

## **Формулы сложения**

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\operatorname{tg}(x+y) = \frac{\operatorname{tg}x + \operatorname{tg}y}{1 - \operatorname{tg}x \operatorname{tg}y}, \quad \text{npru } x, y, x+y \neq \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$$

$$\operatorname{tg}(x-y) = \frac{\operatorname{tg}x - \operatorname{tg}y}{1 + \operatorname{tg}x \operatorname{tg}y}, \quad \text{npru } x, y, x-y \neq \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$$

### **Метод вспомогательного аргумента**

$$a \sin x \pm b \cos x = \sqrt{a^2 + b^2} \times \sin(x \pm \varphi), \quad \varphi = \arcsin \frac{b}{\sqrt{a^2 + b^2}}$$

### **Формулы преобразования суммы в произведение**

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \times \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \times \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \times \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \times \sin \frac{x-y}{2}$$

$$\operatorname{tg} x + \operatorname{tg} y = \frac{\sin(x+y)}{\cos x \cos y} \quad \text{npru } x, y \neq \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$$

$$\operatorname{tg} x - \operatorname{tg} y = \frac{\sin(x-y)}{\cos x \cos y} \quad \text{npru } x, y \neq \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$$

$$\operatorname{ctg} x + \operatorname{ctg} y = \frac{\sin(x+y)}{\sin x \sin y} \quad \text{npru } x, y \neq \pi n, \quad n \in \mathbb{Z}$$

$$\operatorname{ctg} x - \operatorname{ctg} y = \frac{\sin(x-y)}{\sin x \sin y} \quad \text{npru } x, y \neq \pi n, \quad n \in \mathbb{Z}$$

$$\cos x + \sin x = \sqrt{2} \sin\left(\frac{\pi}{4} + x\right)$$

$$\cos x - \sin x = \sqrt{2} \cos\left(\frac{\pi}{4} + x\right)$$

$$1 + \sin 2x = 2 \cos^2\left(\frac{\pi}{4} - x\right)$$

$$1 - \sin 2x = 2 \sin^2\left(\frac{\pi}{4} - x\right)$$

$$1 + \operatorname{tg} x = \frac{\sqrt{2} \sin\left(\frac{\pi}{4} + x\right)}{\cos x}$$

$$1 - \operatorname{tg} x = \frac{\sqrt{2} \sin\left(\frac{\pi}{4} - x\right)}{\cos x}$$

### **Формулы преобразования произведения в сумму**

$$\begin{array}{ll} \sin x \times \cos y = \frac{1}{2} [\sin(x-y) + \sin(x+y)] & \sin mx \times \cos nx = \frac{1}{2} [\sin(m-n)x + \sin(m+n)x] \\ \cos x \times \cos y = \frac{1}{2} [\cos(x-y) + \cos(x+y)] & \cos mx \times \cos nx = \frac{1}{2} [\cos(m-n)x + \cos(m+n)x] \\ \sin x \times \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)] & \sin mx \times \sin nx = \frac{1}{2} [\cos(m-n)x - \cos(m+n)x] \end{array}$$

### **Формулы двойного аргумента**

$$\begin{aligned} \sin 2x &= 2 \sin x \times \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ \cos 2x &= 2 \cos^2 x - 1 \\ \cos 2x &= 1 - 2 \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \quad \text{при } x \neq \frac{\pi}{4} + \frac{\pi n}{2} \quad u \quad x \neq \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z} \end{aligned}$$

### **Формулы тройного аргумента**

$$\begin{aligned} \sin 3x &= 3 \sin x - 4 \sin^3 x \\ \cos 3x &= 4 \cos^3 x - 3 \cos x \\ \tan 3x &= \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \end{aligned}$$

### **Формулы понижения степени**

$$\begin{aligned} \sin^2 x &= \frac{1 - \cos 2x}{2} \\ \cos^2 x &= \frac{1 + \cos 2x}{2} \\ \sin^3 x &= \frac{3 \sin x - \sin 3x}{4} \\ \cos^3 x &= \frac{\cos 3x + 3 \cos x}{4} \end{aligned}$$