Таблица неопределенных интегралов

1. Функции, содержащие a+bx в целой степени

1). $\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) + C$

2). $\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} + C, n \neq -1$

2*). $\int \frac{dx}{(a+bx)^n} = -\frac{1}{b(n-1)(bx+a)^{n-1}} + C, n \neq 1$

3). $\int \frac{xdx}{a+bx} = \frac{1}{b^2}\left[a+bx-a\ln(a+bx)\right] + C$

4). $\int \frac{x^2 dx}{a+bx} = \frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2\ln(a+bx)\right] + C$

5). $\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\ln\left|\frac{a+bx}{x}\right| + C$

6). $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2}\ln\left|\frac{a+bx}{x}\right| + C$

7). $\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}\left[\ln(a+bx) + \frac{a}{a+bx}\right] + C$

8). $\int \frac{x^2 dx}{(a+bx)^2} = \frac{1}{b^3}\left[a+bx-2a\ln(a+bx) - \frac{a^2}{a+bx}\right] + C$

9). $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2}\ln\left|\frac{a+bx}{x}\right| + C$

10). $\int \frac{xdx}{(a+bx)^3} = \frac{1}{b^2}\left[-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2}\right] + C$

2. Функции, содержащие a^2+x^2, a^2-x^2, a^2+bx^2

11). $\int \frac{dx}{1+x^2} = \arctan x + C$

12). $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

13). $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

14). $\int \frac{dx}{a^2-x^2} = -\frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$

15). $\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + C$

16). $\int \frac{dx}{a-bx^2} = \frac{1}{2\sqrt{ab}}\ln\left|\frac{\sqrt{a}+x\sqrt{b}}{\sqrt{a-x\sqrt{b}}}\right| + C$

17). $\int \frac{xdx}{a+bx^2} = \frac{1}{2b}\ln\left|x^2 - \frac{a}{b}\right| + C$

18). $\int \frac{x^2 dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+bx^2}$

19). $\int \frac{dx}{x(a+bx^2)} = \frac{1}{2a}\ln\left|\frac{x^2}{a+bx^2}\right| + C$

20). $\int \frac{dx}{x^2(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{dx}{a+bx^2}$

21). $\int \frac{dx}{(a+bx^2)^2} = \frac{x}{2a(a+bx^2)} + \frac{1}{2a}\int \frac{dx}{a+bx^2}$

3. Функции, содержащие $\sqrt{a+bx}$

22). $\int \sqrt{a+bx}dx = -\frac{2}{3b}\sqrt{(a+bx)^3} + C$

23). $\int x\sqrt{a+bx}dx = -\frac{2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$

24). $\int x^2\sqrt{a+bx}dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$

24*). $\int \frac{x^n dx}{\sqrt{a+bx}} = \frac{2\left((-1)^n 2^{n+n\backslash 2+1}a^n + \sum_{i=1}^{n}(-1)^{n-i}\frac{(2i-1)2^{n+n\backslash 2+1}}{2i}(1+2i)a^{n-i}b^i x^i\right)}{3b^{n+1}\left(2^{n+n\backslash 2+1} + \sum_{i=1}^{n}\frac{(2i-1)}{2i}2^{n+n\backslash 2+1}(1+2i)\right)}\sqrt[2]{(a+bx)^3} + C$

25). $\int \frac{xdx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx} + C$

26). $\int \frac{x^2dx}{\sqrt{a+bx}} = \frac{2(8a^2-4abx+3b^2x^2)}{15b^3}\sqrt{a+bx} + C$

26*). $\int \frac{x^n dx}{\sqrt{a+bx}} = \frac{2\left((-1)^n 2^{n+n\backslash 2+1}a^n + \sum_{i=1}^{n}(-1)^{n-i}\frac{(2i-1)2^{n+n\backslash 2+1}}{2i}a^{n-i}b^i x^i\right)}{b^{n+1}\left(2^{n+n\backslash 2+1} + \sum_{i=1}^{n}\frac{(2i-1)}{2i}2^{n+n\backslash 2+1}\right)}\sqrt{a+bx} + C$

