

Лекция 1

$f(x) \in L[-\pi, \pi]$
 периодическая функция

Фурье (ФФ) $\sum_{k=1}^n (a_k \cos kx + b_k \sin kx)$

$$S_n(x, f) = \frac{a_0}{2} + \sum_{k=1}^n (a_k \cos kx + b_k \sin kx) \quad (1)$$

$$S_0(x, f) = \frac{a_0}{2}$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \cos ky \, dy$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \sin ky \, dy$$

(2)

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \sin ky \, dy, \quad k=1, 2, \dots$$

Удобнее

(2)

$$S_n(x, f) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \left(\frac{1}{2} + \sum_{k=1}^n (\cos ky \cos kx + \sin ky \sin kx) \right) dy$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \left(\frac{1}{2} + \sum_{k=1}^n \cos k(y-x) \right) dy$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x+t) \left(\frac{1}{2} + \sum_{k=1}^n \cos kt \right) dt \quad \left\{ \begin{array}{l} t = y-x \\ y = x+t \end{array} \right.$$

$f(x) \in R$

$$= \int_{-\pi}^{\pi} f(x+t) \left(\frac{1}{2} + \sum_{k=1}^n \cos kt \right) dt$$

$$\frac{1}{2} + \sum_{k=1}^n \cos kt = \frac{2 \sin \frac{t}{2}}{2 \sin \frac{t}{2}} = \frac{1}{2 \sin \frac{t}{2}} \left(\sin \frac{t}{2} + \sum_{k=1}^n 2 \cos k \sin \frac{t}{2} \right) =$$

$$= \frac{1}{2 \sin \frac{t}{2}} \left(\sin \frac{t}{2} + \sum_{k=1}^n (\sin(k+\frac{1}{2})t) - \sin(k-\frac{1}{2})t \right) =$$

$$= \frac{1}{2 \sin \frac{t}{2}} \left(\sin \frac{t}{2} + \sin \frac{3}{2}t - \sin \frac{t}{2} + \sin \frac{5}{2}t - \sin \frac{3}{2}t + \dots + \sin(n-\frac{1}{2})t - \sin(n-\frac{1}{2})t \right) =$$