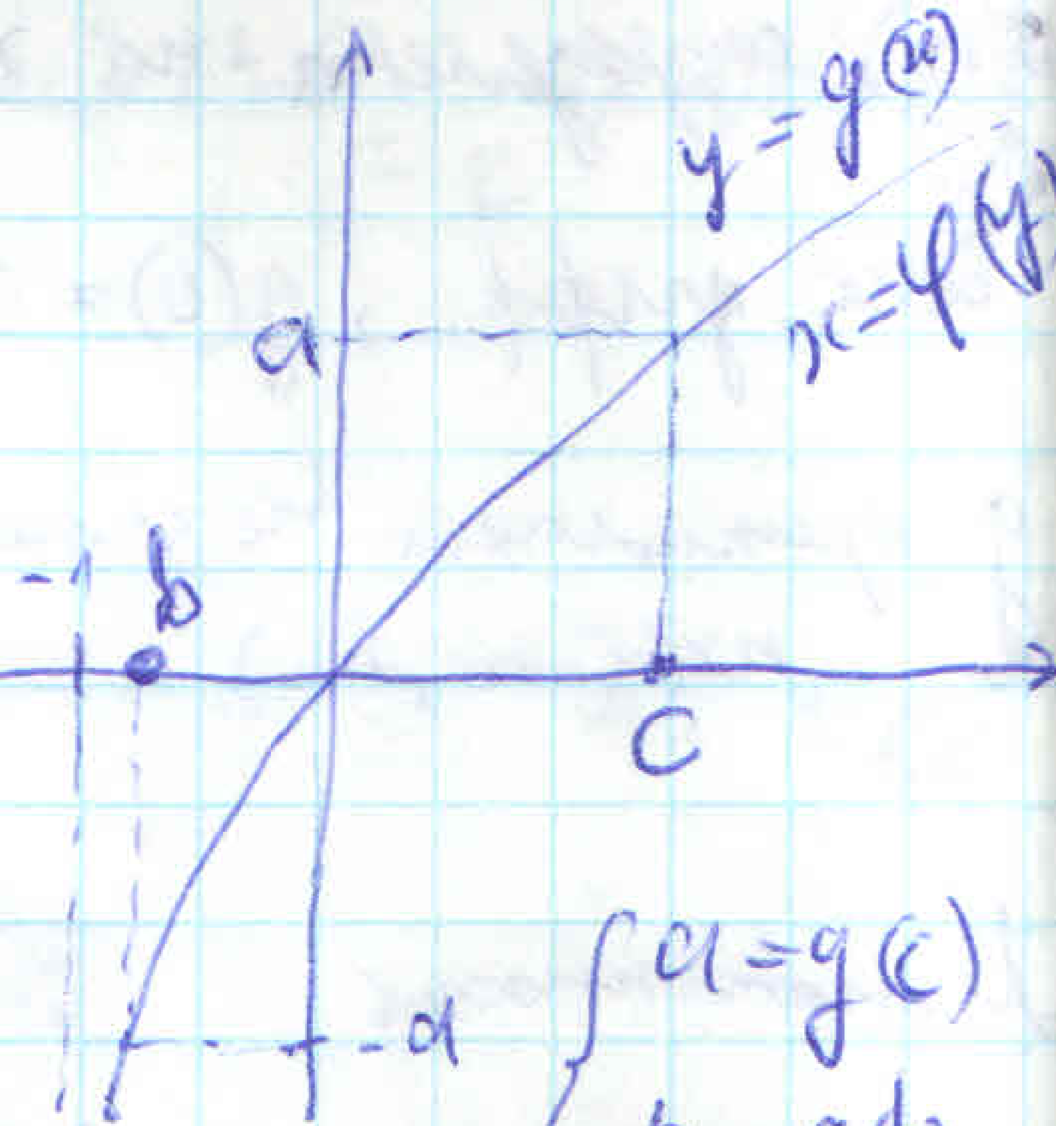
Пусть $\forall a>0$

$$I_\lambda = \int_{-1}^{b} + \int_{b}^{c} + \int_{c}^{+\infty} = I_1 + I_2 + I_3$$



$$\begin{cases} a = g(c) \\ -a = g(b) \end{cases} \Downarrow \begin{cases} c = \varphi(a) \\ b = \varphi(-a) \end{cases}$$

$$I_1 = \int_{-1}^{b} e^{-\lambda g^2(x)}dx \le e^{-\lambda a^2}\int_{-1}^{b} dx = (1-|b|)e^{-\lambda a^2}$$

$x \le b \quad g(x) \le -a \quad g(x) \ge a \quad \lambda > 0$

$$I_1 = \underline{\underline{O}}\left(e^{-\lambda a^2}\right), \forall \lambda > 0$$

$$I_3 = \int_{c}^{+\infty} e^{-\lambda g^2(x)}dx = \left\{ e^{-\lambda g^2(x)} = e^{-(\lambda-1)g^2(x)}\cdot e^{-g^2(x)} \quad \exists \lambda > 1 \right\} \overset{*}{=}$$

$x \ge c \quad g(x) \ge a \quad g^2(x) \ge a^2$

$$\overset{*}{=} \left\{ e^{-\lambda g^2(x)} \le e^{-(\lambda-1)a^2} e^{-g^2(x)} \right\} \le e^{-(\lambda-1)a^2} \underbrace{\int_{c}^{\infty} e^{-\overbrace{g^2(x)}^{=x-\ln(1+x)}}dx}_{\text{сход.}} = ce^{-\lambda a^2}$$

$$c = e^{a^2}\int_{c}^{+\infty} e^{-g^2(x)}dx \qquad I_1 + I_3 = \underline{\underline{O}}\left(e^{-\lambda a^2}\right), \forall \lambda > 1$$

$$I_2 = \int_{b}^{c} e^{-\lambda g^2(x)}dx = \left\{ t = g(x),\ x = \varphi(t),\ dx = \varphi'(t)dt \right\} =$$

$$= \int_{-a}^{a} e^{-\lambda t^2}\varphi'(t)dt$$

Пусть $\forall n \in \mathbb{N}$, $\varphi'(t) \overset{\text{Маклорен}}{=} \sum_{k=0}^{2n-1} \varphi^{(k+1)}(0)\cdot\frac{1}{k!}\cdot t^k + \underline{\underline{O}}\left(t^{2n}\right) =$

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$$\Rightarrow I_2 = \sum_{l=0}^{n-1} \frac{\varphi^{(2l+1)}(0)\cdot\Gamma\left(l+\frac{1}{2}\right)}{(2l)!\ \lambda^{l+\frac{1}{2}}} + \underline{\underline{O}}\left(\lambda^{\frac{1}{n+\frac{1}{2}}}\right) \ge \frac{\underline{\underline{O}}(1)}{\lambda^{n+\frac{1}{2}}} = \underline{\underline{O}}\left(\frac{1}{\lambda^{n+\frac{1}{2}}}\right)$$

$$\forall \lambda > \Lambda \quad e^{-\lambda a^2} \le \frac{1}{\lambda^{n+\frac{1}{2}}}$$

$$\Rightarrow \Gamma(\lambda+1) = \left(\frac{\lambda}{e}\right)^{\lambda}\cdot\lambda\left[\sum_{l=0}^{n-1} \frac{\varphi^{(2l+1)}(0)}{(2l)!}\cdot\frac{\Gamma\left(l+\frac{1}{2}\right)}{\lambda^{l+\frac{1}{2}}} + \underline{\underline{O}}\left(\frac{1}{\lambda^{n+\frac{1}{2}}}\right)\right]$$