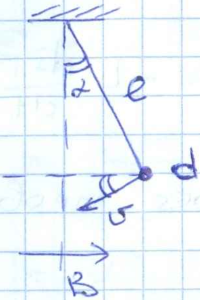
$\boxed{10.1}$ $\boxed{10}$



Известно, что уравнение колебаний матем. маят. выглядит следующ. образом.

$$\ddot{\alpha} + \hat{\omega}^2 \alpha = 0 \quad , \hat{\omega} = \sqrt{\frac{g}{l}}$$

Решение данного дифф. ур:

$$\alpha = \alpha_0 \cos(\hat{\omega} t)$$

$$\Delta U(t) = d \cdot [\bar{B} \times \bar{v}] = d B v(t) \sin \alpha(t) =$$

$$= \left\{ \begin{array}{l} v(t) = \omega(t) l = \frac{d\alpha(t)}{dt} l = \frac{d\alpha}{dt} l \\ \sin\alpha(t) \approx \alpha(t) \text{, т.к. } \alpha(t) \le \alpha_0 \end{array} \right\} =$$

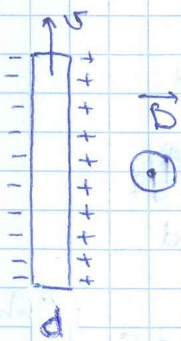
$$= dB \alpha_0 \cos \hat{\omega} t \cdot \frac{d\alpha}{dt} l =$$

$$= - d \cdot B \cdot \alpha_0 \cos \hat{\omega} t \cdot l \cdot \alpha_0 \sin \hat{\omega} t \cdot \hat{\omega} =$$

$$= -\frac{1}{2} B \alpha_0^2 d l \hat{\omega} \sin 2 \hat{\omega} t =$$

$$= -1/2\, \alpha_0^2 B \sqrt{g l}\, d \cdot \sin 2\sqrt{\frac{g}{l}}\, t$$

$\boxed{10.2}$



d – толщина пластины.

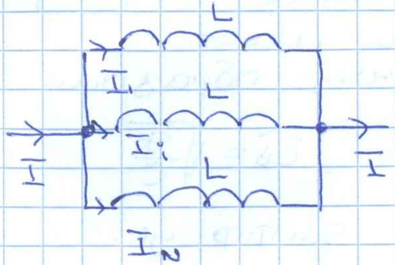
$$\Delta U = d [\bar{B} \times \bar{v}] = B v d$$

$$\left\{ \begin{array}{l} E = \frac{\Delta U}{d} = B v \\ E = \frac{\sigma}{\varepsilon_0} \end{array} \right. \Rightarrow \sigma = \varepsilon_0 B v$$

$$\sigma = \varepsilon_0 B v = 8{,}85 \cdot 10^{-12} \frac{Кл^2}{Н \cdot м^2} \cdot 10 \cdot 10^{-3} Тл \cdot 10 м/с =$$

$$= 8{,}85 \cdot 10^{-13} \frac{Кл}{м^2}$$

$\boxed{10.3}$



$$\Phi_i = L \cdot I_i$$

$$\mathcal{E}_i = U_i = -\frac{d\Phi_i}{dt} = -L\frac{dI_i}{dt}$$

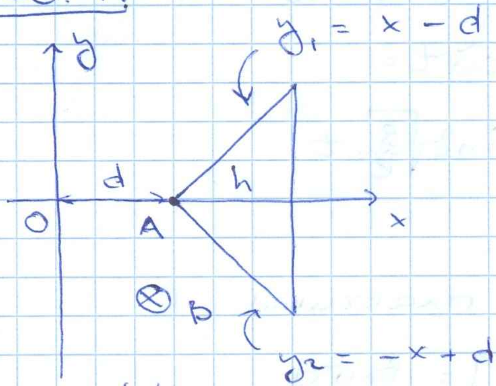
В силу паралл. соед. проводн.

