tau}-1\right)\bar{e}_z$$

$\boxed{11.4}$ $e = -q, \quad q > 0$



1) Выведем распределение потенциала между пластинами (см. пример 11.2)

уравнение Пуассона: $\underbrace{\frac{d^2\varphi}{dx^2}}_{\Delta\varphi} = -\frac{\rho(x)}{\varepsilon_0}$

$\varphi = 0$ $\quad\varphi = U_0$ $\quad\rho(x) = -qn(x)$, где $n(x)$ – концентрация электронов

$$j = \rho(x)\bar{v} = -qn(x)\bar{v} \Rightarrow n(x) = \frac{j}{qv}; \quad j = \text{const}$$

ЗСЭ: $\underbrace{\frac{mv_0^2}{2}}_{=0\ (v_0=0)} - \underbrace{q\varphi(0)}_{=0} = \frac{mv^2}{2} - q\varphi(x) \Rightarrow$

$$\Rightarrow v = \sqrt{\frac{2q\varphi}{m}}$$

$$\frac{d^2\varphi}{dx^2} = \frac{j}{\varepsilon_0}\sqrt{\frac{m}{2q}}\,\varphi^{-1/2} = \alpha\varphi^{-1/2}$$

$$2\cdot\frac{d^2\varphi}{dx^2}\cdot\frac{d\varphi}{dx} = 2\varphi''\cdot\varphi' = \frac{d}{dx}\left(\varphi'^2\right)$$

$$\frac{d}{dx}\left[\left(\frac{d\varphi}{dx}\right)^2\right] = 2\cdot\alpha\varphi^{-1/2}\frac{d\varphi}{dx}$$

$$\left(\frac{d\varphi}{dx}\right)^2 = 4\alpha\varphi^{1/2} + C \qquad \underline{E(0)=0\ ;\ \left.\frac{d\varphi}{dx}\right|_{x=0} = 0}$$

$$\Downarrow$$

$$C = 0$$

$$\frac{d\varphi}{dx} = 2\sqrt{\alpha}\,\varphi^{1/4} \Rightarrow \varphi(x) = \left(\frac{3\sqrt{\alpha}}{2}x\right)^{4/3} + C_1$$

$$\varphi(0) = 0 \qquad \Rightarrow$$

$$\Rightarrow \varphi(x) = \left(\frac{3\sqrt{\alpha}}{2}x\right)^{4/3}$$

$$\varphi(d) = U_0 \qquad \Rightarrow \varphi(x) = U_0\left(\frac{x}{d}\right)^{4/3}$$

2) теперь приступим непосредственно к решению задачи

$$\Delta\varphi = \frac{d^2\varphi}{dx^2} = -\frac{\rho(x)}{\varepsilon_0} \quad \text{(ур-ие Пуассона)} \Rightarrow$$

$$\Rightarrow \rho(x) = -\varepsilon_0\frac{d^2\varphi}{dx^2} = -\varepsilon_0\frac{d}{dx}\left(\frac{4}{3d}\cdot U_0\left(\frac{x}{d}\right)^{1/3}\right) =$$

$$= \frac{-4U_0\varepsilon_0}{3d}\frac{d}{dx}\left(\left(\frac{x}{d}\right)^{1/3}\right) = -\frac{4}{9}\varepsilon_0 U_0\left(\frac{d}{x}\right)^{2/3}\frac{1}{d^2}$$

$$\rho\left(\frac{d}{2}\right) = -\frac{4}{9}\varepsilon_0 U_0\frac{1}{d^2}\,2^{2/3} = -\frac{4}{9}\cdot 8{,}85\cdot 10^{-12}\frac{\text{Кл}^2}{\text{Н}\cdot\text{м}^2}\cdot 100\,\text{В}\cdot$$

$$\cdot\frac{1}{5^2\cdot 10^{-6}\,\text{м}^2}\cdot 2^{2/3} = -25\cdot 10^{-6}\frac{\text{Кл}}{\text{м}^3}$$

