Московский государственный университет имени М. В. Ломоносова
Факультет вычислительной математики и кибернетики
Кафедра системного программирования

Решение задач по курсу математической логики

Москва, 2009
by kayrick

Содержание

1 Формулы логики предикатов

Упражнение 1.1

1. $\Sigma = \{L^2\}$ $\quad L(x,y) - x$ *любит* y

   $\forall x\ L(x,x) \& (\forall x\ L(x,x) \quad \rightarrow \quad \exists y\ \exists z\ L(y,z))$

2. $\Sigma = \{P^1, M^1, S^1, C^2, a, b\}$ $\quad \left\{ \begin{array}{lll} P(x) & - & x \text{ - задача} \\ M(x) & - & x \text{ - математик} \\ S(x) & - & \text{задача } x \text{ - разрешима} \\ C(x,y) & - & \text{математик } x \text{ может решить задача } y \\ a & - & \text{константа «Я»} \\ b & - & \text{константа «эта задача»} \end{array} \right\}$

   $(\forall x(P(x) \& S(x) \ \rightarrow\ \exists y(M(y)\ \&\ C(y,x))))\ \&\ (M(a) \& \neg C(a,b))\ \rightarrow\ \neg S(b)$

3. $\Sigma = \{C^3, a\}$ $\quad \left\{ \begin{array}{lll} C(x,y,t) & - & x \text{ может обмануть } y \text{ в момент времени } t \\ a & - & \text{Константа «Вы»} \end{array} \right\}$

   $(\exists t\ \forall x\ C(a,x,t))\ \&\ (\exists x\ \forall t\ (Ca,x,t))\ \&\ \neg(\forall x\ \forall t\ C(a,x,t))$

Упражнение 1.2

1. $\exists x\ (\forall y\ (B(y)\ \&\ C(y)\ \&\ U(x,y)))\ \&\ S(x)$
2. $\forall x\ \forall y\ (B(x)\ \&\ S(x)\ \&\ W(y)\ \&\ C(y)\ \rightarrow\ \neg U(y,x))$
3. $\forall x\ (B(x)\ \rightarrow\ (S(x)\ \& (\forall y\ (W(y)\ \&\ C(y)\ \rightarrow\ U(y,x)))) \vee (C(x)\ \&\ (\exists\ y\ (S(y)\ \&\ U(x,y)))))$
4. $\forall x\ \forall y\ (B(x)\ \&\ C(x)\ \&\ W(y)\ \&\ S(y)\ \rightarrow \neg(U(x,y) \vee U(y,x)))$
5. $(\forall x\ (S(x)\ \rightarrow\ B(x)))\ \rightarrow\ (\forall y\ (C(y)\ \rightarrow\ \neg W(y)))$
6. $\forall x\ (\neg(C(x)\ \&\ W(x)\ \&\ (\exists z\ (S(z)\ \&\ U(x,z))))\ \rightarrow\ B(x)\ \&\ (\forall z\ (W(x)\ \rightarrow\ U(z,x))))$

Упражнение 1.3

1. $\forall x\ \forall y\ (P(x)\ \&\ P(y)\ \&\ \neg E(x,y)\ \rightarrow$
   $\exists k\ (L(k)\ \&\ B(x,k)\ \&\ B(y,k)\ \&\ (\forall s\ (L(s)\ \&\ B(x,s)\ \&\ B(y,s)\ \rightarrow\ E(k,s)))))$
2. $\forall i\ (P(i)\ \&\ L(x)\ \&\ L(y)\ \&\ B(i,x)\ \rightarrow\ \neg B(i,y))\quad [= Par(x,y)]$
3. $\forall x\ (L(x)\ \rightarrow\ \forall y\ (P(y)\ \&\ \neg B(y,x)\ \rightarrow\ \exists k\ (L(k)\ \&\ B(y,k)\ \&$
   $Par(x,k)\ \&\ \forall s\ (L(s)\ \&\ B(y,s)\ \&\ Par(x,s)\ \rightarrow\ E(k,s)))))$

Упражнение 1.4

1. $Z(x) = \forall y\ S(y,x,y)$
2. $O(x) = \forall y\ P(y,x,y)$
3. $T(x) = \exists k\ \forall y\ (P(y,k,y)\ \&\ S(k,k,x))$
4. $\exists y\ (Z(y)\ \&\ S(x,y,n))$
5. $\exists y\ \exists z\ (T(y)\ \&\ P(z,y,x))$
6. $(\forall k\ \forall l\ (P(k,l,x)\ \rightarrow (O(k) \vee O(l))))\ \&\ \neg(O(x) \vee Z(x)))$

1. $E(x,y) = \exists k\ ((\forall y\ S(y,k,y))\ \&\ S(x,k,y))$
2. $L(x,y) = \exists k\ ((\exists x\ \neg S(k,x,k))\ \&\ S(x,k,y))$
3. $F(x,y) = \exists k\ P(y,k,x)$

2 Вывод семантических таблиц

Упражнение 2.1

1. $\exists x\ P(x)\ \&\ \exists x\ \neg\ P(x)$

   - *Выполнима*
     $D_I = \{0, 1\},\ \overline{\boldsymbol{P}}(0) = \boldsymbol{true}, \overline{\boldsymbol{P}}(1) = \boldsymbol{false}$
   - *Не общезначима*
     $D_I = \{0\},\ \overline{\boldsymbol{P}}(0) = \boldsymbol{true}$

2. $\exists x\ P(x)\ \vee\ \exists x\ \neg\ P(x)$

   - *Общезначима*

$$\langle \emptyset \mid \exists x\ P(x)\ \vee\ \exists x\ \neg\ P(x)\rangle$$
$$\Big\downarrow R\vee$$
$$\langle \emptyset \mid \exists x\ P(x),\ \exists x\ \neg\ P(x)\rangle$$
$$\Big\downarrow R\exists$$
$$\langle \emptyset \mid \exists x\ P(x),\ \exists x\ \neg\ P(x), P(c_1)\rangle$$
$$\Big\downarrow R\exists$$
$$\langle \emptyset \mid \exists x\ P(x),\ \exists x\ \neg\ P(x),\ P(c_1),\ \neg P(c_1)\rangle$$
$$\Big\downarrow R\neg$$
$$\Big\langle \underline{P(c_1)} \mid \exists x\ P(x),\ \exists x\ \neg\ P(x),\ \underline{P(c_1)}\Big\rangle$$

   Закрытая таблица

3. $\exists x\ \forall y\ (P(x)\ \&\ \neg P(y))$

   - *Невыполнима*
     Докажем невыполнимость путем доказательства общезначимости отрицания

$$\langle \exists x\ \forall y\ (P(x)\ \&\ \neg P(y)) | \emptyset\rangle$$
$$\Big\downarrow L\exists$$
$$\langle \forall y\ (P(c_1)\ \&\ \neg P(y)) | \emptyset\rangle$$
$$\Big\downarrow L\forall$$
$$\langle \forall y\ (P(c_1)\ \&\ \neg P(y)), P(c_1)\ \&\ \neg P(c_1) | \emptyset\rangle$$
$$\Big\downarrow L\&$$
$$\langle \forall y\ (P(c_1)\ \&\ \neg P(y)), P(c_1),\ \neg P(c_1) | \emptyset\rangle$$
$$\Big\downarrow L\neg$$
$$\Big\langle \forall y\ (P(c_1)\ \&\ \neg P(y)), \underline{P(c_1)} | \underline{P(c_1)}\Big\rangle$$

   Закрытая таблица

4. $P(x)\ \rightarrow\ \forall x\ P(x)$

   - *Выполнима*
     $D_I = \{0\}, \overline{\boldsymbol{P}}(0) = \boldsymbol{true}$
   - *Не общезначима*
     $D_I = \{0, 1\}, \overline{\boldsymbol{P}}(0) = \boldsymbol{true}, \overline{\boldsymbol{P}}(1) = \boldsymbol{false}$

5. $\forall x\ P(x)\ \rightarrow\ P(x)$

- *Общезначима (очевидно)*

6. $\forall y\ \exists x\ R(x,y)\ \rightarrow\ \exists x\ \forall y\ R(x,y)$

   - *Выполнима*
     $D_I = \{0\},\ \overline{\boldsymbol{R}}(0,0) = \boldsymbol{true}$
   - *Не общезначима*
     $D_I = N,\ \overline{\boldsymbol{R}}(x,y) = \boldsymbol{x > y}$

7. $(\forall x\ P(x)\ \rightarrow\ \forall x\ Q(x))\ \rightarrow\ \forall x\ (P(x)\ \rightarrow\ Q(x))$

   - *Выполнима*
     $D_I = N,\ \overline{\boldsymbol{P}}(x) = \overline{\boldsymbol{Q}}(x)$
   - *Не общезначима*
     $D_I = N, \overline{\boldsymbol{P}}(x) = (\boldsymbol{x\ mod\ 2\ ==\ 0}),\ \overline{\boldsymbol{Q}}(x) = (\boldsymbol{x\ mod\ 4\ ==\ 0})$

Упражнение 2.2

1. $\exists x\ P(x)\ \rightarrow\ \neg\forall x\ \neg P(x)$

$$T_\phi = \langle \emptyset \mid \exists x\ P(x)\ \rightarrow\ \neg\forall x\ \neg P(x)\rangle$$
$$\Big\downarrow R\rightarrow$$
$$T_1 = \langle \exists x\ P(x) \mid \neg\forall x\ \neg P(x)\rangle$$
$$\Big\downarrow L\exists$$
$$T_2 = \langle P(c_1) \mid \neg\forall x\ \neg P(x)\rangle$$
$$\Big\downarrow R\neg$$
$$T_3 = \langle P(c_1),\ \forall x\ \neg P(x) \mid \emptyset\rangle$$
$$\Big\downarrow L\forall$$
$$T_4 = \langle P(c_1),\ \forall x\ \neg P(x),\ \neg P(c_1) \mid \emptyset\rangle$$
$$\Big\downarrow L\neg$$
$$T_5 = \left\langle \underline{P(c_1)},\ \forall x\ \neg P(x) \mid \underline{P(c_1)}\right\rangle$$

Закрытая таблица

2. $\exists x\forall y\ R(x,y)\ \rightarrow\ \forall y\exists x\ R(x,y)$

$$T_\phi = \langle \emptyset \mid \exists x\forall y\ R(x,y)\ \rightarrow\ \forall y\exists x\ R(x,y)\rangle$$
$$\Big\downarrow R\rightarrow$$
$$T_\phi = \langle \exists x\forall y\ R(x,y) \mid \forall y\exists x\ R(x,y)\rangle$$
$$\Big\downarrow L\exists$$
$$T_\phi = \langle \forall y\ R(c_1,y) \mid \forall y\exists x\ R(x,y)\rangle$$
$$\Big\downarrow R\forall$$
$$T_\phi = \langle \forall y\ R(c_1,y) \mid \exists x\ R(x,c_2)\rangle$$
$$\Big\downarrow R\exists$$
$$T_\phi = \langle \forall y\ R(c_1,y) \mid \exists x\ R(x,c_2),\ R(c_1,c_2)\rangle$$
$$\Big\downarrow L\forall$$
$$T_\phi = \left\langle \forall y\ R(c_1,y)\ ,\underline{R(c_1,c_2)} \mid \exists x\ R(x,c_2),\ \underline{R(c_1,c_2)}\right\rangle$$

Закрытая таблица

3. $\forall x\ (P(x)\ \to \exists y\ R(x, f(y)))\ \to\ (\exists x\ \neg P(x)\ \vee\ \forall x \exists z\ R(x,z))$

$$T_\phi = \langle \emptyset \mid \forall x\ (P(x)\ \to \exists y\ R(x, f(y)))\ \to\ (\exists x\ \neg P(x)\ \vee\ \forall x \exists z\ R(x,z))\rangle$$

$$\Big\downarrow R\to$$

$$T_1 = \langle \forall x\ (P(x)\ \to\ \exists y\ R(x, f(y))) \mid \exists x\ \neg P(x)\ \vee\ \forall x \exists z\ R(x,z)\rangle$$

$$\Big\downarrow R\vee$$

$$T_2 = \langle \forall x\ (P(x)\ \to\ \exists y\ R(x, f(y))) \mid \exists x\ \neg P(x),\ \forall x \exists z\ R(x,z)\rangle$$

$$\Big\downarrow R\forall$$

$$T_3 = \langle \forall x\ (P(x)\ \to\ \exists y\ R(x, f(y))) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z)\rangle$$

$$\downarrow L\forall$$

$$T_4 = \left\langle \overbrace{\forall x\ (P(x)\ \to\ \exists y\ R(x, f(y)))}^{\phi_1},\ P(c_1)\ \to\ \exists y\ R(c_1, f(y)) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z) \right\rangle$$

$$L\to \downarrow \qquad\qquad \downarrow L\to$$

$$T_{4.1} = \langle \phi_1,\ \exists y\ R(c_1, f(y)) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z)\rangle \qquad T_{4.2} = \langle \phi_1,\ \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z),\ P(c_1)\rangle$$

$$L\exists \Big\downarrow \qquad\qquad \Big\downarrow R\exists$$

$$T_{5.1} = \langle \phi_1,\ R(c_1, f(c_2)) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z)\rangle \qquad T_{5.2} = \langle \phi_1,\ \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z),\ P(c_1),\ \neg P(c_1)\rangle$$

$$R\exists \Big\downarrow \qquad\qquad \Big\downarrow R/$$

$$T_{5.1} = \left\langle \phi_1,\ \underline{R(c_1, f(c_2))} \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z), \underline{R(c_1, f(c_2))} \right\rangle \qquad T_{5.2} = \left\langle \phi_1,\ \underline{P(c_1)} \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z),\ \underline{P(c_1)} \right\rangle$$