27). $\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right| + C$

28). $\int \frac{dx}{x\sqrt{bx-a}} = \frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bx}{-a}} + C$

29). $\int \frac{dx}{x^2\sqrt{bx+a}} = \frac{-\sqrt{bx+a}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{bx+a}}$

30). $\int \frac{\sqrt{bx+a}dx}{x} = 2\sqrt{bx+a} + a\int \frac{dx}{x\sqrt{bx+a}}$

4. Функции, содержащие $\sqrt{a^2+x^2}$

31). $\int \sqrt{x^2 \pm a^2}dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\int \frac{dx}{\sqrt{x^2 \pm a^2}} + C$

32). $\int \sqrt{\left(x^2 \pm a^2\right)^3}dx = \frac{x}{8}\left(2x^2 \pm 5a^2\right)\sqrt{x^2 \pm a^2} + \frac{3a^4}{8}\int \frac{dx}{\sqrt{x^2 \pm a^2}} + C$

33). $\int x\sqrt{x^2+a^2}dx = \frac{\sqrt{\left(x^2+a^2\right)^3}}{3} + C$

33*). $\int x\sqrt{\left(x^2+a^2\right)^n}dx = \frac{1}{n+2}\sqrt{\left(x^2+a^2\right)^{n+2}} + C$

34). $\int x^2\sqrt{x^2 \pm a^2}dx = \frac{x}{8}\left(2x^2 \pm a^2\right)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\int \frac{dx}{\sqrt{x^2 \pm a^2}} + C$

35). $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

36). $\int \frac{dx}{\sqrt{\left(x^2 \pm a^2\right)^3}} = \pm \frac{x}{a^2\sqrt{x^2 \pm a^2}} + C$

37). $\int \frac{xdx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$

37*). $\int \frac{xdx}{\sqrt{\left(x^2 + a^2\right)^3}} = -\frac{1}{\sqrt{x^2 + a^2}} + C$

37**). $\int \frac{xdx}{\sqrt{\left(x^2 + a^2\right)^n}} = -\frac{1}{(n-2)\sqrt{\left(x^2 + a^2\right)^{n-2}}} + C$

38). $\int \frac{x^2 dx}{\sqrt{x^2 \pm a^2}} = \frac{x}{2}\int \frac{xdx}{\sqrt{x^2 \pm a^2}} \mp \frac{a^2}{2}\int \frac{dx}{\sqrt{x^2 \pm a^2}} + C$

39). $\int \frac{x^2 dx}{\sqrt{\left(x^2 \pm a^2\right)^3}} = -\frac{x}{\sqrt{x^2 \pm a^2}} + \int \frac{dx}{\sqrt{x^2 \pm a^2}} + C$

40). $\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = \frac{1}{a}\ln\left|\frac{x}{a + \sqrt{a^2 \pm x^2}}\right| + C$

41). $\int \frac{dx}{x^2\sqrt{a^2 \pm x^2}} = -\frac{\sqrt{a^2 \pm x^2}}{a^2 x} + C$

42). $\int \frac{dx}{x^3\sqrt{a^2 \pm x^2}} = \frac{1}{2x}\int \frac{dx}{x^2\sqrt{a^2 \pm x^2}} \mp \frac{1}{2a^2}\int \frac{dx}{x\sqrt{a^2 \pm x^2}} + C$

43). $\int \frac{\sqrt{a^2 \pm x^2}}{x} dx = \pm\int \frac{xdx}{\sqrt{a^2 \pm x^2}} + a^2\int \frac{dx}{x\sqrt{a^2 \pm x^2}} + C$

44). $\int \frac{\sqrt{a^2 \pm x^2}}{x^2} dx = -\frac{\sqrt{a^2 \pm x^2}}{x} \pm \int \frac{dx}{\sqrt{a^2 \pm x^2}} + C$