$$U_1 = \ldots = U_N = U_0$$

$\Rightarrow \quad I_1 = \ldots = I_N = I_0$, с другой стороны

$$\begin{cases} U_0 = -\dfrac{d\Phi}{dt} = -L_{эф}\dfrac{dI}{dt} = -L_{эф} N \dfrac{dI_0}{dt} \\ U_0 = -L\dfrac{dI_0}{dt} \end{cases} \Rightarrow L_{эф} = \frac{L}{N}.$$

$\boxed{10.4}$



$$I_1(t) = I_0 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$$

$$B = \frac{\mu_0 I_1(t)}{2\pi x}$$

$$\Phi = \int_S B\,dS =$$

$$= \int_d^{d+h} dx \int_{-x+d}^{x-d} \frac{\mu_0 I_1(t)}{2\pi x} dy = \frac{\mu_0 I_1(t)}{2\pi} \int_d^{d+h} \frac{dx}{x} \int_{-x+d}^{x-d} dy =$$

$$= \frac{\mu_0 I_1(t)}{\pi} \int_d^{d+h} \left(1 - \frac{d}{x}\right) dx = \frac{\mu_0 I_1(t)}{\pi} \Big[x - d\ln x\Big]_d^{d+h}$$

$$= \frac{\mu_0 I_1(t)}{\pi}\left[h - d \ln \frac{d+h}{d}\right]$$

$$\mathcal{E}_i = -\frac{\partial \Phi}{\partial t} = \frac{\mu_0}{\pi}\left[d \ln\left(1+\frac{h}{d}\right) - h\right] \frac{\partial I_1}{\partial t} =$$

$$= \frac{\mu_0}{\pi}\left[d \ln\left(1+\frac{h}{d}\right) - h\right] \cdot \left[I_0 \exp\left(-\frac{t^2}{\tau^2}\right)\left(-\frac{2t}{\tau^2}\right)\right] =$$

$$= \frac{\mu_0 I_0}{\pi \tau^2}\left(h - d \ln\left(1+\frac{h}{d}\right)\right)\left[\exp\left(-\frac{t^2}{\tau^2}\right) 2t\right]$$

$$f(t) = \exp\left(-\frac{t^2}{\tau^2}\right) t$$

$$f'(t) = \exp\left(-\frac{t^2}{\tau^2}\right) + t \exp\left(-\frac{t^2}{\tau^2}\right)\left(-\frac{2t}{\tau^2}\right) =$$

$$= \exp\left(-\frac{t^2}{\tau^2}\right)\left[1 - \frac{2t^2}{\tau^2}\right] = 0$$

$-$ $\quad$ $+$ $\quad$ $-$ $\quad \to t \quad , \quad t > 0$

$\searrow \quad -\frac{\tau}{\sqrt{2}} \quad \nearrow \quad \frac{\tau}{\sqrt{2}} \quad \searrow$

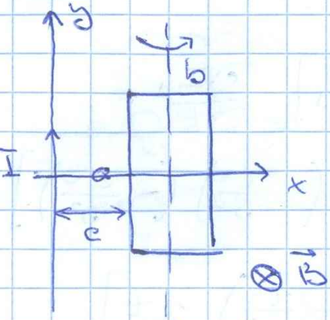
$$I_2(t) = \frac{\mathcal{E}_i}{R} = \frac{2\mu_0 I_0}{\tau^2 \pi R}\left(h - d \ln\left(1+\frac{h}{d}\right)\right)\left[\exp\left(-\frac{t^2}{\tau^2}\right) \cdot t\right]$$

$$I_{max} = I_2\left(\frac{\tau}{\sqrt{2}}\right) = \frac{2\mu_0 I_0}{\tau^2 \pi R}\left(h - d \ln\left(1+\frac{h}{d}\right)\right) \frac{\tau}{\sqrt{2}} \cdot \left(\sqrt{e}\right)^{-1} =$$

$$= \sqrt{\frac{2}{e}} \cdot \frac{\mu_0 I_0}{\pi R \tau} \cdot \left(h - d \cdot \ln\left(1+\frac{h}{d}\right)\right) =$$

$$= \sqrt{\frac{2}{e}} \cdot \frac{4\pi \cdot 10^{-7}\ \mathrm{H/A^2} \cdot 100\ \mathrm{A}}{\pi \cdot 0{,}70\ \Omega \cdot 10^{-3}\ \mathrm{c}} \cdot \left(0{,}17\ \mathrm{m} - 0{,}1\ \mathrm{m} \cdot \ln\left(1 + \frac{0{,}17}{0{,}1}\right)\right) = 5{,}3\ \mathrm{mA}$$