$\boxed{11.5}$ из 11.4 имеем: $q = -e,\ q > 0$

$$\begin{cases} j = \rho(x)\,v(x) \\ v(x) = \sqrt{\dfrac{2q\varphi(x)}{m}} \\ \varphi(x) = U_0\left(\dfrac{x}{d}\right)^{4/3} \\ \rho(x) = -\dfrac{4}{9}\varepsilon_0 U_0 \dfrac{1}{x^{2/3} d^{4/3}} \end{cases} \Rightarrow$$

$$\Rightarrow j = -\frac{4}{9}\varepsilon_0 U_0 \frac{1}{x^{2/3} d^{4/3}} \cdot \sqrt{\frac{2qU_0}{m}} \cdot \frac{x^{2/3}}{d^{2/3}} =$$

$$= -\frac{4}{9}\varepsilon_0 \sqrt{\frac{2qU_0}{m}}\,\frac{U_0}{d^2} =$$

$$= -\frac{4}{9} \cdot 8{,}85 \cdot 10^{-12}\,\frac{\text{Кл}^2}{\text{Н}\cdot\text{м}^2} \sqrt{\frac{2 \cdot 1{,}6 \cdot 10^{-19}\,\text{Кл} \cdot 200\,\text{В}}{9{,}1 \cdot 10^{-31}\,\text{кг}}}\ \frac{200\,\text{В}}{5^2 \cdot 10^{-6}\,\text{м}^2} =$$

$$= 264\,\frac{\text{А}}{\text{м}^2}$$

$\boxed{11.6}$ из 11.4 имеем: $q = -e,\ q > 0$

$$\begin{cases} v(x) = \sqrt{\dfrac{2q\varphi(x)}{m}} \\ \varphi(x) = U_0\left(\dfrac{x}{d}\right)^{4/3} \end{cases} \Rightarrow v(x) = \sqrt{\frac{2qU_0}{m}} \cdot \left(\frac{x}{d}\right)^{2/3}$$

$$v(d/2) = \sqrt{\frac{2qU_0}{m}} \cdot 2^{2/3} = 2^{2/3}\sqrt{\frac{2 \cdot 1{,}6 \cdot 10^{-19}\,\text{Кл} \cdot 100\,\text{В}}{9{,}1 \cdot 10^{-31}\,\text{кг}}} =$$

$$= 3{,}7 \cdot 10^6\,\text{м/с}.$$

$\boxed{11.7}$ Из 11.6 имеем $q=-e, q>0$

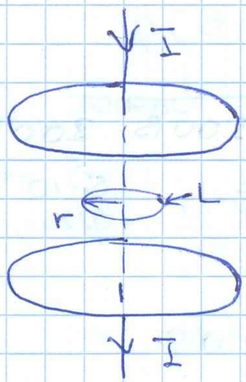
$$\upsilon(x)=\sqrt{\frac{2qU_0}{m}}\left(\frac{x}{d}\right)^{2/3}=\frac{dx}{dt}$$

$$t=\int_0^d\sqrt{\frac{m}{2qU_0}}\left(\frac{d}{x}\right)^{2/3}dx=3d^{2/3}\sqrt{\frac{m}{2qU_0}}\cdot d^{1/3}=$$

$$=3d\sqrt{\frac{m}{2qU_0}}=3\cdot 10\cdot 10^{-3}\text{м}\sqrt{\frac{9{,}1\cdot 10^{-31}\text{кг}}{2\cdot 1{,}6\cdot 10^{-19}\text{Кл}\cdot 100\text{В}}}=$$

$$=5\cdot 10^{-9}\text{с}$$

$\boxed{11.8}$



$U=U_0\cos\omega t$

1) $$\oint_L \bar{H}_0 d\bar{l}=2\pi r\cdot H=I_{см}$$

2) $$I_{см}=\int_S\frac{\partial\bar{D}}{\partial t}d\bar{S}=$$

$$=\frac{\partial}{\partial t}\Big(\underbrace{\varepsilon_0\frac{U}{d}}_{\varepsilon_0 E}\Big)\cdot S=$$