5. Функции, содержащие $\sqrt{a^2 - x^2}$

45). $\int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C$

46). $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$

46*). $\int \frac{dx}{\sqrt{a - bx^2}} = \frac{1}{\sqrt{b}}\arcsin\left(x\frac{\sqrt{b}}{\sqrt{a}}\right) + C$

47). $\int \frac{dx}{\sqrt{\left(a^2 - x^2\right)^3}} = \frac{x}{a^2\sqrt{a^2 - x^2}} + C$

48). $\int \frac{xdx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} + C$

49). $\int \frac{xdx}{\sqrt{\left(a^2 - x^2\right)^3}} = \frac{1}{\sqrt{a^2 - x^2}} + C$

49*). $\int \frac{xdx}{\sqrt{\left(a^2 - x^2\right)^n}} = \frac{1}{(n-2)\sqrt{\left(a^2 - x^2\right)^{n-2}}} + C$

50). $\int \frac{x^2 dx}{\sqrt{a^2 \mp x^2}} = \frac{x}{2}\int \frac{xdx}{\sqrt{x^2 \mp a^2}} \pm \frac{a^2}{2}\int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

51). $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \int \frac{dx}{\sqrt{a^2-x^2}} \right) + C$

52). $\int \sqrt{\left(a^2-x^2\right)^3}\,dx = \frac{1}{8}\left(x\left(5a^2-2x^2\right)\sqrt{a^2-x^2} + 3a^4 \int \frac{dx}{\sqrt{a^2-x^2}} \right) + C$

53). $\int x\sqrt{\pm a^2 \mp x^2}\,dx = \mp\frac{1}{3}\sqrt{\left(\pm a^2 \mp x^2\right)^3} + C$

54). $\int x\sqrt{\left(\pm a^2 \mp x^2\right)^3}\,dx = \mp\frac{1}{5}\sqrt{\left(\pm a^2 \mp x^2\right)^5} + C$

54*). $\int x\sqrt{\left(\pm a^2 \mp x^2\right)^n}\,dx = \mp\frac{1}{n+2}\sqrt{\left(\pm a^2 \mp x^2\right)^{n+2}} + C$

55). $\int x^2\sqrt{a^2 \mp x^2}\,dx = \frac{x}{8}\left(2x^2 \mp a^2\right)\sqrt{x^2 \mp a^2} \pm \frac{a^4}{8}\int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

56). $\int \frac{x^2\,dx}{\sqrt{\left(a^2 \mp x^2\right)^3}} = \pm\frac{x}{\sqrt{x^2 \mp a^2}} \mp \int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

57). $\int \frac{dx}{x\sqrt{a^2 \mp x^2}} = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{a^2 \mp x^2}}\right| + C$

58). $\int \frac{dx}{x^2\sqrt{a^2 \mp x^2}} = -\frac{\sqrt{a^2 \mp x^2}}{a^2 x} + C$

59). $\int \frac{dx}{x^3\sqrt{a^2 \mp x^2}} = \frac{1}{2x}\int \frac{dx}{x^2\sqrt{a^2 \mp x^2}} \pm \frac{1}{2a^2}\int \frac{dx}{x\sqrt{a^2 \mp x^2}} + C$

60). $\int \frac{\sqrt{a^2 \mp x^2}}{x}dx = \sqrt{a^2 \mp x^2} - a\ln\left|\frac{a+\sqrt{a^2 \mp x^2}}{x}\right| + C$

61). $\int \frac{\sqrt{a^2 \mp x^2}}{x^2}dx = -\frac{\sqrt{a^2 \mp x^2}}{x} \mp \int \frac{dx}{\sqrt{a^2 \mp x^2}} + C$

6. Функции, содержащие $\sqrt{x^2-a^2}$

62). $\int \frac{dx}{\sqrt{x^2 \mp a^2}} = \ln\left|x+\sqrt{x^2 \mp a^2}\right| + C$