Закрытые таблицы

4. $\forall x \exists y \forall z (P(x,y)\ \to\ P(y,z))$

$$T_\phi = \langle \emptyset \mid \forall x \exists y \forall z (P(x,y)\ \to\ P(y,z))\rangle$$

$$\Big\downarrow R\forall$$

$$T_1 = \langle \emptyset \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z))\rangle$$

$$\Big\downarrow R\exists$$

$$T_2 = \langle \emptyset \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)), \forall z (P(c_1,c_1)\ \to\ P(c_1,z))\rangle$$

$$\Big\downarrow R\forall$$

$$T_3 = \langle \emptyset \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)), P(c_1,c_1)\ \to\ P(c_1,c_2)\rangle$$

$$\Big\downarrow R\to$$

$$T_4 = \langle P(c_1,c_1) \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ P(c_1,c_2)\rangle$$

$$\Big\downarrow R\exists$$

$$T_5 = \langle P(c_1,c_1) \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ P(c_1,c_2), \forall z (P(c_1,c_2)\ \to\ P(c_2,z))\rangle$$

$$\Big\downarrow R\forall$$

$$T_6 = \langle P(c_1,c_1) \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ P(c_1,c_2), P(c_1,c_2)\ \to\ P(c_2,c_3)\rangle$$

$$\Big\downarrow R\to$$

$$T_7 = \left\langle P(c_1,c_1),\ \underline{P(c_1,c_2)} \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ \underline{P(c_1,c_2)},\ P(c_2,c_3) \right\rangle$$

Закрытая таблица

5. $\exists x \forall y \exists z (P(x,y) \ \to \ P(y,z))$

$$T_\phi = \left\langle \emptyset \mid \underbrace{\exists x \forall y \exists z (P(x,y) \ \to \ P(y,z))}_{\phi_1} \right\rangle$$

$$\downarrow R\exists$$

$$T_1 = \langle \emptyset \mid \phi_1, \ \forall y \exists z (P(c_1,y) \ \to \ P(y,z)) \rangle$$

$$\downarrow R\forall$$

$$T_2 = \left\langle \emptyset \mid \phi_1, \ \underbrace{\exists z (P(c_1,c_2) \ \to \ P(c_2,z))}_{\phi_2} \right\rangle$$

$$\downarrow R\exists$$

$$T_3 = \langle \emptyset \mid \phi_1, \ \phi_2, \ P(c_1,c_2) \ \to \ P(c_2,c_1), \ P(c_1,c_2) \ \to \ P(c_2,c_2) \rangle$$

$$\downarrow R\to$$

$$T_4 = \langle P(c_1,c_2) \mid \phi_1, \ \phi_2, \ P(c_2,c_1), \ P(c_2,c_2) \rangle$$

$$\downarrow R\exists \quad (\phi_1)$$

$$T_5 = \langle P(c_1,c_2) \mid \phi_1, \ \phi_2, \ P(c_2,c_1), \ P(c_2,c_2), \ \forall y \exists z (P(c_2,y) \ \to \ P(y,z)) \rangle$$

$$\downarrow R\forall$$

$$T_6 = \left\langle P(c_1,c_2) \mid \phi_1, \ \phi_2, \ P(c_2,c_1), \ P(c_2,c_2), \ \underbrace{\exists z (P(c_2,c_3) \ \to \ P(c_3,z))}_{\phi_3} \right\rangle$$

$$\downarrow R\exists$$

$$T_7 = \langle P(c_1,c_2) \mid \phi_1, \ \phi_2, \ \phi_3, \ P(c_2,c_1), \ P(c_2,c_2), \ P(c_2,c_3) \ \to \ P(c_3,c_3) \rangle$$

$$\downarrow R\to$$

$$T_8 = \langle P(c_1,c_2), \ P(c_2,c_3) \mid \phi_1, \ \phi_2, \ \phi_3, \ P(c_2,c_1), \ P(c_2,c_2), \ P(c_3,c_3) \rangle$$

$$\downarrow R\exists \quad (\phi_2)$$

$$T_9 = \langle P(c_1,c_2), \ P(c_2,c_3) \mid \phi_1, \ \phi_2, \ \phi_3, \ P(c_2,c_1), \ P(c_2,c_2), \ P(c_3,c_3), \ P(c_1,c_2) \ \to \ P(c_2,c_3) \rangle$$

$$\downarrow R\to$$

$$T_{10} = \left\langle P(c_1,c_2), \ \underline{P(c_2,c_3)} \mid \phi_1, \ \phi_2, \ \phi_3, \ P(c_2,c_1), \ P(c_2,c_2), \ P(c_3,c_3), \ \underline{P(c_2,c_3)} \right\rangle$$

Закрытая таблица

6. $\forall x \ (P(x) \ \& \ R(x)) \ \to \ (\forall x \ P(x) \ \& \ \forall x \ R(x))$

$$T_\phi = \langle \emptyset \mid \forall x\ (P(x)\ \&\ R(x))\ \rightarrow\ (\forall x\ P(x)\ \&\ \forall x\ R(x))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \forall x\ (P(x)\ \&\ R(x)) \mid \forall x\ P(x)\ \&\ \forall x\ R(x)\rangle$$

$$\Big\downarrow R\& \qquad \Big\downarrow R\&$$

$$T_{2.1} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid \forall x\ R(x)\rangle \quad T_{2.2} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid \forall x\ P(x)\rangle$$

$$\Big\downarrow R\forall \qquad \Big\downarrow R\forall$$

$$T_{2.1} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid R(c_1)\rangle \quad T_{2.2} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid P(c_1)\rangle$$

$$\Big\downarrow L\forall \qquad \Big\downarrow L\forall$$

$$T_{3.1} = \langle \forall x\ (P(x)\ \&\ R(x)),\ P(c_1)\ \&\ R(c_1) \mid R(c_1)\rangle \quad T_{3.2} = \langle \forall x\ (P(x)\ \&\ R(x)),\ P(c_1)\ \&\ R(c_1) \mid P(c_1)\rangle$$

$$\Big\downarrow L\& \qquad \Big\downarrow L\&$$

$$T_{3.1} = \Big\langle \forall x\ (P(x)\ \&\ R(x)),\ P(c_1),\ \underline{R(c_1)} \mid \underline{R(c_1)}\Big\rangle \quad T_{3.2} = \Big\langle \forall x\ (P(x)\ \&\ R(x)),\ \underline{P(c_1)},\ R(c_1) \mid \underline{P(c_1)}\Big\rangle$$

Закрытые таблицы

7. $(\forall x\ P(x)\ \&\ \forall x\ R(x))\ \rightarrow\ \forall x\ (P(x)\ \&\ R(x))$

$$T_\phi = \langle \emptyset \mid (\forall x\ P(x)\ \&\ \forall x\ R(x))\ \rightarrow\ \forall x\ (P(x)\ \&\ R(x))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \forall x\ P(x)\ \&\ \forall x\ R(x)) \mid \forall x\ (P(x)\ \&\ R(x))\rangle$$

$$\Big\downarrow L\&$$

$$T_2 = \langle \forall x\ P(x),\ \forall x\ R(x)) \mid \forall x\ (P(x)\ \&\ R(x))\rangle$$

$$\Big\downarrow R\forall$$

$$T_3 = \langle \forall x\ P(x),\ \forall x\ R(x)) \mid P(c_1)\ \&\ R(c_1)\rangle$$

$$\Big\downarrow L\forall$$

$$T_4 = \langle \forall x\ P(x),\ \forall x\ R(x),\ P(c_1),\ R(c_1) \mid P(c_1)\ \&\ R(c_1)\rangle$$

$$\Big\downarrow R\& \qquad \Big\downarrow R\&$$

$$T_{4.1} = \Big\langle \forall x\ P(x),\ \forall x\ R(x),\ P(c_1),\ \underline{R(c_1)} \mid \underline{R(c_1)}\Big\rangle \quad T_{4.2} = \Big\langle \forall x\ P(x),\ \forall x\ R(x),\ \underline{P(c_1)},\ R(c_1) \mid \underline{P(c_1)}\Big\rangle$$

Закрытые таблицы

8. $\exists x\ (P(x)\ \vee\ R(x))\ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ R(x))$

$$T_\phi = \langle \emptyset \ |\exists x\ (P(x)\ \vee\ R(x))\ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ R(x))\ \rangle$$

$$\downarrow R\rightarrow$$

$$T_1 = \langle \exists x\ (P(x)\ \vee\ R(x))\ |\ \exists x\ P(x)\ \vee\ \exists x\ R(x)\ \rangle$$

$$\downarrow L\exists$$

$$T_2 = \langle P(c_1)\ \vee\ R(c_1)\ |\ \exists x\ P(x)\ \vee\ \exists x\ R(x)\ \rangle$$

$$\downarrow R\vee$$

$$T_3 = \langle P(c_1)\ \vee\ R(c_1)\ |\ \exists x\ P(x),\ \exists x\ R(x)\ \rangle$$

$$\downarrow R\exists$$

$$T_4 = \langle P(c_1)\ \vee\ R(c_1)\ |\ \exists x\ P(x),\ \exists x\ R(x),\ P(c_1),\ R(c_1)\ \rangle$$

$$\downarrow L\vee \qquad \downarrow L\vee$$

$$T_{5.1} = \left\langle \underline{R(c_1)}\ |\ \exists x\ P(x),\ \exists x\ R(x),\ P(c_1),\ \underline{R(c_1)}\ \right\rangle \quad T_{5.2} = \left\langle \underline{P(c_1)}\ |\ \exists x\ P(x),\ \exists x\ R(x),\ \underline{P(c_1)},\ R(c_1)\ \right\rangle$$

Закрытые таблицы

9. $(\exists x\ P(x)\ \vee\ \exists x\ R(x))\ \rightarrow\ \exists x\ (P(x)\ \vee\ R(x))$

$$T_\phi = \langle \emptyset\ |\ (\exists x\ (P(x)\ \vee\ \exists x\ R(x))\ \rightarrow\ \exists x\ (P(x)\ \vee\ R(x))\rangle$$

$$\downarrow R\rightarrow$$

$$T_1 = \langle \exists x\ (P(x)\ \vee\ \exists x\ R(x)\ |\ \exists x\ (P(x)\ \vee\ R(x))\rangle$$

$$\downarrow L\vee \qquad \downarrow L\vee$$

$$T_{2.1} = \langle \exists x\ R(x)\ |\ \exists x\ (P(x)\ \vee\ R(x))\rangle \quad T_{2.2} = \langle \exists x\ P(x)\ |\ \exists x\ (P(x)\ \vee\ R(x))\rangle$$

$$\downarrow L\exists \qquad \downarrow L\exists$$

$$T_{3.1} = \langle R(c_1)\ |\ \exists x\ (P(x)\ \vee\ R(x))\rangle \quad T_{3.2} = \langle P(c_1)\ |\ \exists x\ (P(x)\ \vee\ R(x))\rangle$$

$$\downarrow R\exists \qquad \downarrow R\exists$$

$$T_{4.1} = \langle R(c_1)\ |\ \exists x\ (P(x)\ \vee\ R(x)),\ P(c_1)\ \vee\ R(c_1)\rangle \quad T_{4.2} = \langle P(c_1)\ |\ \exists x\ (P(x)\ \vee\ R(x)),\ P(c_1)\ \vee\ R(c_1)\rangle$$

$$\downarrow R\vee \qquad \downarrow R\vee$$

$$T_{5.1} = \left\langle \underline{R(c_1)}\ |\ \exists x\ (P(x)\ \vee\ R(x)),\ P(c_1),\ \underline{R(c_1)}\right\rangle \quad T_{5.2} = \left\langle \underline{P(c_1)}\ |\ \exists x\ (P(x)\ \vee\ R(x)),\ \underline{P(c_1)},\ R(c_1)\right\rangle$$

Закрытые таблицы

10. $(\forall x\ P(x)\ \vee\ R(y))\ \rightarrow\ \forall x\ (P(x)\ \vee\ R(y))$

$$T_\phi = \langle \emptyset \mid (\forall x\ P(x)\ \vee\ R(y))\ \to\ \forall x\ (P(x)\ \vee\ R(y)) \rangle$$

$$\Big\downarrow R\to$$

$$T_1 = \langle \forall x\ P(x)\ \vee\ R(y) \mid \forall x\ (P(x)\ \vee\ R(y)) \rangle$$

$$\Big\downarrow R\forall$$

$$T_2 = \langle \forall x\ P(x)\ \vee\ R(y) \mid P(c_1)\ \vee\ R(y) \rangle$$

$$\Big\downarrow R\vee$$

$$T_3 = \langle \forall x\ P(x)\ \vee\ R(y) \mid P(c_1),\ R(y) \rangle$$

$$\Big\downarrow R\vee \qquad \Big\downarrow R\vee$$

$$T_{4.1} = \langle \forall x\ P(x) \mid P(c_1),\ R(y) \rangle \quad T_{4.2} = \Big\langle \underline{R(y)} \mid P(c_1),\ \underline{R(y)} \Big\rangle$$

$$\Big\downarrow R\forall$$

$$T_{5.1} = \Big\langle \forall x\ P(x),\ \underline{P(c_1)} \mid \underline{P(c_1)},\ R(y) \Big\rangle$$