$\boxed{10.5}$



$$I_i = \frac{\mathcal{E}_i}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = \frac{dq}{dt}$$

$$q(t_2) - \underbrace{q(t_1)}_{=0} = \frac{1}{R}\left(\Phi(t_1) - \Phi(t_2)\right)$$

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right)$$

$$\Phi_н = \int\limits_{-a/2}^{a/2} dy \int\limits_{c}^{c+b} \underbrace{\frac{\mu_0 I}{2\pi x}}_{B(x)} dx = \frac{a\mu_0 I}{2\pi}\ln\left(1+\frac{b}{c}\right)$$

Легко видеть, что абсолютное значение потока через рамку не поменялось при повороте на 180°.

Однако, нормаль поменялась, она тоже повернулась на 180°.

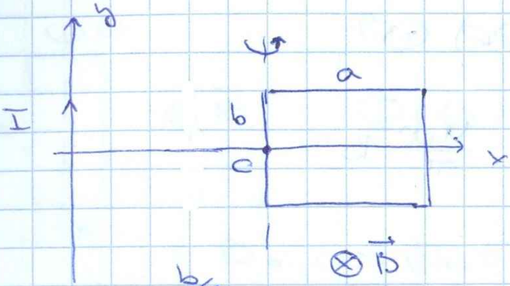
Т.о. $\Phi_к = -\Phi_н$

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right) = \frac{2}{R}\Phi_н = \frac{\mu_0 I a}{\pi R}\ln\left(1+\frac{b}{c}\right) =$$

$$= \frac{4\pi\cdot 10^{-7}\,Н/А^2 \cdot 10\,А \cdot 20\cdot 10^{-2}\,м}{\pi\cdot 0{,}7\,Ом}\cdot\ln\left(1+\frac{17}{10}\right) =$$

$$= 1\,мкКл.$$

$\boxed{10.6}$



$$l = 2(a+b)$$

$$R = \rho \frac{l}{S} \qquad B(x) = \frac{\mu_0 I}{2\pi x}$$

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right) \quad (10.5)$$

$$\Phi_н = \int\limits_{-b/2}^{b/2} dy \int\limits_{c}^{c+a} \frac{\mu_0 I}{2\pi x} dx = \frac{b\mu_0 I}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$

$$\Phi_к = \int\limits_{-b/2}^{b/2} dy \int\limits_{c-a}^{c} \frac{\mu_0 I}{2\pi x} dx = \frac{b\mu_0 I}{2\pi} \ln\left(\frac{c}{c+a}\right)$$

По тем же соображениям, что и в 10.5 знаки у $\Phi_н$ и $\Phi_к$ противоположны.

$$Q = \frac{1}{R}\left(\Phi_н - \Phi_к\right) = \frac{1}{R} \cdot \frac{b\mu_0 I}{2\pi}\left[\ln\frac{a+c}{c} + \ln\frac{c}{c-a}\right] =$$

$$= \frac{\mu_0 I S b}{4\pi(a+b)\rho} \ln\frac{a+c}{c-a} = \frac{\mu_0 I S b \lambda}{4\pi(a+b)} \ln\frac{c+a}{c-a} =$$

$$= \frac{\cancel{4\pi} \cdot 10^{-7}\ Н/А^2 \cdot 10А \cdot 1 \cdot 10^{-6} м^2 \cdot 11 \cdot 10^{-2} м \cdot 6 \cdot 10^{7} \frac{Ом}{м}}{\cancel{4\pi}(19 + 11) \cdot 10^{-2} м} \cdot$$

$$\cdot \ln\frac{21+19}{21-19} = 6{,}6 \text{ мкКл}$$

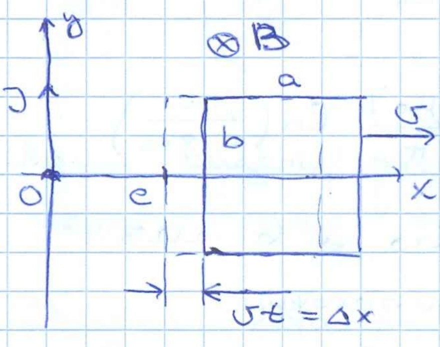
$\boxed{10.7}$ $4a = 2\pi r \Rightarrow r = \frac{2a}{\pi}$