63). $\int \frac{dx}{\sqrt{\left(x^2 \mp a^2\right)^3}} = \mp\frac{x}{a^2\sqrt{x^2 \mp a^2}} + C$

64). $\int \frac{x\,dx}{\sqrt{x^2 \mp a^2}} = \sqrt{x^2 \mp a^2} + C$

65). $\int \sqrt{x^2 \mp a^2}\,dx = \frac{x}{2}\sqrt{x^2 \mp a^2} \mp \frac{a^2}{2}\int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

66). $\int \sqrt{\left(x^2 \mp a^2\right)^3}\,dx = \frac{x}{8}\left(2x^2 \mp 5a^2\right)\sqrt{x^2 \mp a^2} + \frac{3a^4}{8}\int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

67). $\int x\sqrt{\pm a^2 \mp x^2}\,dx = \mp\frac{1}{3}\sqrt{\left(\pm a^2 \mp x^2\right)^3} + C$

68). $\int x\sqrt{\left(\pm a^2 \mp x^2\right)^3}\,dx = \mp\frac{1}{5}\sqrt{\left(\pm a^2 \mp x^2\right)^5} + C$

68*). $\int x\sqrt{\left(\pm a^2 \mp x^2\right)^n}\,dx = \mp\frac{1}{n+2}\sqrt{\left(\pm a^2 \mp x^2\right)^{n+2}} + C$

69). $\int x^2\sqrt{x^2 \mp a^2}\,dx = \frac{x}{8}\left(2x^2 \mp a^2\right)\sqrt{x^2 \mp a^2} - \frac{a^4}{8}\int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

70). $\int \frac{x^2 dx}{\sqrt{x^2 \mp a^2}} = \frac{x}{2}\int \frac{xdx}{\sqrt{x^2 \mp a^2}} \pm \frac{a^2}{2}\int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

71). $\int \frac{x^2 dx}{\sqrt{\left(x^2 \mp a^2\right)^3}} = -\frac{x}{\sqrt{x^2 \mp a^2}} + \int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

72). $\int \frac{dx}{x\sqrt{x^2 - 1}} = \sec^{-1} x + C$

73). $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + C$

74). $\int \frac{dx}{x^2\sqrt{x^2 \mp a^2}} = \pm\frac{\sqrt{x^2 \mp a^2}}{a^2 x} + C$

75). $\int \frac{dx}{x^3\sqrt{x^2 \mp a^2}} = \frac{1}{2x}\int \frac{dx}{x^2\sqrt{a^2 \mp x^2}} \pm \frac{1}{2a^2}\int \frac{dx}{x\sqrt{a^2 \mp x^2}} + C$

76). $\int \frac{\sqrt{x^2 - a^2}}{x} dx = \int \frac{xdx}{\sqrt{x^2 - a^2}} - a\arccos\frac{a}{x} + C$

77). $\int \frac{\sqrt{x^2 \mp a^2}}{x^2} dx = -\frac{\sqrt{x^2 \mp a^2}}{x} + \int \frac{dx}{\sqrt{x^2 \mp a^2}} + C$

7. Функции, содержащие $\sqrt{2ax \pm x^2}$

Функция содержащая $\sqrt{2ax \pm x^2}$, интегрируется подстановкой $t = x \pm a$. Тогда $\sqrt{2ax \pm x^2}$ получит вид $\sqrt{2a(t \mp a) \pm (t \mp a)^2} = \sqrt{2at \mp 2a^2 \pm (t^2 \mp 2at + a^2)} = \sqrt{\mp a^2 \pm t^2}$ и интеграл находят из таблицы.