Закрытые таблицы

11. $\forall x\ (P(x)\ \vee\ R(y))\ \to\ (\forall x\ P(x)\ \vee\ R(y))$

$$T_\phi = \langle \emptyset \mid \forall x\ (P(x)\ \vee\ R(y))\ \to\ (\forall x\ P(x)\ \vee\ R(y)) \rangle$$

$$\Big\downarrow R\to$$

$$T_1 = \langle \forall x\ (P(x)\ \vee\ R(y)) \mid (\forall x\ P(x)\ \vee\ R(y)) \rangle$$

$$\Big\downarrow R\vee$$

$$T_2 = \langle \forall x\ (P(x)\ \vee\ R(y)) \mid \forall x\ P(x),\ R(y) \rangle$$

$$\Big\downarrow R\forall$$

$$T_3 = \langle \forall x\ (P(x)\ \vee\ R(y)) \mid P(c_1),\ R(y)) \rangle$$

$$\Big\downarrow L\forall$$

$$T_4 = \langle \forall x\ (P(x)\ \vee\ R(y)),\ P(c_1)\ \vee\ R(y) \mid P(c_1),\ R(y)) \rangle$$

$$\Big\downarrow R\vee \qquad \Big\downarrow R\vee$$

$$T_{5.1} = \Big\langle \forall x\ (P(x)\ \vee\ R(y)),\ \underline{P(c_1)} \mid \underline{P(c_1)},\ R(y) \Big\rangle \quad T_{5.2} = \Big\langle \forall x\ (P(x)\ \vee\ R(y)),\ \underline{R(y)} \mid P(c_1),\ \underline{R(y)} \Big\rangle$$

Закрытые таблицы

12. $\exists y \forall x\ Q(x,y)\ \to\ \forall x \exists y\ Q(x,y)$

$$T_\phi = \langle \emptyset \mid \exists y \forall x\ Q(x,y)\ \rightarrow\ \forall x \exists y\ Q(x,y) \rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \exists y \forall x\ Q(x,y) \mid \forall x \exists y\ Q(x,y) \rangle$$

$$\Big\downarrow R\forall$$

$$T_2 = \langle \exists y \forall x\ Q(x,y) \mid \exists y\ Q(c_1,y) \rangle$$

$$\Big\downarrow L\exists$$

$$T_3 = \langle \forall x\ Q(x,c_2) \mid \exists y\ Q(c_1,y) \rangle$$

$$\Big\downarrow L\forall$$

$$T_4 = \langle \forall x\ Q(x,c_2),\ Q(c_1,c_2) \mid \exists y\ Q(c_1,y) \rangle$$

$$\Big\downarrow R\exists$$

$$T_4 = \left\langle \forall x\ Q(x,c_2),\ \underline{Q(c_1,c_2)} \mid \exists y\ Q(c_1,y),\ \underline{Q(c_1,c_2)} \right\rangle$$

Закрытая таблица

Упражнение 2.3

1. $\forall x\ (P(x)\ \vee\ Q(x))\ \rightarrow\ (\forall x\ P(x)\ \vee\ \forall x\ Q(x))$
   Вывод не будет успешным так как формула не общезначима.
   $D_I = N, \overline{\boldsymbol{P}}(x) = (\boldsymbol{x\ mod\ 2 == 0}),\ \overline{\boldsymbol{Q}}(x) = (\boldsymbol{x\ mod\ 2 == 1})$

2. $\exists x\ (P(x)\ \vee\ Q(x))\ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ Q(x))$
   Построим вывод

$$\langle \emptyset | \exists x\ (P(x)\ \vee\ Q(x))\ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ Q(x)) \rangle$$

$$\Big\downarrow R\rightarrow$$

$$\langle \exists x\ (P(x)\ \vee\ Q(x)) | \exists x\ P(x)\ \vee\ \exists x\ Q(x) \rangle$$

$$\Big\downarrow \exists$$

$$\langle P(c_1)\ \vee\ Q(c_1) | \exists x\ P(x)\ \vee\ \exists x\ Q(x) \rangle$$

$$\Big\downarrow R\vee$$

$$\langle P(c_1)\ \vee\ Q(c_1) | \exists x\ P(x),\ \exists x\ Q(x) \rangle$$

$$\Big\downarrow R\exists$$

$$\langle P(c_1)\ \vee\ Q(c_1) | \exists x\ P(x),\ \exists x\ Q(x),\ P(c_1),\ Q(c_1) \rangle$$

$$L\vee \Big\downarrow \quad \Big\downarrow L\vee$$

$$\left\langle \underline{P(c_1)} | \exists x\ P(x),\ \exists x\ Q(x),\ \underline{P(c_1)},\ Q(c_1) \right\rangle \left\langle \underline{Q(c_1)} | \exists x\ P(x),\ \exists x\ Q(x),\ P(c_1),\ \underline{Q(c_1)} \right\rangle$$

Упражнение 2.4 *Пусть такая формула существует. Рассмотрим ее на интерпретации, область которой содержит три элемента. На данной интерпретации формула истинна. То есть для любой подстановки она истинна. Следовательно существует подстановка, состоящая из 1 или 2 объектов, на которой формула так же истинна. Следовательно формула истинна на интерпретации, область которой содержит только эти 2 объекта, следовательно такой формулы нет.*

Упражнение 2.5

$$\exists x_1 \exists x_2 \exists x_3 \exists x_4 \exists x_5 \left(\underset{y_i \in \{x_1,x_2,x_3,x_4,x_5\}}{\&} P(y_1,y_2,y_3,y_4,y_5) \right)\ \rightarrow\ \forall x_1 \forall x_2 \forall x_3 \forall x_4 \forall x_5 P(x_1,x_2,x_3,x_4,x_5)$$

3 Нормальные формы и унификация

3.1 Приведение к ССФ

1. Переименование переменных
   $\models \ \substack{\exists\\\forall} \ x \ F(x) \ \equiv \ \substack{\exists\\\forall} \ y \ F(y)$

2. Уничтожение импликаций $\models \ (A \ \rightarrow \ B) \ \equiv \ (\neg A \ \vee \ B)$

3. Отрицания

   (a) $\models \ \neg(A \substack{\&\\\vee} B) \ \equiv \ (\neg A \substack{\vee\\\&} \neg B)$

   (b) $\models \ (\neg \substack{\exists\\\forall} x F(x)) \equiv (\substack{\forall\\\exists} x \neg F(x)$

   (c) $\models \ \neg\neg A \ \equiv \ A$

4. Вынос кванторов
   $\models \ \substack{\exists\\\forall} x F(x) \ \substack{\&\\\vee} \ B \ \equiv \ \substack{\exists\\\forall} x (F(x) \ \substack{\&\\\vee} \ B)$

5. $\models \ A \& B \ \vee \ C \ \equiv (A \vee C) \ \& \ (B \vee C)$

3.2 Нахождение НОУ

$$P(t_1, t_2, \dots t_n) \quad = \quad P(s_1, s_2, \dots s_n) \qquad \rightarrow \quad \begin{cases} t_1 & = & s_1 \\ t_2 & = & s_2 \\ \dots & \dots & \dots \\ t_n & = & s_n \end{cases}$$

1. $\{ \ f(t_1, t_2, \dots t_n) \quad = \quad f(s_1, s_2, \dots s_n) \qquad \rightarrow \quad \begin{cases} t_1 & = & s_1 \\ t_2 & = & s_2 \\ \dots & \dots & \dots \\ t_n & = & s_n \end{cases}$

2. $\{ \ f(t_1, t_2, \dots t_n) \quad = \quad f(s_1, s_2, \dots s_k) \qquad \rightarrow$ НОУ не существует

3. $t_i = x_i \quad \rightarrow \quad x_1 = t_i \quad (t_i \neq x_i)$

4. $t_i = t_i \quad \rightarrow \quad \emptyset$

5. $x_i = t_i \quad (x_i \notin Var_{t_i}, \ \exists k \ x_i \in Var_{t_k}) \quad \rightarrow$ Во все t_k подствить вместо x_i t_i

6. $x_i = t_i \quad (x_i \in Var_{t_i}) \quad \rightarrow$ НОУ не существует

3.3 Задачи

Упражнение 3.1

1. $\exists x \forall y \ P(x, y) \ \& \ \forall x \exists y \ P(y, x)$

$\exists x \forall y \ P(x, y) \ \& \ \forall x \exists y \ P(y, x) \ \xrightarrow{1} \ \exists x_1 \forall y_1 \ P(x_1, y_1) \ \& \ \forall x_2 \exists y_2 \ P(y_2, x_2)$
$\exists x_1 \forall y_1 \ P(x_1, y_1) \ \& \ \forall x_2 \exists y_2 \ P(y_2, x_2) \ \xrightarrow{4} \ \exists x_1 \forall y_1 \forall x_2 \exists y_2 \ P(x_1, y_1) \ \& \ P(y_2, x_2)$

2. $\forall x((\exists y \ P(y, x) \ \rightarrow \ \exists y \ P(x, y)) \ \rightarrow \ Q(x)) \ \rightarrow \ \exists x \ Q(x)$

$\forall x((\exists y \ P(y, x) \ \rightarrow \ \exists y \ P(x, y)) \ \rightarrow \ Q(x)) \ \rightarrow \ \exists x \ Q(x) \ \xrightarrow{1}$
$\forall x_1((\exists y_1 \ P(y_1, x_1) \ \rightarrow \ \exists y_2 \ P(x_1, y_2)) \ \rightarrow \ Q(x_1)) \ \rightarrow \ \exists x_2 \ Q(x_2) \ \xrightarrow{2,3}$
$\exists x_1 \ ((\forall y_1 \ \neg P(y_1, x_1) \ \vee \ \exists y_2 \ P(x_1, y_2)) \ \& \ \neg Q(x_1)) \ \vee \ \exists x_2 \ Q(x_2) \ \xrightarrow{4}$
$\exists x_1 \forall y_1 \exists y_2 \forall x_2 \ ((\neg P(y_1, x_1) \ \vee \ P(x_1, y_2)) \ \& \ \neg Q(x_1) \ \vee \ Q(x_2)) \ \xrightarrow{5}$
$\exists x_1 \forall y_1 \exists y_2 \forall x_2 \ ((\neg P(y_1, x_1) \ \vee \ P(x_1, y_2) \ \vee \ Q(x_2)) \ \& \ (\neg Q(x_1) \ \vee \ Q(x_2)))$

3. $\neg\forall y\ (\exists x\ P(x,y) \ \rightarrow\ \forall u\ (R(y,u) \ \rightarrow\ \forall z\ (P(z,u) \ \vee\ \neg R(z,y))))$

$$\neg\forall y\ (\exists x\ P(x,y) \ \rightarrow\ \forall u\ (R(y,u) \ \rightarrow\ \forall z\ (P(z,u) \ \vee\ \neg R(z,y)))) \xrightarrow{2,3}$$
$$\exists y\ (\exists x\ P(x)\ \&\ (\exists u(R(z,u)\ \&\ \forall z\ (P(z,u) \ \vee\ \neg R(z,y))))) \xrightarrow{4}$$
$$\exists y \exists x \exists u \forall z\ (P(x,y)\ \&\ R(y,u)\ \&\ (P(z,u) \ \vee\ R(z,y)))$$

4. $\exists x \exists y\ (P(x,y) \ \rightarrow\ R(x)) \ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y) \ \vee R(x))$

$$\exists x \exists y\ (P(x,y) \ \rightarrow\ R(x)) \ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y) \ \vee R(x)) \xrightarrow{1}$$
$$\exists x_1 \exists y_1\ (P(x_1,y_1) \ \rightarrow\ R(x_1)) \ \rightarrow\ \forall x_2\ (\neg\exists y_2\ P(x_2,y_2) \ \vee R(x_2)) \xrightarrow{2,3}$$
$$\forall x_1 \forall x_2\ (P(x_1,y_2)\ \&\ \neg R(x_1)) \ \vee\ \forall x_2 (\forall y_2\ \neg P(x_2,y_2) \ \vee\ R(x_2)) \xrightarrow{4}$$
$$\forall x_1 \forall x_2 \forall x_2 \forall y_2\ (P(x_1,y_2)\ \&\ \neg R(x_1) \ \vee\ \neg P(x_2,y_2) \ \vee\ R(x_2)) \xrightarrow{5}$$
$$\forall x_1 \forall x_2 \forall x_2 \forall y_2\ ((P(x_1,y_2) \ \vee\ \neg P(x_2,y_2) \ \vee\ R(x_2))\ \&\ (\neg R(x_1) \ \vee\ \neg P(x_2,y_2) \ \vee\ R(x_2)))$$

5. $\exists x \forall y\ (P(x,y) \ \rightarrow\ (\neg P(y,x) \ \rightarrow\ (P(x,x) \ \rightarrow\ P(y,y))\ \&(P(y,y) \ \rightarrow\ P(x,x))))$

$$\exists x \forall y\ (P(x,y) \ \rightarrow\ (\neg P(y,x) \ \rightarrow\ (P(x,x) \ \rightarrow\ P(y,y))\ \&(P(y,y) \ \rightarrow\ P(x,x)))) \xrightarrow{2,3}$$
$$\exists x \forall y\ (\neg P(x,y) \ \vee\ P(y,x) \ \vee (\neg P(x,x) \ \vee\ P(y,y))\ \&\ (\neg P(y,y) \ \vee\ P(x,x))) \xrightarrow{5}$$
$$\exists x \forall y\ ((\neg P(x,y) \ \vee\ P(y,x) \ \vee\ \neg P(x,x) \ \vee\ P(y,y))\ \&\ (\neg P(x,y) \ \vee\ P(y,x) \ \vee\ \neg P(y,y) \ \vee\ P(x,x)))$$