$$=\frac{\partial}{\partial t}\left(\varepsilon_0\frac{U_0\cos\omega t}{d}\right)\pi r^2=$$

$$=-\frac{U_0\varepsilon_0\omega\sin\omega t}{d}\pi r^2$$

$$H=\frac{I_{см}}{2\pi r}=-\frac{\varepsilon_0 U_0\omega\sin\omega t}{2d}r\Rightarrow$$

$$\Rightarrow H_0=\frac{\varepsilon_0 U_0\omega r}{2d};\quad B_0=\frac{\mu_0\varepsilon_0 U_0\omega r}{2d}$$

$$H_0 = \frac{\varepsilon_0 U_0 \omega r}{2d} = \frac{8{,}85\cdot 10^{-12}\,\frac{Кл^2}{Н\cdot м^2}\cdot 300В\cdot 3\cdot 10^{-6}с\cdot 1\cancel{см}}{2\cdot 1\cancel{см}} = $$

$$= 4\cdot 10^{-3}\,\frac{А}{м}$$

$$B_0 = \mu_0 H_0 = 4\pi\cdot 10^{-7}\,\frac{Н}{А^2}\cdot 4\cdot 10^{-3}\,\frac{А}{м} = 5\cdot 10^{-9}\,Тл$$

Аналогично выкладкам выше можно получить:

$$H_1 = \frac{\varepsilon\varepsilon_0 U_0 \omega r}{2d} = 4\cdot 10^{-2}\,\frac{А}{м}$$

$$B_1 = \frac{\mu\mu_0\varepsilon\varepsilon_0 U_0\omega r}{2d} = 5\cdot 10^{-6}\,Тл$$

$\boxed{11.9}$ Пусть I – сила тока в искре, тогда

$$\oint H\,dl = H\cdot 2\pi r = I + I_{см}$$

Пусть R – радиус обкладки конденсатора, тогда

$$I_{см} = \frac{\partial}{\partial t}\int_S \bar{D}\,d\bar{S} = \frac{\partial}{\partial t}(\pi r^2\cdot D) = \left\{D = \varepsilon_0 E = \varepsilon_0\,\frac{\sigma}{\varepsilon_0} = \sigma\right\} =$$

$$= \pi r^2\,\frac{\partial}{\partial t}\sigma = \frac{r^2}{R^2}\frac{\partial}{\partial t}Q = \frac{r^2}{R^2}I$$

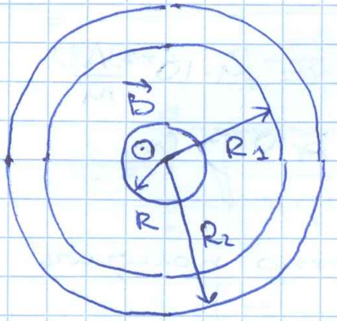
$$H = \frac{I + I_{см}}{2\pi r} = \frac{\left(1 - \frac{r^2}{R^2}\right)I}{2\pi r} \quad (*)$$

В силу симметрии ~~магнитные~~ силовые линии будут коаксиальными окружностями с осью конденсат.

Знак минус в (*) связан с тем, что электрическое поле убывает.

(и чё дальше ?!)

$\boxed{11.2}$



$$B(t) = \begin{cases} B_0 \sin\omega t, & 0 \le r \le R \\ 0, & r > R \end{cases}$$

В цилиндрических коорд. (r, φ, z):

$$\vec{B} = (0, 0, B_0 \sin\omega t)$$

1) $\operatorname{rot} \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

Внутри: в силу симметрии $\dfrac{\partial E}{\partial \varphi} = \dfrac{\partial E}{\partial z} = 0$. (*)

$$\begin{cases} \dfrac{1}{r}\dfrac{\partial E_z}{\partial \varphi} - \dfrac{\partial E_\varphi}{\partial z} = 0 \\ \dfrac{\partial E_r}{\partial z} - \dfrac{\partial E_z}{\partial r} = 0 \\ \dfrac{1}{r}\dfrac{\partial (r E_\varphi)}{\partial r} - \dfrac{\partial E_r}{\partial \varphi} = -\omega B_0 \cos\omega t \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{\partial E_z}{\partial r} = 0 \\ \dfrac{\partial (r E_\varphi)}{\partial r} = -\omega r B_0 \cos\omega t \end{cases} \Rightarrow$$

$$\Rightarrow \quad E_z = C_1 \quad ; \quad E_\varphi = -\frac{\omega r B_0 \cos\omega t}{2} + C_2$$