8. Функции, содержащие $a + bx + cx^2$, c > 0

78). $\int \frac{dx}{a + bx + cx^2} = \begin{cases} \frac{2}{\sqrt{4ac - b^2}}\arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) + C, & \text{если } b^2 < 4ac \\ \frac{1}{\sqrt{b^2 - 4ac}}\ln\left(\frac{2cx + b - \sqrt{b^2 - 4ac}}{2cx + b + \sqrt{b^2 - 4ac}}\right) + C, & \text{если } b^2 > 4ac \end{cases}$

79). $\int \frac{dx}{\sqrt{a + bx + cx^2}} = \frac{1}{\sqrt{c}}\ln\left|2cx + b + 2\sqrt{c(a + bx + cx^2)}\right| + C$

80). $\int \sqrt{a + bx \pm cx^2}\, dx = \frac{2cx \pm b}{4c}\sqrt{a + bx \pm cx^2} \mp \frac{b^2 \mp 4ac}{8c}\int \frac{dx}{\sqrt{a + bx \pm cx^2}} + C$

81). $\int \frac{xdx}{\sqrt{a + bx \pm cx^2}} = \pm\frac{1}{c}\left(\sqrt{a + bx \pm cx^2} - \frac{b}{2}\int \frac{dx}{\sqrt{a + bx \pm cx^2}}\right) + C$

9. Функции, содержащие $a + bx - cx^2$, c > 0

82). $\int \frac{dx}{a + bx - cx^2} = \frac{1}{\sqrt{b^2 + 4ac}}\ln\left|\frac{\sqrt{b^2 + 4ac} + 2cx - b}{\sqrt{b^2 + 4ac} - 2cx + b}\right| + C$

83). $\int \frac{dx}{\sqrt{a + bx - cx^2}} = \frac{1}{\sqrt{c}}\arcsin\left(\frac{2cx - b}{\sqrt{b^2 + 4ac}}\right) + C$

84). $\int \sqrt{a + bx \mp cx^2}\, dx = \frac{2cx \mp b}{4c}\sqrt{a + bx \mp cx^2} \pm \frac{b^2 \pm 4ac}{8c}\int \frac{dx}{\sqrt{a + bx \mp cx^2}} + C$

85). $\int \frac{xdx}{\sqrt{a + bx \mp cx^2}} = \mp\frac{1}{c}\left(\sqrt{a + bx \mp cx^2} - \frac{b}{2}\int \frac{dx}{\sqrt{a + bx \mp cx^2}}\right) + C$

10. Другие алгебраические функции

86). $\int \sqrt{\frac{a+x}{b+x}}dx = \sqrt{(a+x)(b+x)} + (a-b)\ln\left|\sqrt{a+x} + \sqrt{b+x}\right| + C$

87). $\int \sqrt{\frac{a-x}{b+x}}dx = \sqrt{(a-x)(b+x)} + (a+b)\arcsin\sqrt{\frac{x+b}{a+b}} + C$

88). $\int \sqrt{\frac{a+x}{b-x}}dx = \sqrt{(a+x)(b-x)} - (a+b)\arcsin\sqrt{\frac{b-x}{a+b}} + C$

89). $\int \sqrt{\frac{1\pm x}{1\mp x}}dx = \pm\sqrt{1-x^2} + \arcsin x + C$

90). $\int \frac{dx}{\sqrt{(x-a)(b-x)}} = 2\arcsin\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right) + C$

90*). $\int \frac{dx}{\sqrt{(x-a)(x-b)}} = \ln\left|\frac{\sqrt{x-a}+\sqrt{x-b}}{\sqrt{x-a}-\sqrt{x-b}}\right| + C$