из 10.5: $Q = \frac{1}{R}(\Phi_н - \Phi_к) =$

$\Phi_н = B \cdot a^2 \qquad \Phi_к = B \cdot \pi r^2 = \frac{4a^2}{\pi} B$

$= \frac{Ba^2}{R}\left(1 - \frac{4}{\pi}\right)$

$\boxed{10.8}$

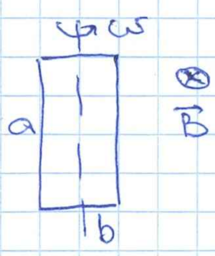


$$\Phi(t) = \int_{-b/2}^{b/2} dy \int_{c+\Delta x}^{c+\Delta x+a} \frac{\mu_0 I}{2\pi x} dx = \frac{b\mu_0 I}{2\pi} \ln\left(1 + \frac{a}{c+\Delta x}\right) = \frac{b\mu_0 I}{2\pi} \ln\left(1 + \frac{a}{c+vt}\right)$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = +\frac{b\mu_0 I}{2\pi} \cdot \frac{c+vt}{c+vt+a} \cdot \frac{av}{(c+vt)^2} =$$

$$= \frac{\mu_0 I ab v}{(c+vt+a)(c+vt)} \cdot \frac{1}{2\pi}$$

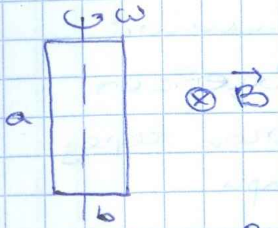
$\boxed{10.9}$



$$\Phi(t) = \int_{S(t)} B\, dS = BS\cos\alpha(t) = Bab\cos\omega t$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = Bab\omega \sin\omega t$$

$\boxed{10.10}$



$$B = B_0 \cos(\Omega t)$$

$$\Phi(t) = \int_{S(t)} B(t)\, dS = B(t)\, S \cos\alpha(t) =$$

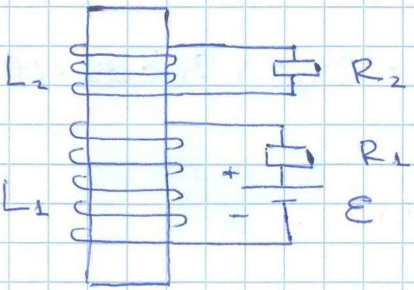
$$= B_0 \cos(\Omega t)\, ab \cos(\omega t)$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = -B_0 ab \left[-\Omega \sin\Omega t \cos\omega t - \right.$$

$$\left. - \omega \sin\omega t \cos\Omega t \right] =$$

$$= B_0 ab \left[\Omega \sin\Omega t \cos\omega t + \omega \sin\omega t \cos\Omega t \right]$$

10.13



Коэффициент взаимной индукции L_{12} показывает как изменяется поток через вторую катушку при изменении тока в первой.

$$\mathcal{E}_{21} = -L_{12}\frac{dI_1}{dt}$$

с другой стороны $\mathcal{E}_{21} = I_2 R_2 = \frac{dQ}{dt} R_2$

$$-L_{12}\int_{t_1}^{t_2}\frac{dI_1}{\cancel{dt}} = R_2\int_{t_1}^{t_2}\frac{dQ}{\cancel{dt}} \xRightarrow{\text{интегрир.}} Q = -L_{12} I_1 \frac{1}{R_2}$$

$Q = -\mathcal{E}\dfrac{L_{12}}{R_1 R_2}$, где $L_{12} = \sqrt{L_1 L_2}$ (пример 9.3)