В силу ограниченности E при $r = 0 \Rightarrow C_2 = 0$

Снаружи: аналогично имеем:

$$\begin{cases} \dfrac{\partial E_z}{\partial r} = 0 \\ \dfrac{\partial (r E_\varphi)}{\partial r} = 0 \end{cases} \Rightarrow \begin{cases} \widehat{E_z} = \widehat{C_1} \\ \widehat{E_\varphi} = \dfrac{\widehat{C_2}}{r} \end{cases}$$

Запишем граничные условия на R:

$$E_{\tau 1} = E_{\tau 2} \Rightarrow E_z = \hat{E}_z \Rightarrow C_1 = \hat{C}_1$$

$$E_\varphi = \hat{E}_\varphi \Rightarrow$$

$$\Rightarrow -\frac{\omega R B_0 \cos\omega t}{2} = \frac{\hat{C}_2}{R} \Rightarrow$$

$$\Rightarrow \hat{C}_2 = -\frac{\omega R^2 B_0 \cos\omega t}{2}$$

2) $\operatorname{div} D = \rho \Rightarrow \operatorname{div} \varepsilon_0 E = 0$

$$\frac{1}{r}\frac{\partial (r E_r)}{\partial r} + \frac{1}{r}\frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z} = 0 \quad \text{в силу } (*):$$

$$\frac{\partial (r E_r)}{\partial r} = 0 \Rightarrow E_r = \frac{C_3}{r} \quad (\text{внутри})$$

аналогично $\hat{E}_r = \frac{\hat{C}_3}{r}$ (снаружи)

В силу гранич. условий: $D_{n1} = D_{n2} \Rightarrow C_3 = \hat{C}_3$

В силу ограниченности E при $r = 0 \Rightarrow C_3 = 0$.

т.о. $E = \left(0, -\frac{\omega r B_0 \cos\omega t}{2}, C_1\right)$

$$\hat{E} = \left(0, -\frac{\omega R^2 B_0 \cos\omega t}{2r}, C_1\right)$$

$\vec{j} = \lambda \hat{E}$ и в силу ограниченности цилиндре ток не явл. замкн. $\Rightarrow$ не может поддерживаться $\Rightarrow C_1 = 0$.

$$\vec{j} = \left(0, -\frac{\omega \lambda R^2 B_0 \cos\omega t}{2r}, 0\right)$$

3) $\text{Ɔ} = \vec{j}\vec{E} = \dfrac{j^2}{\lambda} = \lambda \dfrac{\omega^2 R^4 B_0^2 \cos^2\omega t}{4r^2}$

$$N = \int_V \text{Ɔ}\, dV = \int_0^{2\pi} d\varphi \int_0^h dz \int_{R_1}^{R_2} \lambda \frac{\omega^2 R^4 B_0^2 \cos^2\omega t}{4r^2}\, r\, dr =$$

$$= \frac{\lambda R^4 \cdot 2\pi h \cdot B_0^2 \cos^2\omega t}{4} \ln\frac{R_2}{R_1}$$

Рассмотрим $f(t) = \cos^2\omega t$ на $[0, \frac{2\pi}{\omega}]$

среднее значение $\hat{f}(t) = \dfrac{\omega}{2\pi} \cdot \displaystyle\int_0^{2\pi/\omega} \cos^2\omega t\, dt =$

$$= \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \left[\frac{1+\cos 2\omega t}{2}\right] dt = \frac{1}{2}$$

$$\overline{N} = \frac{\lambda R^4 \pi h B_0^2}{4} \ln\frac{R_2}{R_1}$$

$\boxed{11.10}$

1) $I_{см} = j_{см} S = \dfrac{\partial}{\partial t} \displaystyle\int_S D\, dS = \dfrac{\partial}{\partial t}(DS) =$