6. $\exists x\ (\forall x\ P(x,x) \ \vee\ \exists x\ \neg R(x)) \ \rightarrow\ \exists x\ (R(x) \ \rightarrow\ \exists y\ P(f(x),y))$

$$\exists x\ (\forall x\ P(x,x) \ \vee\ \exists x\ \neg R(x)) \ \rightarrow\ \exists x\ (R(x) \ \rightarrow\ \exists y\ P(f(x),y)) \xrightarrow{1}$$
$$\exists k\ (\forall x_1\ P(x_1,x_1) \ \vee\ \exists x_2\ \neg R(x_2)) \ \rightarrow\ \exists x_3\ (R(x_3) \ \rightarrow\ \exists y\ P(f(x_3),y)) \xrightarrow{2,3}$$
$$\exists k\ (\exists x_1\ \neg P(x_1,x_1)\ \&\ \forall x_2\ \neg R(x_2) \ \vee\ \exists x_3 (\neg R(x_3) \ \vee \exists y\ R(f(x_3),y))) \xrightarrow{4}$$
$$\exists k \exists x_1 \forall x_2 \exists x_3 \exists y\ (\neg P(x_1,x_1)\ \&\ \neg R(x_2) \ \vee\ \neg R(x_3) \ \vee\ R(f(x_3),y)) \xrightarrow{5}$$
$$\exists k \exists x_1 \forall x_2 \exists x_3 \exists y\ ((\neg P(x_1,x_1) \ \vee\ \neg R(x_3) \ \vee\ R(f(x_3),y))\ \&\ (\neg R(x_2) \ \vee\ \neg R(x_3) \ \vee\ R(f(x_3),y)))$$

Упражнение 3.2

1. $\forall x \exists y \forall z \exists u\ R(x,y,z,u)$

$$\forall x \exists y \forall z \exists u\ R(x,y,z,u) \longrightarrow$$
$$\forall x \forall z \exists u\ R(x,f(x),z,u) \longrightarrow$$
$$\forall x \forall z\ R(x,f(x),z,g(x,z))$$

2. $\neg\forall x\ (\exists y\ R(x,y) \ \rightarrow\ \forall z\ P(z,x)) \xrightarrow{2,3}$
$$\exists z\ (\exists y\ R(x,y)\ \& \exists z\ \neg P(z,x)) \xrightarrow{4}$$
$$\exists x \exists y \exists z\ (R(x,y)\ \&\ \neg P(z,x)) \longrightarrow$$
$$\exists y \exists z\ (R(c_1,y)\ \&\ \neg P(z,c_1)) \longrightarrow$$
$$\exists z\ (R(c_1,c_2)\ \&\ \neg P(z,c_1)) \longrightarrow$$
$$R(c_1,c_2)\ \&\ \neg P(_3,c_1)$$

3. $\neg\forall y\ (\exists x\ P(x,y) \ \rightarrow\ \forall u\ (R(y,u) \ \rightarrow\ \forall z\ (P(z,u) \ \vee\ \neg R(z,y))))$

$$\neg\forall y\ (\exists x\ P(x,y) \ \rightarrow\ \forall u\ (R(y,u) \ \rightarrow\ \forall z\ (P(z,u) \ \vee\ \neg R(z,y)))) \xrightarrow{2,3}$$
$$\exists y\ (\exists x\ P(x)\ \&\ (\exists u(R(z,u)\ \&\ \forall z\ (P(z,u) \ \vee\ \neg R(z,y))))) \xrightarrow{4}$$
$$\exists y \exists x \exists u \forall z\ (P(x,y)\ \&\ R(y,u)\ \&\ (P(z,u) \ \vee\ R(z,y))) \longrightarrow$$
$$\exists x \exists u \forall z\ (P(x,c_1)\ \&\ R(c_1,u)\ \&\ (P(z,u) \ \vee\ R(z,c_1))) \longrightarrow$$
$$\exists u \forall z\ (P(c_2,c_1)\ \&\ R(c_1,u)\ \&\ (P(z,u) \ \vee\ R(z,c_1))) \longrightarrow$$
$$\forall z\ (P(c_2,c_1)\ \&\ R(c_1,c_3)\ \&\ (P(z,c_3) \ \vee\ R(z,c_1)))$$

4. $\exists x \exists y\ (P(x,y) \ \rightarrow\ R(x)) \ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y) \ \vee R(x))$

$$\exists x \exists y\ (P(x,y) \ \rightarrow\ R(x)) \ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y) \ \vee R(x)) \xrightarrow{1}$$

$\exists x_1 \exists y_1\ (P(x_1, y_1)\ \to\ R(x_1))\ \to\ \forall x_2\ (\neg\exists y_2\ P(x_2, y_2)\ \vee R(x_2)) \xrightarrow{2,3}$
$\forall x_1 \forall x_2\ (P(x_1, y_2)\ \&\ \neg R(x_1))\ \vee\ \forall x_2 (\forall y_2\ \neg P(x_2, y_2)\ \vee\ R(x_2)) \xrightarrow{4}$
$\forall x_1 \forall x_2 \forall x_2 \forall y_2\ (P(x_1, y_2)\ \&\ \neg R(x_1)\ \vee\ \neg P(x_2, y_2)\ \vee\ R(x_2)) \xrightarrow{5}$
$\forall x_1 \forall x_2 \forall x_2 \forall y_2\ ((P(x_1, y_2)\ \vee\ \neg P(x_2, y_2)\ \vee\ R(x_2))\ \&\ (\neg R(x_1)\ \vee\ \neg P(x_2, y_2)\ \vee\ R(x_2)))$

5. $\exists x \forall y\ (P(x, y)\ \to\ (\neg P(y, x)\ \to\ (P(x, x)\ \to\ P(y, y))\ \&(P(y, y)\ \to\ P(x, x))))$

$\exists x \forall y\ (P(x, y)\ \to\ (\neg P(y, x)\ \to\ (P(x, x)\ \to\ P(y, y))\ \&(P(y, y)\ \to\ P(x, x)))) \xrightarrow{2,3}$
$\exists x \forall y\ (\neg P(x, y)\ \vee\ P(y, x)\ \vee (\neg P(x, x)\ \vee\ P(y, y))\ \&\ (\neg P(y, y)\ \vee\ P(x, x))) \xrightarrow{5}$
$\exists x \forall y\ ((\neg P(x, y)\ \vee\ P(y, x)\ \vee\ \neg P(x, x)\ \vee\ P(y, y))\ \&\ (\neg P(x, y)\ \vee\ P(y, x)\ \vee\ \neg P(y, y)\ \vee\ P(x, x))) \longrightarrow$
$\forall y\ ((\neg P(c, y)\ \vee\ P(y, c)\ \vee\ \neg P(c, c)\ \vee\ P(y, y))\ \&\ (\neg P(c, y)\ \vee\ P(y, c)\ \vee\ \neg P(y, y)\ \vee\ P(c, c)))$

6. $\exists x\ (\forall x\ P(x, x)\ \vee\ \exists x\ \neg R(x))\ \to\ \exists x\ (R(x)\ \to\ \exists y\ P(f(x), y))$

$\exists x\ (\forall x\ P(x, x)\ \vee\ \exists x\ \neg R(x))\ \to\ \exists x\ (R(x)\ \to\ \exists y\ P(f(x), y)) \xrightarrow{1}$
$\exists k\ (\forall x_1\ P(x_1, x_1)\ \vee\ \exists x_2\ \neg R(x_2))\ \to\ \exists x_3\ (R(x_3)\ \to\ \exists y\ P(f(x_3), y)) \xrightarrow{2,3}$
$\exists k\ (\exists x_1\ \neg P(x_1, x_1)\ \&\ \forall x_2\ \neg R(x_2)\ \vee\ \exists x_3 (\neg R(x_3)\ \vee \exists y\ R(f(x_3), y))) \xrightarrow{4}$
$\exists k \exists x_1 \forall x_2 \exists x_3 \exists y\ (\neg P(x_1, x_1)\ \&\ \neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y)) \xrightarrow{5}$
$\exists k \exists x_1 \forall x_2 \exists x_3 \exists y\ ((\neg P(x_1, x_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y))) \longrightarrow$
$\exists x_1 \forall x_2 \exists x_3 \exists y\ ((\neg P(x_1, x_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y))) \longrightarrow$
$\forall x_2 \exists x_3 \exists y\ ((\neg P(c_1, c_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3), y))) \longrightarrow$
$\forall x_2 \exists y\ ((\neg P(c_1, c_1)\ \vee\ \neg R(g(x_2))\ \vee\ R(g(x_2), y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(g(x_2))\ \vee\ R(f(g(x_2)), y))) \longrightarrow$
$\forall x_2\ ((\neg P(c_1, c_1)\ \vee\ \neg R(g(x_2))\ \vee\ R(g(x_2), h(x_2))\ \&\ (\neg R(x_2)\ \vee\ \neg R(g(x_2))\ \vee\ R(f(g(x_2)), h(x_2))))$

Упражнение 3.3

1. $\theta_1 = \{x/f(x), y/g(x, z), u/v, v/f(c)\}, \quad \theta_2 = \{x/f(y), y/c, z/g(y, v), v/u\}$

$\theta = \{x/f(x)\theta_2, y/g(x, z)\theta_2, u/v\theta_2, v/f(c)\theta_2\}\ \cup\ \{z/g(y, v)\}$

$\theta = \{x/f(f(y)), y/g(f(y), g(y, v)), u/u, v/f(c)\}\ \cup\ \{z/g(y, v)\}$

$\theta = \{x/f(f(y)), y/g(f(y), g(y, v)), v/f(c), z/g(y, v)\}$

2. $\theta_1 = \{x/y\}, \quad \theta_2 = \{y/z, z/x, x/y\}$

$\theta = \{x/y\theta_2\}\ \cup\ \{y/z, z/x\}$

$\theta = \{x/z\}\ \cup\ \{y/z, z/x\}$

$\theta = \{x/z, y/z, z/x\}$

Упражнение 3.4

1. $P(c, X, f(X)) \qquad P(c, Y, Y)$

$$\left\{\begin{array}{ccc} c & = & c \\ X & = & Y \\ f(X) & = & Y \end{array}\right. \xrightarrow{4} \left\{\begin{array}{ccc} X & = & Y \\ f(X) & = & Y \end{array}\right. \xrightarrow{3}$$

$$\left\{\begin{array}{ccc} X & = & Y \\ Y & = & f(X) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} X & = & Y \\ \underline{Y} & = & \underline{f(Y)} \end{array}\right. \quad \textit{НОУ Нет}$$

2. $P(f(X, Y), Z, h(Z, Y)) \qquad P(f(Y, X), g(Y), V)$

$$\left\{\begin{array}{ccc} f(X, Y) & = & f(Y, X) \\ Z & = & g(Y) \\ h(Z, Y) & = & V \end{array}\right. \xrightarrow{1} \left\{\begin{array}{ccc} X & = & Y \\ Y & = & X \\ Z & = & g(Y) \\ h(Z, Y) & = & V \end{array}\right. \xrightarrow{1}$$

$$\begin{cases} X &= Y \\ Y &= Y \\ Z &= g(Y) \\ h(Z,Y) &= V \end{cases} \xrightarrow{4} \begin{cases} X &= Y \\ Z &= g(Y) \\ h(Z,Y) &= V \end{cases} \xrightarrow{3}$$

$$\begin{cases} X &= Y \\ Z &= g(Y) \\ V &= h(Z,Y) \end{cases} \xrightarrow{3} \begin{cases} X &= Y \\ Z &= g(Y) \\ V &= h(g(Y),Y) \end{cases} \quad \text{НОУ построен}$$

3. $R(Z, f(X,b,Z)) \qquad R(h(X), f(g(a), Y, Z))$

$$\begin{cases} Z &= h(X) \\ f(X,b,Z) &= f(g(a),Y,Z) \end{cases} \xrightarrow{1} \begin{cases} Z &= h(X) \\ X &= g(a) \\ b &= Y \\ Z &= Z \end{cases} \xrightarrow{4}$$

$$\begin{cases} Z &= h(X) \\ X &= g(a) \\ b &= Y \end{cases} \xrightarrow{3} \begin{cases} Z &= h(X) \\ X &= g(a) \\ Y &= b \end{cases} \xrightarrow{5}$$

$$\begin{cases} Z &= h(g(a)) \\ X &= g(a) \\ Y &= b \end{cases} \quad \text{НОУ построен}$$

4. $P(X, f(Y), h(Z,X)) \qquad P(f(Y), X, h(f(Y), f(Z)))$

$$\begin{cases} X &= f(Y) \\ f(Y) &= X \\ h(Z,X) &= h(f(Y),f(Z)) \end{cases} \xrightarrow{1} \begin{cases} X &= f(Y) \\ f(Y) &= X \\ Z &= f(Y) \\ X &= f(Z) \end{cases} \xrightarrow{5}$$

$$\begin{cases} X &= f(Y) \\ f(Y) &= f(Y) \\ Z &= f(Y) \\ f(Y) &= f(Z) \end{cases} \xrightarrow{4} \begin{cases} X &= f(Y) \\ Z &= f(Y) \\ f(Y) &= f(Z) \end{cases} \xrightarrow{1}$$

$$\begin{cases} X &= f(Y) \\ Z &= f(Y) \\ Y &= Z \end{cases} \xrightarrow{5} \begin{cases} X &= f(Y) \\ \underline{Z} &= \underline{f(Z)} \\ Y &= Z \end{cases} \quad \text{НОУ Нет}$$