10.12 Аналогично соображениям 10.13

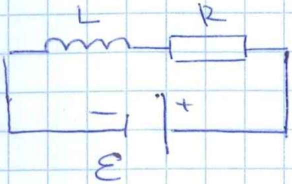
$$\mathcal{E}_{21} = -L_{12}\frac{dI_1}{dt}$$

с другой стороны $\mathcal{E}_{21} = I_2 R_2 = \frac{dQ}{dt} R_2$

$$-L_{12}\int_{t_1}^{t_2}\frac{dI_1}{\cancel{dt}} = R_2\int_{t_1}^{t_2}\frac{dQ}{\cancel{dt}} \xRightarrow{\text{интегр.}} Q = L_{12} I_1 \frac{1}{R_2}$$

$Q = L_{12} I_1 \dfrac{1}{R_2} = \dfrac{\mu_0 n S I}{R}$ (см. 9.6)

$\boxed{10.14}$ (исходное условие)



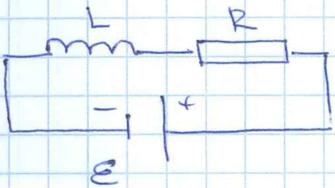
$$I(t) = I_0 e^{-t/\tau}$$

$$\mathcal{E}_i = -L\frac{dI}{dt} = \frac{LI_0}{\tau} e^{-t/\tau}$$

$$\mathcal{E}(t) + \mathcal{E}_i = I(t)R$$

$$\mathcal{E}(t) = RI_0 e^{-t/\tau} - \frac{LI_0}{\tau} e^{-t/\tau} = I_0 e^{-t/\tau}\left(R + \frac{L}{\tau}\right)$$

$\boxed{10.14}$ (немного другое условие)



$$I(t) = I_0\left(1 - e^{-t/\tau}\right)$$

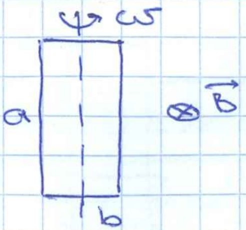
$$\mathcal{E}_i = -L\frac{dI}{dt} = -\frac{LI_0}{\tau} e^{-t/\tau}$$

$$\mathcal{E}(t) + \mathcal{E}_i = I(t)R$$

$$\mathcal{E}(t) = RI_0 - Re^{-t/\tau} I_0 + \frac{LI_0}{\tau} e^{-t/\tau} =$$

$$= RI_0 - I_0 e^{-t/\tau}\left(R - \frac{L}{\tau}\right)$$

$\boxed{10.11}$



$$\omega(t) = \omega_0 \left(1 - e^{-\delta t}\right) = \frac{d\alpha(t)}{dt}$$

$$\alpha(t) = \omega_0 t + \omega_0 \frac{e^{-\delta t}}{\delta} = $$

$$= \omega_0 \left(t + \frac{e^{-\delta t}}{\delta}\right)$$

$$\Phi(t) = \int_{S(t)} B\,dS = BS\cos\alpha(t) = Bab\cos\left[\omega_0\left(t + \frac{e^{-\delta t}}{\delta}\right)\right]$$

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = Bab\,\sin\left[\omega_0\left(t + \frac{e^{-\delta t}}{\delta}\right)\right] \cdot$$

$$\cdot \left[\omega_0\left(1 - e^{-\delta t}\right)\right] = \mathcal{E}_i(t)$$

$$\omega_0\left(1 - e^{-\delta t_0}\right) = \omega_0/2 \Rightarrow t_0 = \frac{\ln 2}{\delta}$$

$$\tilde{\mathcal{E}}_i = \mathcal{E}_i(t_0) = Bab\,\sin\left[\omega_0\left(\frac{\ln 2}{\delta} - \frac{1}{2\delta}\right)\right]\omega_0 \cdot \frac{1}{2} =$$

$$= \frac{1}{2} abB\omega_0 \sin\left[\omega_0\left(\frac{\ln 2}{\delta} - \frac{1}{2\delta}\right)\right]$$