$= S \dfrac{\partial}{\partial t} D \Rightarrow j_{см} = \dfrac{\partial}{\partial t} D = \dfrac{\partial}{\partial t}\sigma = 0$

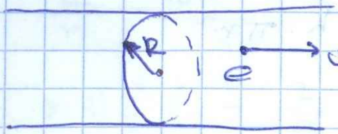
см. 11.9

2) $I_{см} = j_{см} S = S \dfrac{\partial}{\partial t} D$

$$j_{см} = \frac{\partial}{\partial t} D = \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{U}{d+vt}\right) = -\varepsilon_0 U \frac{v}{(d+vt)^2} =$$

$$= -\varepsilon_0 U \frac{v}{d^2}$$

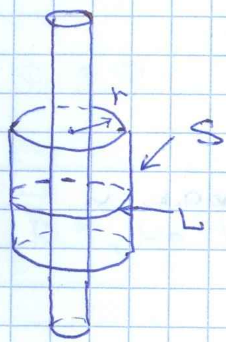
$\boxed{11.12}$ $e = -q$, $q > 0$



$$W = \frac{m\upsilon^2}{2} \Rightarrow \upsilon = \sqrt{\frac{2W}{m}}$$

$$j = -qn(x)\upsilon = -qn\sqrt{\frac{2W}{m}}$$

1) Вне пучка:



$$\oint\limits_S \bar{D}\, d\bar{S} = \varepsilon_0 E \cdot 2\pi r \cdot h = $$
$$= Q = -nqh\pi R^2$$

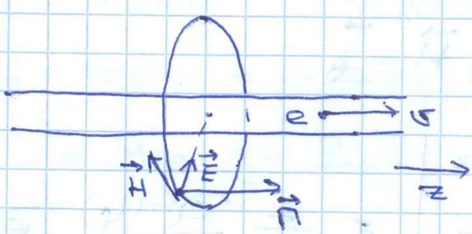
$$E = \frac{-nqR^2}{2\varepsilon_0 r}$$

$$\oint\limits_L \bar{H}\, d\bar{l} = H \cdot 2\pi r = I = j\pi R^2 =$$
$$= -qn\sqrt{\frac{2W}{m}}\pi R^2$$

$$H = \frac{-qnR^2\sqrt{\frac{2W}{m}}}{2r}$$

$\vec{E}$ направлено радиально пучку; $\vec{E} \perp \vec{H}$

$\vec{H}$ направлено по касательным к окружностям, с центрами на оси пучка



$$\vec{\Pi} = [\vec{E}, \vec{H}] =$$
$$= \frac{q^2 n^2 R^4}{4\varepsilon_0 r^2}\sqrt{\frac{2W}{m}}\,\vec{z}$$

2) Внутри пучка:

$$\oint_S \bar{D}\, d\bar{S} = \varepsilon_0 \cdot E \cdot 2\pi r \cdot h = Q = -nqh \cdot \pi r^2$$

$$E = \frac{-nqr}{2\varepsilon_0}$$

$$\oint_L \bar{H}\, d\bar{l} = H \cdot 2\pi r = j\pi r^2 = -nq\sqrt{\frac{2W}{m}}\,\pi r^2$$

$$H = -\frac{nqr}{2}\sqrt{\frac{2W}{m}}$$

Векторы $\vec{E}$ и $\vec{H}$ направлены так же, как и в случае „вне пучка":

$$\vec{\Pi} = [\vec{E} \times \vec{H}] = \frac{n^2 q^2 r^2}{4\varepsilon_0}\sqrt{\frac{2W}{m}}\,\vec{z}$$

