2) симметричность

$B(p,q) = B(q,p) \quad \forall p,q>0.$

$$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx = -\int_1^0 (1-t)^{p-1} t^{q-1} dt =$$

$$= \int_0^1 t^{q-1}(1-t)^{p-1} dt = B(q,p)$$

3) Формулы приведения

$B(p,q+1)$ для $\forall p,q>0$

$$B(p,q+1) = \int_0^1 x^{p-1}(1-x)^q dx =$$

$$= \frac{x^p}{p}(1-x)^q \Big|_{x=0}^{1} + \frac{q}{p}\int_0^1 x^p (1-x)^{q-1}dx =$$

$$= \{ x^p = x^{p-1} - x^{p-1} + x^p = x^{p-1} - x^{p-1}(1-x) \} =$$

$$= \frac{q}{p}\underbrace{\int_0^1 x^{p-1}(1-x)^{q-1}dx}_{B(p,q)} - \frac{q}{p}\underbrace{\int_0^1 x^{p-1}(1-x)^{q}dx}_{B(p,q+1)} =$$

$$= B(p,q+1) = \frac{q}{p}\cdot\frac{p}{p+q} B(p,q) = \frac{q}{p+q} B(p,q)$$

$$\frac{q}{p} B(p+1,q) = \cancel{B(p,q+1)} \quad B(p,q+1) = \frac{q}{p+q} B(p,q)$$

15.03.06 [Лекция 5]

п.3. Ф-ла связывающая Г и B - ф-ии Эйлера

$B(p,q) \quad (0,1) \to (0,+\infty)$

$x = \frac{1}{1+t}, \quad t = \frac{1-x}{x}$

$$1-x = 1 - \frac{1}{1+t} = \frac{t}{1+t}; \quad dx = -\frac{dt}{(1+t)^2}$$

$$B(p,q) = -\int_{+\infty}^0 \ldots dt = \int_0^{+\infty} \left(\frac{1}{1+t}\right)^{p-1}\left(\frac{t}{1+t}\right)^{q-1}\frac{dt}{(1+t)^2} =$$

$$= \int_0^{+\infty} \frac{t^{q-1}}{(1+t)^{p+q}}dt = \{B(p,q) = B(q,p)\} =$$

$$= \int_0^{+\infty} \frac{t^{p-1}}{(1+t)^{p+q}}dt, \quad \forall p,q>0$$

$\Gamma(p)$: $x = ty$, $t>0$ — параметр, $dx = t\,dy$

$$\Gamma(p) = \int_0^{+\infty} e^{-x}x^{p-1}dx = t\int_0^{+\infty} e^{-ty}(ty)^{p-1}dy =$$

$$= t^p \int_0^{+\infty} e^{-ty} y^{p-1}dy \Rightarrow$$

$$\frac{\Gamma(p)}{t^p} = \int_0^{+\infty} e^{-ty}y^{p-1}dy \quad \forall p>0, t>0 \qquad (1)$$

$$\frac{\Gamma(p+q)}{(1+t)^{p+q}} = \int_0^{+\infty} e^{-(1+t)y}y^{p+q-1}dy \qquad (2) \quad [\cdot t^{p-1}] \int_0^{+\infty} dt$$

$$\int_0^{+\infty} \Gamma(p+q)\frac{t^{p-1}}{(1+t)^{p+q}}dt = \Gamma(p+q)B(p,q) =$$

$$= \int_0^{+\infty} t^{p-1}dt \int_0^{+\infty} e^{-(1+t)y} y^{p+q-1}dy \overset{?}{=} \qquad (*)$$

$$= \int_0^{+\infty} y^{p+q-1}e^{-y}dy \int_0^{+\infty} t^{p-1}e^{-ty}dt \overset{(1)}{=} \int_0^{+\infty} y^{p+q-1}e^{-y}\frac{\Gamma(p)}{y^p}dy = \Gamma(p)\cdot\Gamma(q) \Rightarrow$$

$$\Rightarrow \boxed{B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}} \qquad (3)$$