$\forall R>0 \quad \left|\int_R^\infty e^{-\alpha x}\frac{\sin x}{x}dx\right| = \left|\frac{\varphi(\alpha,x)}{x}\Big|_{x=R}^{\infty} + \int_R^\infty \frac{\varphi(\alpha,x)}{x^2}dx\right| = \left|\frac{\varphi(\alpha,R)}{R} + \int_R^\infty \frac{\varphi(\alpha,x)}{x}dx\right| \leq$

$\leq \frac{1}{R} + \int_R^\infty \frac{dx}{x^2} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} < \varepsilon.$

$R > \frac{2}{\varepsilon}$ фикс $\forall \varepsilon>0 \ \exists A = \frac{2}{\varepsilon} > 0 : \forall R > A$

$\left|\int_R^\infty f(x,\alpha)dx\right| < \varepsilon \quad \forall \alpha \geq 0 \Rightarrow$

$\Rightarrow J(\alpha)$ сх. равн. на $\alpha \geq 0$

$\underset{\text{т.5}}{\Rightarrow} J(\alpha)$ — непр. ф-я на $\alpha \geq 0$.

$\Rightarrow J(\alpha) \xrightarrow[\alpha\to 0+0]{} J(0) = J_0.$

(II) Посчитаем $J(\alpha)$ при $\alpha > 0$

$J(\alpha) = \int_0^\infty e^{-\alpha x}\frac{\sin x}{x}dx \qquad J'(\alpha) - ?$

~~Непр.~~ по опред.: $f(x,\alpha)$, $\frac{\partial f(x,\alpha)}{\partial \alpha}$ — непр. в $P_\infty = [0,\infty) \times [c,d]$

$J(\alpha)$ — сх. $\forall \alpha$ — выпол. $\int^\infty \frac{\partial f(x,\alpha)}{\partial \alpha}dx$ — равн.

сх. $\alpha \in [c,d] \Rightarrow \exists J'(\alpha) = \int_0^\infty \frac{\partial f(x,\alpha)}{\partial \alpha}dx$

f — непр. $x \geq 0$, $\alpha \geq 0$. $\quad \frac{\partial f(x,\alpha)}{\partial \alpha} = -e^{-\alpha x}\sin x$

непр. $\alpha, x \geq 0$.

$J(\alpha)$ — сх. $\forall \alpha > 0 \quad \frac{\partial f}{\partial \alpha} ?$

$\int_0^\infty e^{-\alpha x}\sin x dx \quad \begin{matrix}\alpha = 0 & \text{— расх.} \\ \alpha > 0 & \text{— нет равном. сх-ти}\end{matrix}$

фикс. $\forall \alpha_0 > 0 \ \exists [c,d] : 0 < c < \alpha_0 < d \quad \int_0^\infty e^{-\alpha x}\sin x dx$ на $\alpha \in [c,d]$

$P_\infty = [0,+\infty) \times [c,d]$ (=) ?

$|e^{-\alpha x}\sin x| \underset{\substack{\alpha\in[c,d] \\ x\geq 0}}{\leq} e^{-cx} \quad \int_0^\infty e^{-cx}dx$ — сх-ся. $\Rightarrow$

по пр. Вейерштрасса инт. сх-ся равномерно на $[c,d]$.

$\underset{\text{т.10}}{\Rightarrow} \exists J'(\alpha) = \int_0^\infty \frac{\partial f(x,\alpha)}{\partial \alpha}dx \Rightarrow$

$\Rightarrow$ в т. $\alpha_0 \ \exists J'(\alpha_0) = \int_0^\infty \frac{\partial f}{\partial \alpha}(x,\alpha_0)dx$

$\forall$ т. $\alpha_0 \Rightarrow$ для $\forall \alpha \ \exists J'(\alpha) = \int_0^\infty \frac{\partial f}{\partial \alpha}(x,\alpha)dx =$

$= -\int_0^\infty e^{-\alpha x}\sin x dx = -\varphi(\alpha,x)\Big|_{x=0}^{+\infty} = \varphi(\alpha,0) =$

$= \varphi(\alpha,0) = -\frac{1}{1+\alpha^2}$

$J'(\alpha) = -\frac{1}{1+\alpha^2}$ для $\forall \alpha > 0.$

$J(\alpha) = -\int\frac{d\alpha}{1+\alpha^2} = -\operatorname{arctg}\alpha + C$

$C - ?$ $\quad \alpha \to +\infty \quad J(\alpha) = \int_0^\infty e^{-\alpha x}\frac{\sin x}{x}dx$

Док.: $J(\alpha) \to 0$ при $\alpha \to +\infty$

$|J(\alpha)| \leq \int_0^\infty e^{-\alpha x}dx = -\frac{1}{\alpha}e^{-\alpha x}\Big|_0^\infty = \frac{1}{\alpha} \xrightarrow[\alpha\to+\infty]{} 0 \Rightarrow$

$\Rightarrow J(+\infty) = 0$

$C = J(\alpha) + \operatorname{arctg}\alpha \xrightarrow[\alpha\to+\infty]{} 0 + \frac{\pi}{2} \Rightarrow C = \frac{\pi}{2}.$

$J(\alpha) = \frac{\pi}{2} - \operatorname{arctg}\alpha$ для $\forall \alpha > 0.$

$J_0 = J(0) = \lim_{\alpha\to+0} J(\alpha) = \frac{\pi}{2}.$

$\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$