10*. Функции содержащие $\sqrt{ax^2 \pm b^2}$

$$\int \frac{dx}{\sqrt{a^2x^2 \pm b^2}} = \frac{1}{a}\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right| + C$$

$$\int \frac{dx}{\sqrt{b^2 - a^2x^2}} = \frac{1}{a}\arcsin\frac{ax}{b} + C$$

$$\int \frac{xdx}{\sqrt{a^2x^2 \pm b^2}} = \frac{1}{a^2}\sqrt{a^2x^2 \pm b^2} + C$$

$$\int \frac{xdx}{\sqrt{b^2 - a^2x^2}} = -\frac{1}{a^2}\sqrt{b^2 - a^2x^2} + C$$

$$\int \frac{x^2dx}{\sqrt{a^2x^2 \pm b^2}} = \frac{1}{2a^3}\left(ax\sqrt{a^2x^2 \pm b^2} \mp b^2\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right|\right) + C$$

$$\int \frac{x^2dx}{\sqrt{b^2 - a^2x^2}} = \frac{1}{2a^3}\left(-ax\sqrt{b^2 - a^2x^2} + b^2\arcsin\frac{ax}{b}\right) + C$$

$$\int \sqrt{a^2x^2 \pm b^2}dx = \frac{1}{2}\left(x\sqrt{a^2x^2 \pm b^2} \pm \frac{b^2}{a}\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right|\right) + C$$

$$\int \frac{x^2dx}{\sqrt{b^2 - a^2x^2}} = \frac{1}{2}\left(x\sqrt{b^2 - a^2x^2} + \frac{b^2}{a}\arcsin\frac{ax}{b}\right) + C$$

$$\int x\sqrt{a^2x^2 \pm b^2}dx = \frac{1}{3a^2}\sqrt{\left(a^2x^2 \pm b^2\right)^3} + C$$

$$\int x\sqrt{b^2 - a^2x^2}dx = -\frac{1}{3a^2}\sqrt{\left(b^2 - a^2x^2\right)^3} + C$$

$$\int x^2\sqrt{a^2x^2 \pm b^2}dx = \frac{1}{8a^3}\left(ax\left(2a^2x^2 \pm b^2\right)\sqrt{a^2x^2 \pm b^2} - b^4\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right|\right) + C$$

$$\int \frac{\sqrt{a^2x^2 + b^2}}{x}dx = \sqrt{a^2x^2 + b^2} - \frac{b}{2}\ln\left|\frac{\sqrt{a^2x^2 + b^2} - b}{\sqrt{a^2x^2 + b^2} + b}\right| + C = \sqrt{a^2x^2 + b^2} - b\ln\left|\frac{\sqrt{a^2x^2 + b^2} - b}{ax}\right| + C$$

$$\int \frac{\sqrt{a^2x^2 - b^2}}{x}dx = \sqrt{a^2x^2 - b^2} - b\arcsin\left|\frac{b}{ax}\right| + C$$

$$\int \frac{\sqrt{b^2 - a^2x^2}}{x}dx = \sqrt{b^2 - a^2x^2} - b\ln\left|\frac{b - \sqrt{b^2 - a^2x^2}}{x}\right| + C$$

$$\int \frac{\sqrt{a^2x^2 \pm b^2}}{x^2}dx = -\frac{\sqrt{a^2x^2 \pm b^2}}{x} + a^2\int \frac{dx}{\sqrt{a^2x^2 \pm b^2}} + C$$

$$\int \frac{\sqrt{b^2 - a^2x^2}}{x^2}dx = -\frac{\sqrt{b^2 - a^2x^2}}{x} - a^2\int \frac{dx}{\sqrt{b^2 - a^2x^2}} + C$$

$$\int \frac{dx}{x\sqrt{a^2x^2 + b^2}} = \frac{1}{b}\ln\left|\frac{x}{b + \sqrt{a^2x^2 + b^2}}\right| + C$$

$$\int \frac{dx}{x\sqrt{a^2x^2 - b^2}} = -\frac{1}{b}\arcsin\left|\frac{b}{ax}\right| + C$$

$$\int \frac{dx}{x\sqrt{b^2 - a^2x^2}} = \frac{1}{b}\ln\left|\frac{x}{b + \sqrt{b^2 - a^2x^2}}\right| + C$$

$$\int \frac{dx}{x^2\sqrt{a^2x^2 \pm b^2}} = \mp\frac{\sqrt{a^2x^2 \pm b^2}}{b^2x} + C$$

$$\int \frac{dx}{x^2\sqrt{b^2 - a^2x^2}} = -\frac{\sqrt{b^2 - a^2x^2}}{b^2x} + C$$