5. $P(X_1, X_2, X_3.X_4) \qquad P(f(c,c), f(X_1,X_1), f(X_2,X_2), f(X_3,X_3))$

$$\begin{cases} X_1 &= f(c,c) \\ X_2 &= f(X_1,X_1) \\ X_3 &= f(X_2,X_2) \\ X_4 &= f(X_3,X_3) \end{cases} \xrightarrow{5} \begin{cases} X_1 &= f(c,c) \\ X_2 &= f(f(c,c),f(c,c)) \\ X_3 &= f(X_2,X_2) \\ X_4 &= f(X_3,X_3) \end{cases} \xrightarrow{5}$$

$$\begin{cases} X_1 &= f(c,c) \\ X_2 &= f(f(c,c),f(c,c)) \\ X_3 &= f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c))) \\ X_4 &= f(X_3,X_3) \end{cases} \xrightarrow{5}$$

$$\begin{cases} X_1 &= f(c,c) \\ X_2 &= f(f(c,c),f(c,c)) \\ X_3 &= f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c))) \\ X_4 &= f(f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c))),f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c)))) \end{cases} \quad \text{НОУ построен}$$

4 Метод резолюций

Упражнение 4.1

1. $\neg P(f(x,y),z,h(z,y)) \ \vee \ R(z,v), \ Q(x) \ \vee \ P(f(y,x),g(y),v)$

   $D_1 = \neg P(f(x,y),z,h(z,y)) \ \vee \ R(z,v)$

   $D_2 = Q(x) \ \vee \ P(f(y,x),g(y),v)$

   НОУ$(P(f(y,x),g(y),v), \neg P(f(x,y),z,h(z,y)))$

$$\left\{\begin{array}{ccc} f(y,x) & = & f(x,y) \\ g(y) & = & z \\ v = & = & h(z,y) \end{array}\right. \xrightarrow{3} \left\{\begin{array}{ccc} f(y,x) & = & f(x,y) \\ z & = & g(y) \\ v = & = & h(z,y) \end{array}\right. \xrightarrow{1}$$

$$\left\{\begin{array}{ccc} y & = & x \\ x & = & y \\ z & = & g(y) \\ v = & = & h(z,y) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} y & = & x \\ x & = & x \\ z & = & g(x) \\ v = & = & h(z,x) \end{array}\right. \xrightarrow{4}$$

$$\left\{\begin{array}{ccc} y & = & x \\ z & = & g(x) \\ v = & = & h(z,x) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} y & = & x \\ z & = & g(x) \\ v = & = & h(g(x),x) \end{array}\right.$$

   $\Theta = \{y/x, z/g(x), v/h(g(x),x)\}$

   $D_3 \ \overset{D_1,D_2}{\underset{\Theta}{=}} \ R(g(x),h(g(x),x)) \ \vee \ Q(x)$

2. $P(x,y,h(y,x)) \ \vee \ R(y,f(x)), \ \neg P(x,f(x),h(x,y)) \ \vee \ P(y,g(x),h(y,y))$

   $D_1 = P(x_1,y_1,h(y_1,x_1)) \ \vee \ R(y_1,f(x_1))$

   $D_2 = \neg P(x_2,f(x_2),h(x_2,y_2)) \ \vee \ P(y_2,g(x_2),h(y_2,y_2))$

   НОУ$(P(x_1,y_1,h(y_1,x_1)), \vee P(y_2,g(x_2),h(y_2,y_2)))$

$$\left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ h(y_1,x_1) & = & h(y_2,y_2) \end{array}\right. \xrightarrow{1} \left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ y_1 & = & y_2 \\ x_1 & = & y_2 \end{array}\right. \xrightarrow{3}$$

$$\left\{\begin{array}{ccc} y_2 & = & x_1 \\ y_1 & = & g(x_2) \\ y_1 & = & y_2 \\ x_1 & = & y_2 \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} y_2 & = & x_1 \\ y_1 & = & g(x_2) \\ y_1 & = & x_1 \\ x_1 & = & x_1 \end{array}\right. \xrightarrow{4}$$

$$\left\{\begin{array}{ccc} y_2 & = & x_1 \\ y_1 & = & g(x_2) \\ y_1 & = & y_2 \end{array}\right. \xrightarrow{3} \left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ y_2 & = & y_1 \end{array}\right. \xrightarrow{5}$$

$$\left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ y_2 & = & g(x_2) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} x_1 & = & g(g_2) \\ y_1 & = & g(x_2) \\ y_2 & = & g(x_2) \end{array}\right.$$

   $\Theta = \{x_1/g(x_2), y_1/g(x_2), y_2/g(x_2)\}$

   $D_3 \ \overset{D_1,D_2}{\underset{\Theta}{=}} \ R(g(x_2),f(g(x_2))) \ \vee \ \neg P(x_2,f(x_2),,h(x_2,g(x_2)))$

Упражнение 4.2

1. $S = \{D_1, D_2, D_3, D_4, D_5\}$

   $D_1 = P(X_1, f(X_1))$
   $D_2 = R(Y_2, Z_2) \ \lor \ \neg P(Y_2, f(a))$
   $D_3 = \lor R(c, X_3)$
   $D_4 = R(X_4, Y_4) \ \lor \ R(Z_4, f(Z_4)) \ \lor \ \neg P(Z_4, Y_4)$
   $D_5 = P(X_5, X_5)$

   $D_6 \quad \underset{\{X_1/a, Y_2/a\}}{\overset{D_1, D_2}{=}} \quad R(a, Z_6)$

   $D_7 \quad \underset{\{X_4/Z_4, Y_4/f(Z_4)\}}{\overset{D_4}{=}} \quad R(Z_7, f(Z_7)) \ \lor \ \neg P(Z_7, f(Z_7))$

   $D_8 \quad \underset{\{X_3/f(c), Z_7/c\}}{\overset{D_7, D_3}{=}} \quad \neg P(c, f(c))$

   $D_9 \quad \underset{\{X_1/c\}}{\overset{D_1, D_8}{=}} \quad \square$

2. $S = \{D_1, D_2.D_3, D_4, D_5, D_6, D_7\}$

   $D_1 = E(x_1) \ \lor \ V(y_1) \ \lor \ C(f(x_1))$
   $D_2 = E(x_2) \ \lor \ S(x_2, f(x_2))$
   $D_3 = \neg E(a)$
   $D_4 = P(a)$
   $D_5 = P(f(x_5)) \ \lor \ \neg S(y_5, z_5)$
   $D_6 = \neg P(x_6) \ \lor \ \neg V(g(x_6)) \neg \ \lor V(y_6)$
   $D_7 = \neg P(x_7) \ \lor \ \neg C(y_7)$

   $D_8 \quad \underset{\{y_6/g(x)\}}{\overset{D_6}{=}} \quad \neg P(x_8) \ \lor \ \neg V(g(x_8))$

   $D_9 \quad \underset{\{x_7/a\}}{\overset{D_4, D_7}{=}} \quad \neg C(y_9)$

   $D_{10} \quad \underset{\{x_8/a\}}{\overset{D_8, D_4}{=}} \quad \neg V(g(a))$

   $D_{11} \quad \underset{\{y_9/f(x_1)\}}{\overset{D_1, D_9}{=}} \quad E(x_{11}) \ \lor \ V(y_{11})$

   $D_{12} \quad \underset{\{y_{11}/g(a))\}}{\overset{D_{11}, D_{10}}{=}} \quad E(x_{12})$

   $D_{13} \quad \underset{\{x_{12}/a\}}{\overset{D_{12}, D_3}{=}} \quad \square$

3. $S = \{D_1, D_2, D_3, D_4\}$

   $D_1 = P(y_1, f(x_1))$
   $D_2 = \neg Q(y_2) \ \lor \ \neg Q(z_2) \ \lor \ \neg P(y, f(z)) \ \lor \ Q(v)$
   $D_3 = Q(b)$
   $D_4 = \neg Q(a)$

   $D_5 \quad \underset{\{x_1/z_2, y_1/y_2\}}{\overset{D_1, D_2}{=}} \quad \neg Q(y_5) \ \lor \ \neg Q(z_5) \ \lor \ Q(v_5)$

   $D_6 \quad \underset{\{z_5/y_5\}}{\overset{D_5}{=}} \quad \neg Q(y_6) \ \lor \ Q(v_6)$

   $D_7 \quad \underset{\{v_6/a\}}{\overset{D_6, D_4}{=}} \quad \neg Q(y_7)$

   $D_8 \quad \underset{\{y_7/b\}}{\overset{D_7, D_3}{=}} \quad \square$

Упражнение 4.3

1. $\exists x\ P(x)\ \rightarrow\ \neg\forall x\ \neg P(x)$

   $\phi_0 = \neg(\exists x\ P(x)\ \rightarrow\ \neg\forall y\ \neg P(x))$
   $\phi_{01} = \exists x\ P(x)\ \&\ \forall y\ \neg P(y)$
   $\phi_{02} = \exists x\forall y\ P(x)\ \&\ \neg P(y)$
   $\phi_1 = \forall y\ P(c)\ \&\ \neg P(y)$

   $S = \{P(c), \neg P(y)\}$

   $D_1 = P(c)$
   $D_2 = \neg P(y)$
   $D_3 \overset{D_1,D_2}{\underset{\{y/c\}}{=}} \square$

2. $\exists x\forall y R(x,y)\ \rightarrow\ \forall y\exists x R(x,y)$
   $\phi_0 = \neg(\exists x_1\forall y_1 R(x_1,y_1)\ \rightarrow\ \forall y_2\exists x_2 R(x_2,y_2))$
   $\phi_{01} = \exists x_1\forall y_1 R(x_1,y_1)\ \&\ \exists y_2\forall x_2\neg R(x_2,y_2)$
   $\phi_{02} = \exists x_1\forall y_1\exists y_2\forall x_2\ R(x_1,y_1)\ \&\ \neg R(x_2,y_2)$
   $\phi_1 = \forall y_1\forall x_2\ R(c,y_1)\ \&\ \neg R(x_2,f(y_1))$
   $S = \{R(c,y_1), \neg R(x_2,f(y_1))\}$

   $D_1 = R(c,y_1)$
   $D_2 = \neg R(x_2,f(y_2))$ *переименование переменных*
   $D_3 \overset{D_1,D_2}{\underset{\{x_2/c, y_1/f(y_2)\}}{=}} \square$

3. $\forall x(P(x)\ \rightarrow\ \exists y\ R(x,f(y)))\ \rightarrow\ (\exists x\ \neg P(x)\ \vee\ \forall x\exists z R(x,z))$

   $\phi_0 = \neg(\forall x_1(P(x_1)\ \rightarrow\ \exists y_1\ R(x_{,1} f(y_1)))\ \rightarrow\ (\exists x_2\ \neg P(x_2)\ \vee\ \forall x_3\exists z_1 R(x_3,z_1)))$
   $\phi_{01} = \forall x_1(\neg P(x_1)\ \vee\ \exists y_1\ R(x_1,f(y_1)))\ \&\ \forall x_2\ P(x_2)\ \&\ \exists x_3\forall z_1\neg R(x_3,z_1)$
   $\phi_{02} = \forall x_1\exists y_1\forall x_2\exists x_3\forall z_1(\neg P(x_1)\ \vee\ R(x_1,f(y_1)))\ \&\ P(x_2)\ \&\ \neg R(x_3,z_1)$
   $\phi_1 = \forall x_1\forall x_2\forall z_1(\neg P(x_1)\ \vee\ R(x_1,f(g(x_1))))\ \&\ P(x_2)\ \&\ \neg R(h(x_1,x_2),z_1)$
   $S = \{\neg P(x_1)\ \vee\ R(x_1,f(g(x_1))), P(x_2), \neg R(h(x_1,x_2),z_1)\}$

   $D_1 = \neg P(x_1)\ \vee\ R(x_1,f(g(x_1)))$
   $D_2 = P(x_2)$
   $D_3 = \neg R(h(x_{31},x_{32}),z_3)$
   $D_4 \overset{D_1,D_2}{\underset{\{x_1/x_2\}}{=}} R(x_4,f(g(x_4))$
   $D_5 \overset{D_3,D_4}{\underset{\{x_4/h(x_{31},x_{32}), z_3/f(g(h(x_{31},x_{32})))\}}{=}} \square$

4. $\forall x\exists y\forall z(P(x,y)\ \rightarrow\ P(y,z))$

   $\phi_0 = \neg(\forall x\exists y\forall z(P(x,y)\ \rightarrow\ P(y,z)))$
   $\phi_{01} = \exists x\forall y\exists z(P(x,y)\ \&\ \neg P(y,z)))$
   $\phi_1 = \forall y(P(c,y)\ \&\ \neg P(y,f(y)))$
   $S = \{P(c,y), \neg P(y,f(y))\}$

   $D_1 = P(c,y_1)$
   $D_2 = \neg P(y_2,f(y_2))$
   $D_3 \overset{D_1,D_2}{\underset{\{y_2/c, y_1/f(c)\}}{=}} \square$

5. $\exists x \forall y \exists z (P(x,y) \ \rightarrow \ P(y,z))$

$\phi_0 = \neg(\exists x \forall y \exists z (P(x,y) \ \rightarrow \ P(y,z)))$
$\phi_{01} = \forall x \exists y \forall z (P(x,y) \ \& \ \neg P(y,z)))$
$\phi_{02} = \forall x \forall z (P(x,f(x)) \ \& \ \neg P(y,z)))$
$S = \{P(x,f(x)), \neg P(y,z)\}$

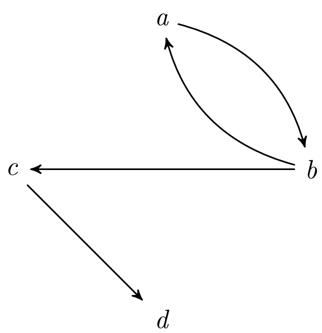
$D_1 = P(x_1, f(x_1))$
$D_2 = \neg P(y_2, z_2)$
$D_3 \quad \overset{D_1,D_2}{\underset{\{y_2/x_1, z_2/f(x_1)\}}{=}} \quad \square$

