$$\sigma_n(x_0,t) - \tilde{f}(x_0) \underset{\text{лемма 4}}{=} \int_{-\pi}^{\pi} f(x_0+t)\,\varphi_n(t)\,dt -$$

$$- \tilde{f}(x_0) \underbrace{\int_{-\pi}^{\pi} \varphi_n(t)\,dt}_{=1\ (\text{лемма } 4)} = \int_{\delta\le|t|\le\pi} \left(f(x_0+t) - \tilde{f}(x_0)\right)\varphi_n(t)\,dt +$$

$$+ \int_{-\delta}^{\delta} f(x_0+t)\,\varphi_n(t)\,dt - \frac{f(x_0+0)}{2}\int_{-\delta}^{\delta}\varphi_n\,dt -$$

$$- \frac{f(x_0-0)}{2}\int_{-\delta}^{\delta}\varphi_n\,dt =$$

$$= \left\{\int_{-\delta}^{\delta}\varphi_n(t)\,dt \underset{\text{лемма}}{=} 2\int_{0}^{\delta}\varphi_n(t)\,dt = 2\int_{-\delta}^{\delta}\varphi_n(t)\,dt\right\} =$$

$$= \int_{\delta\le|t|\le\pi}\left(f(x_0+t) - \tilde{f}(x_0)\right)\varphi_n(t)\,dt + \int_{0}^{\delta}\left(f(x_0+t) - f(x_0+0)\right)\varphi_n(t)\,dt$$

$$+ \int_{-\delta}^{\delta}\left(f(x_0+t) - f(x_0-0)\right)\varphi_n(t)\,dt \equiv J_1 + J_2 + J_3.$$

$$|J_1| \le \int_{\delta\le|t|\le\pi}\left(|f(x_0+t)| + |\tilde{f}(x_0)|\right)\varphi_n(t)\,dt \le$$

$$\le 2M\int_{\delta\le|t|\le\pi}\varphi_n(t)\,dt = 2M\left(\int_{-\pi}^{-\delta}\varphi_n(t)\,dt +$$

$$+ \int_{\delta}^{\pi}\varphi_n(t)\,dt\right) = 2M\left(\eta_n(\delta) + \eta_n(\delta)\right) =$$

$$= 4M\,\eta_n(\delta) \xrightarrow[\text{лемма 4}]{n\to\infty} 0 \Rightarrow \forall\varepsilon>0\ \exists N:$$

$$\forall n>N \quad |J_1| < \frac{\varepsilon}{2}.$$

$$|J_2 + J_3| \underset{(2)}{\le} \frac{\varepsilon}{2}\left(\int_{0}^{\delta}\varphi_n(t)\,dt + \int_{-\delta}^{0}\varphi_n(t)\,dt\right) =$$

$$= \frac{\varepsilon}{2}\int_{-\delta}^{\delta}\varphi_n(t)\,dt \underset{\varphi_n\ge 0}{\le} \frac{\varepsilon}{2}\underbrace{\int_{-\pi}^{\pi}\varphi_n(t)\,dt}_{1\ (\text{л.4})} = \frac{\varepsilon}{2}. \quad \forall n = 1,2,\ldots$$

$\forall\varepsilon>0\ \exists N;\ \forall n>N$

$$|\sigma_n(x_0,f) - \tilde{f}(x_0)| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \Rightarrow (1)$$

$\blacksquare$

<u>Следствие</u> $f \in L[-\pi,\pi]$ $2\pi\sim R$, $\exists f(x_0+0)$, $\sigma_n(x_0,f)$ сх $\Rightarrow$ $\Rightarrow \sigma_n(x_0,f) \to \dfrac{f(x_0+0)+f(x_0-0)}{2}$ (из рез-та непрер. теоремы)