11. Показательные и тригонометрические функции

91). $\int a^{bx}dx = \frac{a^{bx}}{b\ln a} + C$

92). $\int e^x dx = e^x + C$

92*). $\int e^{a/x}\left(1 - \frac{a}{x}\right)dx = e^{a/x} + C$

92*). $\int \frac{e^x}{x}dx = \mathrm{Ei}\,x + C$, экспоненциальный интеграл

93). $\int e^{ax}dx = \frac{e^{ax}}{a} + C$

93*). $\int xe^x dx = xe^x - e^x + C$

93*). $\int x^n e^{ax}dx = \frac{x^n e^{ax}}{a} + \sum_{i=n}^{1}(-1)^{n-i+1}\frac{x^{i-1}\prod_{j=n}^{i} j}{a^{n-i+1}}e^{ax} + C$

93*). $\int \frac{dx}{e^x} = -e^{-x} + C$

94). $\int \sin x dx = -\cos x + C$

94*). $\int \frac{\sin x}{x}dx = \mathrm{Si}\,x + C$

95). $\int \cos x dx = \sin x + C$

95*). $\int \frac{\sin x}{x}dx = \mathrm{Ci}\,x + C$

96). $\int \tan x dx = -\ln\left|\cos x\right| + C$

97). $\int \cot x dx = \ln\left|\sin x\right| + C$

$$\int \sec x dx = \int \frac{dx}{\cos x} = \ln\left|\sec x + \tan x\right| + C$$

98).
$$= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$$

$$= \ln\left|\cos\frac{x}{2} + \sin\frac{x}{2}\right| - \ln\left|\cos\frac{x}{2} - \sin\frac{x}{2}\right| + C$$

99). $\int \csc x dx = \ln\left|\csc x + \cot x\right| + C = \ln\left|\tan\frac{x}{2}\right| + C$

100). $\int \sec^2 x dx = \int \frac{dx}{\cos^2 x} = \tan x + C$

101). $\int \csc^2 x dx = \int \frac{dx}{\sin^2 x} = -\cot x + C$

102). $\int \sec x \tan x dx = \int \frac{\sin x}{\cos^2 x} dx = \sec x + C = \frac{1}{\cos x} + C$

103). $\int \csc x \cot x dx = \int \frac{\cos x}{\sin^2 x} dx = -\csc x + C = -\frac{1}{\sin x} + C$

104). $\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x + C$

105). $\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$

106). $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx + C$

107). $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x dx + C$

108). $\int \frac{dx}{\sin^n x} = -\frac{1}{n-1}\frac{\cos x}{\sin^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} x} + C$

109). $\int \frac{dx}{\cos^n x} = \frac{1}{n-1}\frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x} + C$

110). $\int \sin x \cos^n x dx = -\frac{\cos^{n+1} x}{n+1} + C$

111). $\int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1} + C$

112). $\int \sin^n x \cos^m x dx = \begin{cases} \frac{\cos^{m-1} x \sin^{m+1} x}{m+n} + \frac{m-1}{m+n}\int \cos^{m-2} x \sin^n x dx + C, \text{если } m < n \\ -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n}\int \cos^m x \sin^{n-2} x dx + C, \text{если } n < m \end{cases}$

114). $\int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(n+m)} + \frac{\sin(m-n)x}{2(m-n)} + C, (m \neq n)$

115). $\int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(n+m)} + \frac{\sin(m-n)x}{2(m-n)} + C, (m \neq n)$

116). $\int \sin mx \cos nx dx = -\frac{\cos(m+n)x}{2(n+m)} - \frac{\cos(m-n)x}{2(m-n)} + C, (m \neq n)$