6. $\exists x \forall y (\forall z (P(y,z) \ \rightarrow \ P(x,z)) \ \rightarrow \ (P(x,x) \ \rightarrow \ PP(y,z)))$

$\phi_0 = \neg(\exists x \forall y (\forall z (P(y,z) \ \rightarrow \ P(x,z)) \ \rightarrow \ (P(x,x) \ \rightarrow \ PP(y,z))))$
$\phi_{01} = \forall x \exists y ((\forall z (\neg P(y,z) \ \vee \ P(x,z))) \ \& \ P(x,x) \ \& \ \neg P(y,z))$
$\phi_{02} = \forall x \exists y \forall z ((\neg P(y,z) \ \vee \ P(x,z)) \ \& \ P(x,x) \ \& \ \neg P(y,z))$
$S = \{\neg P(y,z) \ \vee \ P(x,z), P(x,x), \neg P(y,z)\}$

$D_1 = \neg P(y_1, z_1) \ \vee \ P(x_1, z_1)$
$D_2 = P(x_2, x_2)$
$D_3 = \neg P(y_3, z_3)$
$D_4 \quad \overset{D_1,D_2}{\underset{\{y_1/x_2, z_1/x_2\}}{=}} \quad P(x_{31}, x_{32})$
$D_5 \quad \overset{D_4,D_3}{\underset{\{y_3/x_{31}, z_3/x_{32}\}}{=}} \quad \square$

Упражнение 4.4 *Граф*



База знаний

1. $\phi_1 = R(a,b)$
2. $\phi_2 = R(b,c)$
3. $\phi_3 = R(b,a)$
4. $\phi_4 = R(c,d$
5. $\psi_2 = \forall x Q(x,x)$
6. $\psi_2 = \forall x \forall y \forall z (Q(x,y) \& R(y,z) \ \rightarrow Q(x,z))$

Запрос
$\Phi_1 = Q(a, d)$

Система дизъюнктов
$S = \{D_1, D_2, D_3, D_4, D_5, D_6, D_7\}$

$D_1 = R(a, b)$
$D_2 = R(b, c)$
$D_3 = R(b, a)$
$D_4 = R(c, d)$
$D_5 = Q(X_5, X_5)$
$D_6 = \neg Q(X_6, Y_6) \ \vee \ \neg R(Y_6, Z_6) \ \vee \ Q(X_6, Z_6)$
$D_7 = \neg Q(a, d)$

Покажем противоречивость данной системы дизъюнктов

$D_8 \underset{\{X_6/a, Z_6/d\}}{\overset{D_7, D_6}{=}} \neg Q(a, Y_8) \ \vee \ \neg R(Y_8, d)$

$D_9 \underset{\{Y_8/c\}}{\overset{D_8, D_4}{=}} \neg Q(a, c)$

$D_{10} \underset{\{X_6/a, Z_6/c\}}{\overset{D_9, D_6}{=}} \neg Q(a, Y_{10}) \ \vee \ \neg R(Y_{10}, c)$

$D_{11} \underset{\{Y_{10}/b\}}{\overset{D_{10}, D_2}{=}} \neg Q(a, b)$

$D_{12} \underset{\{X_6/a, Z_6/b\}}{\overset{D_{11}, D_6}{=}} \neg Q(a, Y_{12}) \ \vee \ \neg R(Y_{12}, b)$

$D_{13} \underset{\{Y_{12}/a\}}{\overset{D_{12}, D_1}{=}} \neg Q(a, a)$

$D_{14} \underset{\{X_5/a\}}{\overset{D_{13}, D_5}{=}} \square$

5 Хорновские логические программы. Декларативные и операционные семантики.

Упражнение 5.1

1. $parent(X, Y) \leftarrow father(X, Y).$
   $parent(X, Y) \leftarrow mather(X, Y).$

2. $grandfather(X, Y) \leftarrow father(X, Z), parent(Z, Y).$

3. $to\_be\_a\_father(X) \leftarrow father(X, Z).$

4. $brother(X, Y) \leftarrow parent(Z, X), man(X), parent(Z, Y), X \neq Y.$

5. $offspring(X, Y) \leftarrow parent(Y, X).$
   $offspring(X, Y) \leftarrow parent(Z, X), offspring(X, Z).$