$\boxed{11.13}$ из 11.5 имеем: $-q = e$, $q > 0$

$$j = -\frac{4}{9}\varepsilon_0\sqrt{\frac{2qU_0}{m}}\,\frac{U_0}{d^2}$$

из 11.4 известно, что $\rho(x)$ не зависит от t

$\Rightarrow$ из $\operatorname{div}\vec{D} = \rho$ следует, что $\vec{D}$ не зависит от t $\Rightarrow$

$$\Rightarrow \frac{\partial D}{\partial t} = 0 \Rightarrow \operatorname{rot}\vec{H} = \vec{j}$$

$$\operatorname{rot} H = \begin{vmatrix} \frac{1}{r}\bar{e}_r & \bar{e}_\varphi & \frac{1}{r}\bar{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & rH_\varphi & 0 \end{vmatrix} = \frac{1}{r}\frac{\partial (rH_\varphi)}{\partial r}\,\bar{e}_z$$

$H_r = H_z = 0$ в силу симметрии

$$\frac{1}{r}\frac{\partial(rH_\varphi)}{\partial r} = -\frac{4}{9}\varepsilon_0\sqrt{\frac{2qU_0}{m}}\frac{U_0}{d^2}$$

$$rH_\varphi = -\frac{2}{9}\varepsilon_0\sqrt{\frac{2qU_0}{m}}\frac{U_0}{d^2}r^2 + C$$

$$H_\varphi = -\frac{2}{9}\varepsilon_0\sqrt{\frac{2qU_0}{m}}\frac{U_0}{d^2}r + \frac{C}{r}$$

$C = 0$ в силу того, что $H = 0$ при $r = 0$, (т. о циркуляции)

$$\vec{B} = \mu_0\vec{H} = \Big(0,\ -\frac{2}{9}\underbrace{\varepsilon_0\mu_0}_{1/c^2}\sqrt{\frac{2qU_0}{m}}\frac{U_0}{d^2}r,\ 0\Big)$$

$$B = -\frac{2}{9}\frac{1}{c^2}\sqrt{\frac{2qU_0}{m}}\frac{U_0}{d^2}r =$$

$$= -\frac{2}{9}\cdot\frac{1}{9\cdot10^{12}\ м^2/с^2}\cdot\sqrt{\frac{2\cdot1{,}6\cdot10^{-19}\ Кл\cdot200\ В}{9{,}1\cdot10^{-31}\ кг}}\cdot\frac{200\ В}{25\cdot10^{-6}\ м^2}\cdot10^{-2}\ м$$

$$= -1{,}4\cdot10^{-19}\ Тл$$

$\boxed{11.14}$ $e = -q$, $q > 0$

из 11.13: $B = -\frac{2}{9}\varepsilon_0\mu_0\sqrt{\frac{2qU_0}{m}}\frac{U_0}{d^2}R$

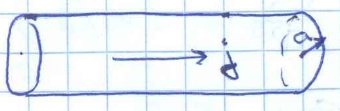
из 11.6: $v(d) = \sqrt{\frac{2qU_0}{m}}$

$$F_л = Bve = -Bvq = \frac{2}{9}\cdot\frac{1}{c^2}\cdot\frac{2q^2U_0}{m}\frac{U_0}{d^2}R =$$

$$= 4/9\,\frac{1}{c^2}\frac{q^2U_0^2}{m}\frac{R}{d^2} \qquad \left(\vec{F}_л = q[\vec{v}\times\vec{B}]\right)$$

$\boxed{11.11}$

$\vec{j} = \lambda \vec{E}$, поэтому



$\vec{\Pi} = [\vec{E} \times \vec{H}]$

$\vec{E} \perp \vec{H}$

П сонаправлено с радиусом.

$$\Pi S_{бок} = EH \cdot l \cdot 2\pi a = \left\{ \oint_L H dl = H \cdot 2\pi a = I \right\} =$$

$$= El \cdot I = \Delta U \cdot I = N$$

$\boxed{11.9}$ (продолжение)

$\Phi_{\omega} = \int_S \vec{\Pi}\, d\vec{S}$, но $\vec{\Pi} = [\vec{E} \times \vec{H}] = 0$, т.к. $\vec{H}(R) = 0$.

S – цилиндр, с осью на оси конденсатора и радиусом R (радиус обкладок конденс.).