117). $\int \frac{dx}{a + b\cos x} = \begin{cases} \frac{2}{\sqrt{a^2 - b^2}} \arctan\left(\sqrt{\frac{a-b}{a+b}} \tan\frac{x}{2}\right) + C, \text{если } a > b \\ \frac{1}{\sqrt{b^2 - a^2}} \ln\left|\frac{\sqrt{b-a}\tan\frac{x}{2} + \sqrt{b+a}}{\sqrt{b-a}\tan\frac{x}{2} - \sqrt{b+a}}\right| + C, \text{если } a < b \end{cases}$

119). $\int \frac{dx}{a + b\sin x} = \begin{cases} \frac{2}{\sqrt{a^2 - b^2}} \arctan\left(\frac{a\tan\frac{x}{2} + b}{\sqrt{a^2 - b^2}}\right) + C, \text{если } a > b \\ \frac{1}{\sqrt{b^2 - a^2}} \ln\left|\frac{a\tan\frac{x}{2} + b - \sqrt{b^2 - a^2}}{a\tan\frac{x}{2} + b + \sqrt{b^2 - a^2}}\right| + C, \text{если } a < b \end{cases}$

121). $\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{1}{ab}\arctan\left(\frac{b\tan x}{a}\right) + C$

122). $\int e^x \sin x dx = \frac{e^x(\sin x - \cos x)}{2} + C$

123). $\int e^{ax} \sin nx dx = \frac{e^{ax}(a\sin nx - n\cos nx)}{a^2 + n^2} + C$

124). $\int e^x \cos x dx = \frac{e^x(\sin x + \cos x)}{2} + C$

125). $\int e^{ax} \cos nx dx = \frac{e^{ax}(n\sin nx + a\cos nx)}{a^2 + n^2} + C$

126). $\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + C$

127). $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax} + C$

128). $\int xa^{mx} dx = \frac{xa^{mx}}{m\ln x} - \frac{a^{mx}}{m(\ln a)^2} + C$

129). $\int xa^{mx} dx = \frac{a^{mx} x^n}{n\ln a} - \frac{n}{m\ln a}\int a^{mx} x^{n-1} dx + C$

130). $\int e^{ax} \cos^n xdx = \frac{e^{ax}\cos^{n-1} x(n\sin x + a\cos x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2}\int e^{ax} \cos^{n-2} xdx + C$

131). $\int \sinh x dx = \cosh x + C$

132). $\int \cosh x dx = \sinh x + C$

133). $\int \tanh x dx = \ln|\cosh x| + C$

134). $\int \coth x dx = \ln|\sinh x| + C$

135). $\int \operatorname{sech} x dx = 2\arctan e^x + C$

136). $\int \operatorname{csch} x dx = \ln\left|\tanh\frac{x}{2}\right| + C$

137). $\int \operatorname{sech}^2 x dx = \tanh x + C$

138). $\int \operatorname{csch}^2 x dx = -\coth x + C$

139). $\int \operatorname{sech} x \tan x dx = \operatorname{sech} x + C$

140). $\int \operatorname{csch} x \cot x dx = -\operatorname{csch} x + C$

141). $\int \sinh^2 x dx = -\frac{x}{2} + \frac{1}{4}\sinh 2x + C$

142). $\int \cosh^2 x dx = \frac{x}{2} + \frac{1}{4}\sinh^2 2x + C$

12. Логарифмические функции

143). $\int \ln x dx = x\ln x - x + C$

144). $\int \frac{dx}{x\ln x} = \ln(\ln x) + C$

145). $\int x^n \ln x dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right) + C$

146). $\int \ln^n x dx = x\ln^n x - n\int \ln^{n-1} x dx$

147). $\int x^m \ln^n x dx = \frac{x^{m+1}}{m+1}\ln^n x - \frac{n}{m+1}\int x^m \ln^{n-1} x dx$