Упражнение 5.2

1. $list(X)$

   $list(nil) \leftarrow ;$
   $list(X.Y) \leftarrow list(Y).$

2. $elem(X, Y)$

   $elem(X, X.Y) \leftarrow ;$
   $elem(X, Z.Y) \leftarrow elem(X, Y);$

1. *True.*

2. *X - любой атом.*

3. *False.*

4. $X = a, X = b, X = c.$

5. *X - любой список, содержащий атом a.*

Упражнение 5.3

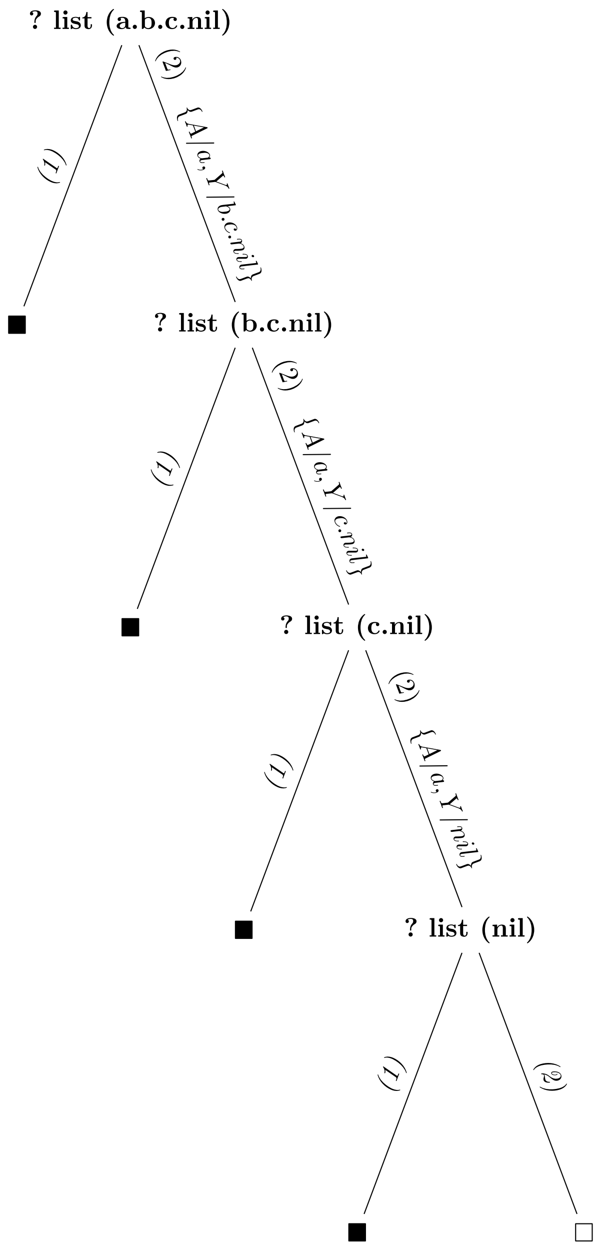
1. **list(nil).**

2. **list(A.Y) ← list(Y).**

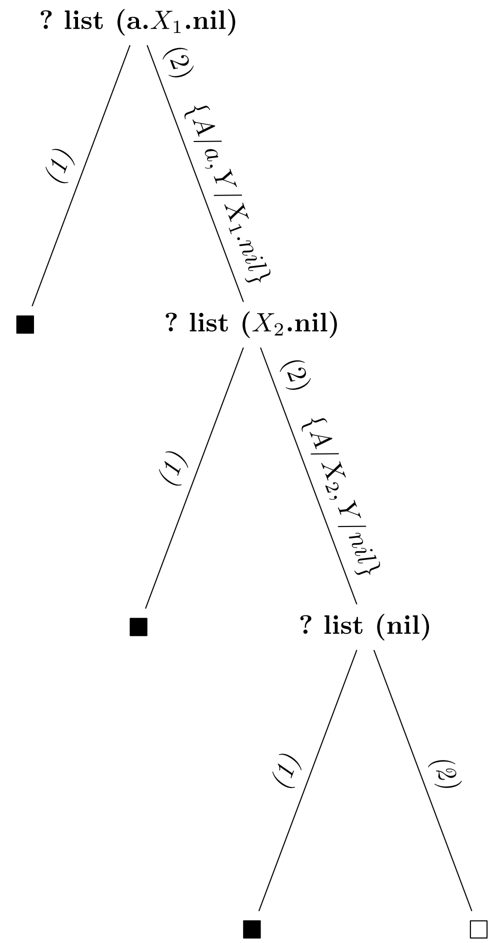
1. **elem(X, X.Y).**

2. **elem(X, Z.Y) ← elem(X, Y).**

1. **? list(a.b.c.nil)**

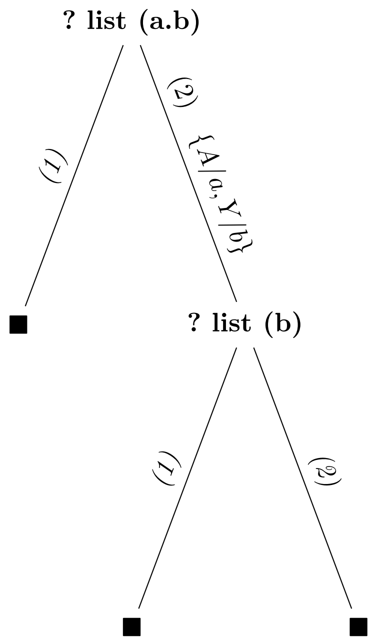


2. **? list(a.X.nil)**

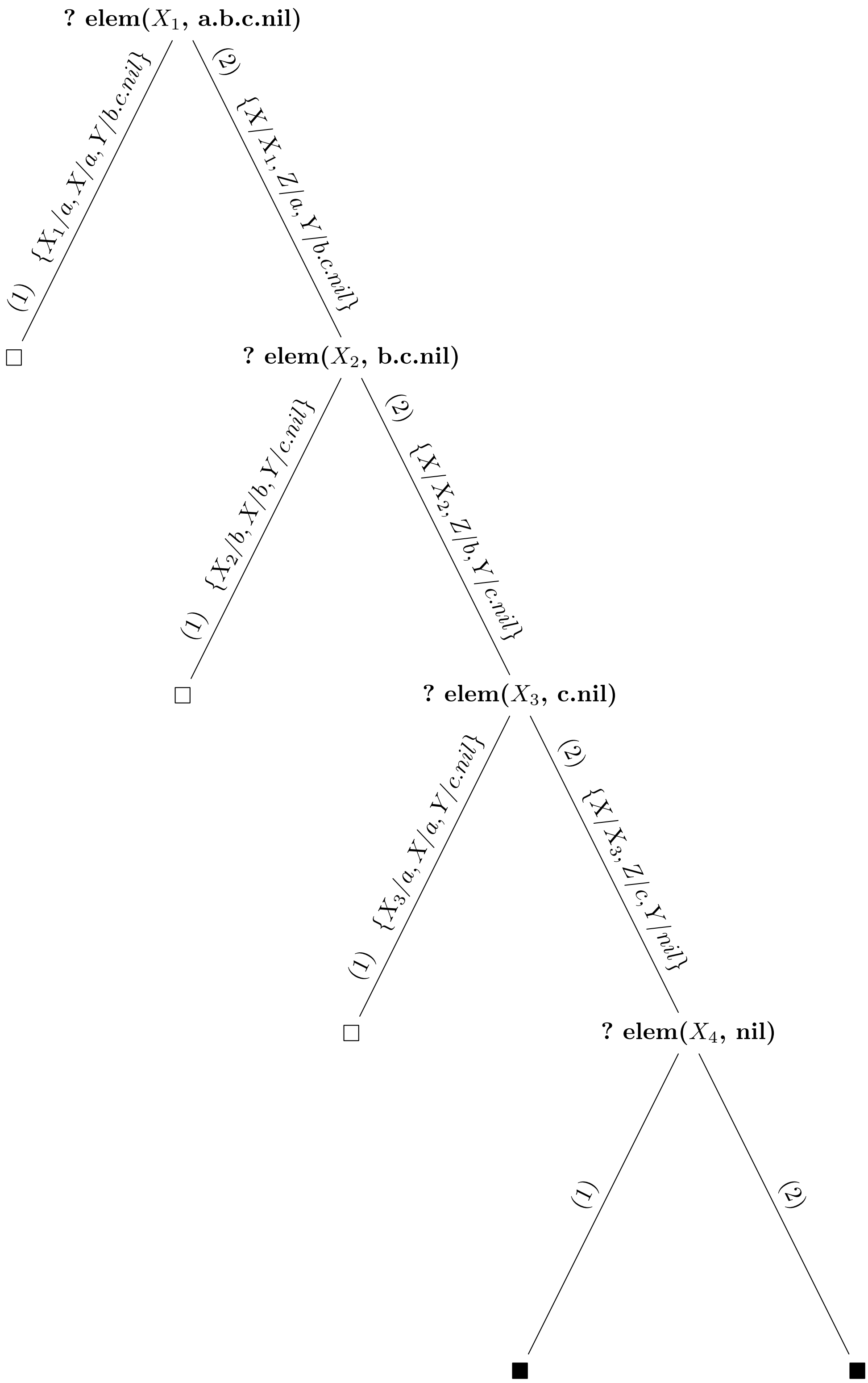


$X = A$

3. **? list(a.b)**

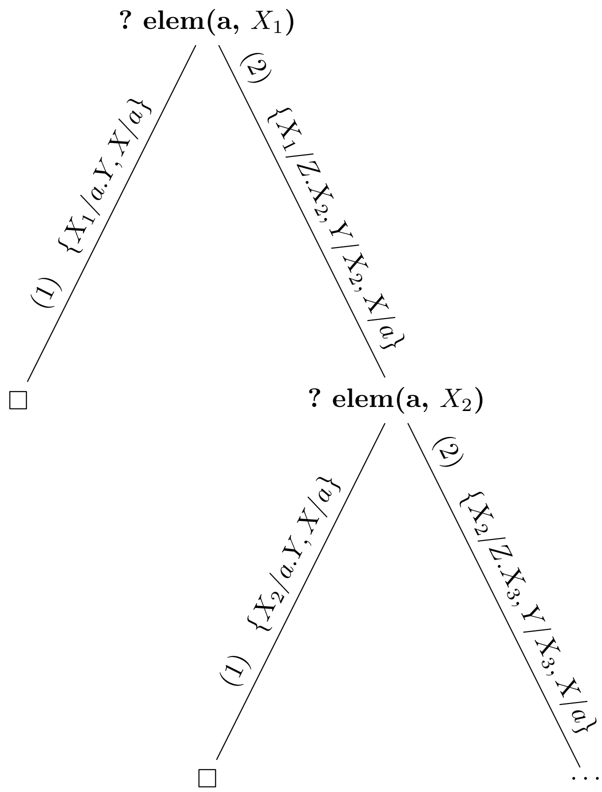


4. **? elem(X, a.b.c.nil)**



$X = a,\ X = b,\ X = c$

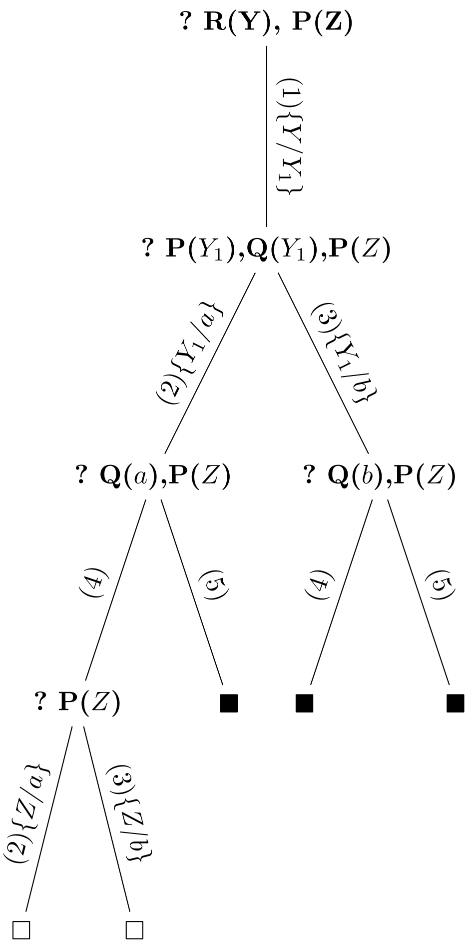
5. **? elem(a,X)**


? elem(a, X_1)
(1) $\{X_1/a.Y, X/a\}$
(2) $\{X_1/Z.X_2, Y/X_2, X/a\}$
□
? elem(a, X_2)
(1) $\{X_2/a.Y, X/a\}$
(2) $\{X_2/Z.X_3, Y/X_3, X/a\}$
□
…

Упражнение 5.4

1. $\mathbf{R}(Y_1) \leftarrow \mathbf{P}(Y_1),\mathbf{Q}(Y_1)$;
2. $\mathbf{P}(\mathbf{a}) \leftarrow$;
3. $\mathbf{P}(\mathbf{b}) \leftarrow$;
4. $\mathbf{Q}(\mathbf{a}) \leftarrow$;
5. $\mathbf{Q}(\mathbf{f}(X_5)) \leftarrow \mathbf{Q}(X_5)$

? R(Y), P(Z)



$\{\mathbf{Z}/a,\ \mathbf{Y}/a\}$ $\{\mathbf{Z}/b,\ \mathbf{Y}/a\}$

Упражнение 5.5

```
%Sub
elem(X, [X|_]).
elem(X, [_|Y]) :- elem(X, Y).

%1
head([X| _], X).

%2
tail([_| Tail], Z) :-tail(Tail, Z).
tail([_| B], B).

%3
prefix([Head| Tail_1], [Head| Tail_2]) :- prefix(Tail_1, Tail_2).
prefix(_, []).

%4
sublist(List, Sublist) :- prefix(List, Sublist).
sublist([_| Tail], Sublist) :- sublist(Tail, Sublist).

%5
less([], [_|_]).
less([_|Tail_1], [_|Tail_2]) :- less(Tail_1, Tail_2).

%6
subset([], _).
subset([Head| Tail], Y) :- elem(Head, Y),subset(Tail, Y).

%7
concat(X, [], X).
concat([Head|Tail_1], [Head|Tail_2], X) :- concat(Tail_1, Tail_2, X).

%8

reverse(X, Y) :- reverse_loop([], X, Y).
reverse_loop(Rev, [], Rev).
reverse_loop(Rev, [Head| Tail], Goal) :- reverse_loop([Head| Rev], Tail, Goal).

%9
period(X, Y) :- loop_period(X, Y, Y).
loop_period([], [], _).
loop_period(Main, [], Base) :- loop_period(Main, Base, Base).
loop_period([Head| Main], [Head| Curr], Base) :- loop_period(Main,Curr, Base).
```

Упражнение 5.6

```
%1
main_less([], [], 1).
main_less([], [_], _).
main_less([_|Tail_X], [_|Tail_Y], -1) :- main_less(Tail_X, Tail_Y, -1).
main_less([_|Tail_X], [_|Tail_Y], 1) :- main_less(Tail_X, Tail_Y, 1).
main_less([A|Tail_X], [A|Tail_Y], 0) :- main_less(Tail_X, Tail_Y, 0).
main_less([0|Tail_X], [1|Tail_Y], 0) :- main_less(Tail_X, Tail_Y, 1).
main_less([1|Tail_X], [0|Tail_Y], 0) :- main_less(Tail_X, Tail_Y, -1).
less([0|Tail_X], Y) :- less(Tail_X, Y).
less(X, [0|Tail_Y]) :- less(X, Tail_Y).
less([], [1|_]).
less([1|Tail_X], [1|Tail_Y]) :- main_less(Tail_X, Tail_Y, 0).
%2 Z = X + Y
sum(X,Y,Z) :- reverse(X, R_X), reverse(Y, R_Y), reverse(Z, R_Z), r_sum(R_X, R_Y, R_Z, 0)
r_sum([A|Tail_X], [A|Tail_Y], [B|Tail_Z], B) :- r_sum(Tail_X, Tail_Y, Tail_Z, A).
r_sum([_|Tail_X], [_|Tail_Y], [1|Tail_Z], 0) :- r_sum(Tail_X, Tail_Y, Tail_Z, 0).
r_sum([_|Tail_X], [_|Tail_Y], [0|Tail_Z], 1) :- r_sum(Tail_X, Tail_Y, Tail_Z, 1).
r_sum([], [1|Tail_Y], [0|Tail_Z], 1) :- r_sum([], Tail_Y, Tail_Z, 1).
r_sum([], [0|Tail_Y], [1|Tail_Z], 1) :- r_sum([], Tail_Y, Tail_Z, 0).
r_sum([1|Tail_X], [], [0|Tail_Z], 1) :- r_sum(Tail_X, [], Tail_Z, 1).
r_sum([0|Tail_X], [], [1|Tail_Z], 1) :- r_sum(Tail_X, [], Tail_Z, 0).
r_sum([], N, N, 0).
r_sum(N, [], N, 0).
```

6 Встроенные функции и предикаты

Упражнение 6.1

```
elem([X|_],X).
elem([_|A],X) :- elem(A,X).

not_elem([], _).
not_elem([A|Y], X) :- A \= X, not_elem(Y, X).

%make_ordered(L_1,L_2).
%insert
insert_sort(List,Sorted):-i_sort(List,[],Sorted).
i_sort([],Acc,Acc).
i_sort([H|T],Acc,Sorted):-insert(H,Acc,NAcc),i_sort(T,NAcc,Sorted).

insert(X,[Y|T],[Y|NT]):-X>Y,insert(X,T,NT).
insert(X,[Y|T],[X,Y|T]):-X=<Y.
insert(X,[],[X]).

%bubble
bubble_sort(List,Sorted):-b_sort(List,[],Sorted).
b_sort([],Acc,Acc).
b_sort([H|T],Acc,Sorted):-bubble(H,T,NT,Max),b_sort(NT,[Max|Acc],Sorted).

bubble(X,[],[],X).
bubble(X,[Y|T],[Y|NT],Max):-X>Y,bubble(X,T,NT,Max).
bubble(X,[Y|T],[X|NT],Max):-X=<Y,bubble(Y,T,NT,Max).


%quick
quick_sort(List,Sorted):-q_sort(List,[],Sorted).
q_sort([],Acc,Acc).
q_sort([H|T],Acc,Sorted):-
        pivoting(H,T,L1,L2),
        q_sort(L1,Acc,Sorted1),q_sort(L2,[H|Sorted1],Sorted).

pivoting(_,[],[],[]).
pivoting(H,[X|T],[X|L],G):-X=<H,pivoting(H,T,L,G).
pivoting(H,[X|T],L,[X|G]):-X>H,pivoting(H,T,L,G).

%single(L_1,L_2)
single([],[]).
single([A|X], Y) :- single(X,Y), elem(Y, A).
single([A|X], [A|Y]) :- single(X,Y), not_elem(Y,A).

%common(L_1,L_2,L_3)
common([],X,Y) :- single(X,Y).
common([A|X], Z, Y) :- common(X,Z,Y), elem(Y,A).
common([A|X], Z, [A|Y]) :- common(X,Z,Y), not_elem(Y,A).

%intersect (L_1, L_2, L_3)
intersect([],_,[]).
intersect([A|X], Z, [A|Y]) :- elem(Z,A), intersect(X,Z,Y).
intersect([A|X], Z, Y) :- not_elem(Z,A), intersect(X,Z,Y).
```

Упражнение 6.2

```
%length (L, X)
m_length([],0).
m_length([_|L], X) :- m_length(L, Z), X is Z + 1.

%sum (L, X)
sum([X], X).
sum([X|L], S) :- sum(L, Z), S is X + Z.

a_sum(L, X) :- acc_sum(L, X, 0).
acc_sum([], X, X).
acc_sum([A|L], X, ACC) :- W is A + ACC, acc_sum(L, X, W).

%mult (L, X, Y)
mult([], _, 0).
mult([X|L], X, Y) :- mult(L, X, Z), Y is Z + 1.
mult([A|L], X, Y) :- A \= X,mult(L, X, Y).

%most_of (L,X)
most_of([X], X).
most_of([X|L], X) :- most_of(L, Y), mult(L, X, M_X), mult(L, Y, M_Y), M_X>=M_Y.
most_of([Z|L], X) :- most_of(L, X), mult(L, X, M_X), mult([Z|L], Z , M_Z), M_X>M_Z.

% i_most_of (L, X) iterative
i_most_of(L, X) :- msort(L, S_L), max_mult(S_L, CURR, 1, 0, X).
max_mult([_], M_M, CURR, MAX_MULT, M_M) :- CURR =< MAX_MULT.
max_mult([A,A|L], M_M, CURR, MAX_COUNT, X) :- NEW_CURR is CURR + 1, max_mult([A|L], M_M,
max_mult([A,B|L], M_M, CURR, MAX_COUNT, X) :- A \= B, MAX_COUNT >= CURR, max_mult([B|L],
max_mult([A,B|L], M_M, CURR, MAX_COUNT, X) :- A \= B, MAX_COUNT < CURR, max_mult([B|L],


%prime(L,X)
prime([2],2).
prime([X|L], X) :- Y is X - 1, prime(L, Y), pr(X, L).
prime(L, X) :- Y is X - 1, prime(L, Y), antipr(X, L).
pr(_, []).
pr(X, [Y|L]) :- gcd(X, Y, 1), pr(X, L).
antipr(X, [Y|_]) :- gcd(X, Y, Z), Z > 1.
antipr(X, [Y|L]) :- gcd(X, Y, 1), antipr(X, L).


%gcd(X,Y,Z)
gcd(X,X,X).
gcd(X,Y,Z) :- X > Y, N_X is X - Y, gcd(N_X, Y, Z).
gcd(X,Y,Z) :- X < Y, N_Y is Y - X, gcd(X, N_Y, Z).
```

7 Операторы отсечения и отрицания

Упражнение 7.1

1. $\mathbf{A}(Y_1) \leftarrow \mathbf{B}(Y_1),\ \mathbf{C}(a_2,\ Y_1);$
2. $\mathbf{A}(X_2) \leftarrow \mathbf{D}(a_1,\ X_2),\ \mathbf{C}(X_2, Y_2);$
3. $\mathbf{B}(U_3) \leftarrow \mathbf{D}(U_3,\ V_3),\ \mathbf{!},\ \mathbf{E}(V_3);$
4. $\mathbf{B}(V_4) \leftarrow \mathbf{E}(a_5);$
5. $\mathbf{E}(a_2) \leftarrow ;$
6. $\mathbf{E}(a_3) \leftarrow ;$
7. $\mathbf{E}(Z_7) \leftarrow ;$
8. $\mathbf{D}(U_8,\ a_1) \leftarrow \mathbf{C}(U_8,\ f(U_8));$
9. $\mathbf{D}(U_9,\ U_9) \leftarrow ;$
10. $\mathbf{D}(X_{10},\ a_2) \leftarrow ;$
11. $\mathbf{C}(Z_{11},\ a_3) \leftarrow ;$

? A(X)

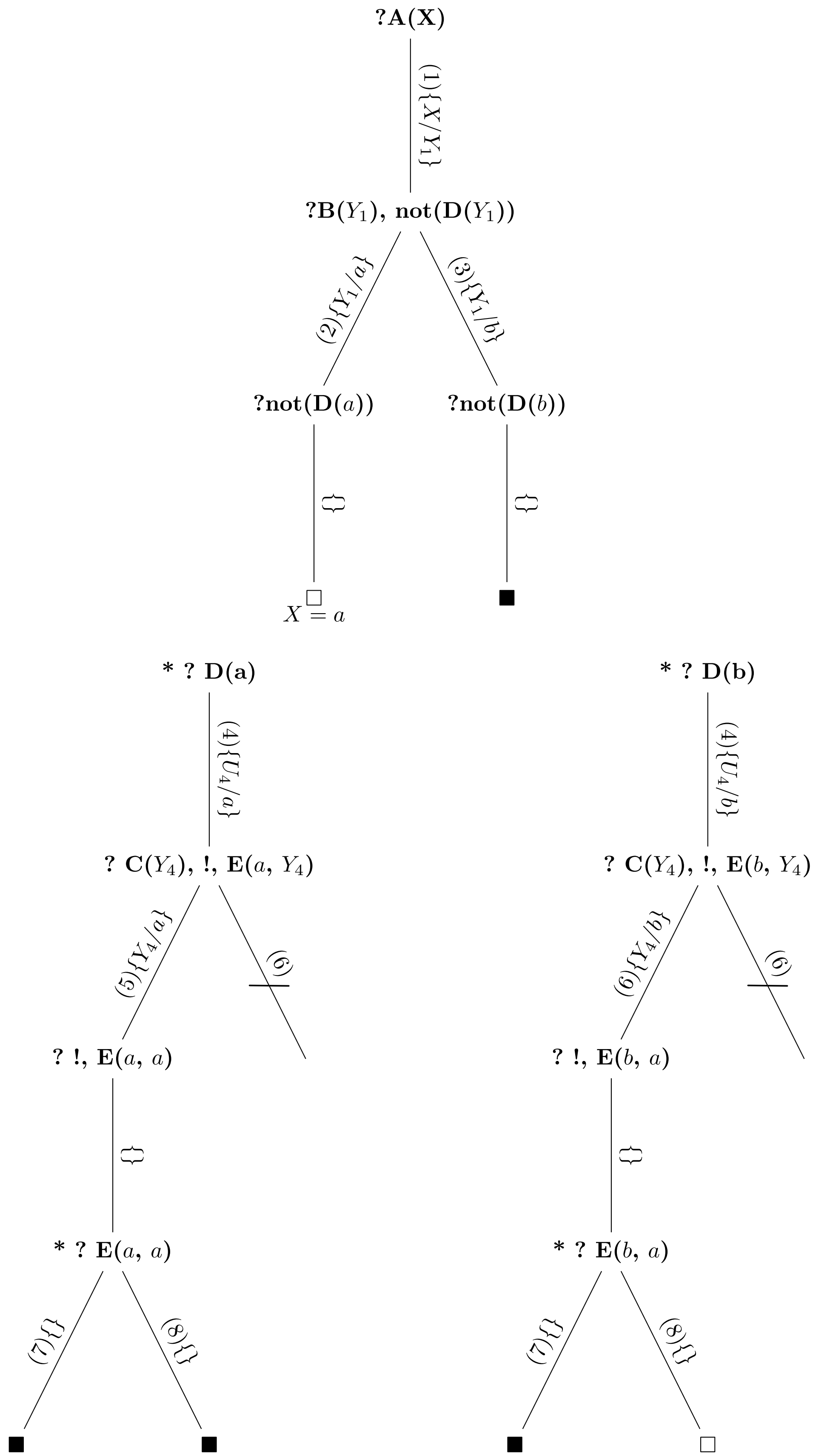
Дерево не вмещается. Но там все просто ;=).

Упражнение 7.2

```
elem([X|_],X).
elem([_|A],X) :- elem(A,X).

%max(X, Y, Z)
max(X, Y, X) :- X>Y, !.
max(_, Y, Y).

%common (L1, L2, L3).
common([], _, []).
common([A|L1], L2, [A|L3]) :- elem(L2, A), !, common(L1, L2, L3).
common([_|L1], L2, L3) :- common(L1, L2, L3).

%nonsquare (L1, L2)
nonsquare(L1, L2) :- nonsquare_seq(L1, L1, L2).
nonsquare_seq([], _, []).
nonsquare_seq([A|L1], SL1, L2) :- M is A * A, elem(SL1, M), !,
        nonsquare_seq(L1, SL1, L2).

nonsquare_seq([A|L1], SL1, [A|L2]) :- nonsquare_seq(L1, SL1, L2).
```

Упражнение 7.3

1. $\mathbf{A}(Y_1) \leftarrow \mathbf{B}(Y_1),\ \mathbf{not}(\mathbf{D}(Y_1));$
2. $\mathbf{B}(a) \leftarrow ;$
3. $\mathbf{B}(b) \leftarrow ;$
4. $\mathbf{D}(U_4) \leftarrow \mathbf{C}(Y_4),\ \mathbf{!},\ \mathbf{E}(U_4,\ Y_4);$
5. $\mathbf{C}(a) \leftarrow ;$
6. $\mathbf{C}(b) \leftarrow ;$
7. $\mathbf{E}(a,\ b) \leftarrow ;$
8. $\mathbf{E}(b,\ a) \leftarrow ;$


?A(X)
(1){X/Y1}
?B(Y1), not(D(Y1))
(2){Y1/a}
(3){Y1/b}
?not(D(a))
?not(D(b))
{}
{}
X = a
* ? D(a)
(4){U4/a}
? C(Y4), !, E(a, Y4)
(5){Y4/a}
(6)
? !, E(a, a)
{}
* ? E(a, a)
(7){}
(8){}
* ? D(b)
(4){U4/b}
? C(Y4), !, E(b, Y4)
(6){Y4/b}
(6)
? !, E(b, a)
{}
* ? E(b, a)
(7){}
(8){}

Упражнение 7.4

```
%max (L, X)
m_max([X], X).
m_max([A|L], A) :- m_max(L, Y), A >= Y, !.
m_max([_|L], A) :- m_max(L, A).

%max_occur(L1, L2).
max_occur([W], W).
max_occur([X|L], W) :- max_occur(L, S), max_list(X, S, W).
max_list(X, S, X) :- length(X, XL), length(S, SL), XL >= SL, !.
max_list(_, S, S).

%short_path(V1, V2, G, L).
short_path(V1, V2, G, [V1|L]) :- short_path_with_len(V1, V2, G, L, _).
short_path_with_len(V1, V2, G, L, New_len) :-
        mark_possible_ways(V1, G, New_G, Beg),
        try_some_ways(Beg, New_G, V2, L, Len),
        New_len is Len + 1.
short_path_with_len(F, F, _, [], 0).


mark_possible_ways(V1, [[V1, T]|G],  New_G, [T|Beg]) :-
        mark_possible_ways(V1, G, New_G, Beg), !.

mark_possible_ways(V1, [[_, V1]|G],  New_G, Beg) :-
        mark_possible_ways(V1, G, New_G, Beg), !.

mark_possible_ways(V1, [[F, T]|G], [[F, T]|New_G], Beg) :-
        V1 \= F, mark_possible_ways(V1, G, New_G, Beg), !.

mark_possible_ways(_, [], [], []).



try_some_ways([P], G, V2, [P|L], Len) :-
        short_path_with_len(P, V2, G, L, Len).

try_some_ways([P|Beg], G, V2, [P|L], Len) :-
        short_path_with_len(P, V2, G, L, Len),
        try_some_ways(Beg, G, V2, _, T_Len), Len < T_Len.

try_some_ways([P|Beg], G, V2, T_L, T_Len) :-
        short_path_with_len(P, V2, G, _, Len),
        try_some_ways(Beg, G, V2, T_L, T_Len), Len >= T_Len.

try_some_ways([P|Beg], G, V2, T_L, T_Len) :-
        not(short_path_with_len(P, V2, G, _, Len)),
        try_some_ways(Beg, G, V2, T_L, T_Len).

try_some_ways([P|Beg], G, V2, [P|L], Len) :-
        short_path_with_len(P, V2, G, L, Len),
        not(try_some_ways(Beg, G, V2, _, T_Len)).
```

Упражнение 7.5

```
%reach(V, E, x, y).
reach(_, E, X, Y) :- short_path(X, Y, E, _).

%short_path(V, E, x, y, L).
short_path(_, E, X, Y, L) :- short_path(X, Y, E, L).

%color(V, E, R).
```

8 Экзаменационные задачи

```
%% Дан текст, разбить его на 2 множества слов так, что слова из разных множеств
%% не имеют общих букв.

%% elem(L, X) %%
elem([X|_],X).
elem([_|A],X) :- elem(A,X).

%% mult (L, X, Y)
mult([], _, 0).
mult([X|L], X, Y) :- !, mult(L, X, Z), Y is Z + 1.
mult([_|L], X, Y) :- mult(L, X, Y).


text_split(L, X, Y) :- sublist(L, X), delete(L, X, Y), no_lett(X, Y).

sublist(_, []).
sublist([C|L], [C|X]) :- !, sublist(L, X).
sublist([_|L], X) :- sublist(L, X).

delete(L, [], L).
delete([C|L], [C|X], Y) :- delete(L, X, Y), !.
delete([C|L], X, [C|Y]) :- delete(L, X, Y).

no_lett([], _).
no_lett([X|L], Y) :- no_lett1(X, Y), no_lett(L, Y).

no_lett1(_, []).
no_lett1(X, [Y|L]) :- cap(X, Y, []), no_lett1(X, L).

cap([], _, []).
cap([C|X], Y, []) :- not(elem(Y, C)), cap(X, Y, []).

%% Для данного текста построить список наиболее встречающихся
%% в нем слов.

max_text(L, L1) :- max_count(X, L), get_words(L, L1, X).

max_count(X, L) :- elem(L, C), mult(L, C, X), not(exists_gt(L, X)).

exists_gt(L, M) :- elem(L, C), mult(L, C, N), N > M.

get_words([], [], _).
get_words([X|L], [X|L1], N) :- mult([X|L], X, N), !, get_words(L, L1, N).
get_words([_|L], L1, N) :- get_words(L, L1, N).

%% Для данного графа построить кратчайший путь между двумя вершинами
%% в нем.

s_path(V1, V2, G, L) :- path(V1, V2, G, L), m_length(L, N),
        not(exists_lt(V1, V2, G, N)).

path(V, V, _,  []).
path(V1, V2, G, [[V1, V3] | L]) :- elem(G, [V1, V3]), path(V3, V2, G, L).
```

```
exists_lt(V1, V2, G, N) :- path(V1, V2, G, L), m_length(L, M), M < N.

m_length([], 0).
m_length([_|L], M) :- m_length(L, N), M is N + 1.


%% Для данного множества точек построить список наиболее удаленных друг
%% от друга пар

g_pair(L, X) :- max_dist(L, N), get_pairs(L, X, N).

max_dist(L, N) :- elem(L, X), elem(L, Y), dest(X, Y, N),
        not(exists_gt_point(L, N)).

dest([X1, X2], [Y1, Y2], N) :- N is sqrt((X1 - Y1)^2  + (X2 - Y2)^2).

exists_gt_point(L, N) :- elem(L, X), elem(L, Y), dest(X, Y, M), M > N.

m_concat([], L, L).
m_concat([A|L1], L2, [A|RES]) :- m_concat(L1, L2, RES).

get_pairs([], [], _).
get_pairs([X|L], RES, N) :- build_pairs(X, L, TT_RES, N),
        get_pairs(L, T_RES, N), m_concat(TT_RES, T_RES, RES).

build_pairs(_, [], [], _).
build_pairs(X, [C|L], [[C, X]|RES], N) :- dest(X, C, N),!, build_pairs(X, L, RES, N).
build_pairs(X, [_|L], RES, N) :- build_pairs(X, L, RES, N).

%% Для данного графа построить его максимальную клику
clique(G, V,  X) :- sublist(V, X), is_clique(G, X), m_length(X, N),
        not(exists_gt_clique(G, V, N)).

exists_gt_clique(G, V, N) :- sublist(V, X), is_clique(G, X),
        m_length(X, M), M > N.


is_clique(_, []).
is_clique(G, [X|L]) :- edge_to_all(X, L, G), is_clique(G, L).

edge_to_all(_,[], _).
edge_to_all(X, [C|L], G) :- elem(G, [X, C]), !, edge_to_all(X, L, G).
edge_to_all(X, [C|L], G) :- elem(G, [C, X]), edge_to_all(X, L, G).

%% Для данного графа построить его максимальное независимое подмножество
%% (ни какие две вершины не соеденины ребром).

indep_set(G, V,  X) :- sublist(V, X), is_indep(G, X), m_length(X, N),
        not(exists_gt_indep(G, V, N)).

exists_gt_indep(G, V, N) :- sublist(V, X), is_indep(G, X),
        m_length(X, M), M > N.


is_indep(_, []).
is_indep(G, [X|L]) :- no_edges(X, L, G), is_indep(G, L).
```

```
no_edges(_,[], _).
no_edges(X, [C|L], G) :- not(elem(G, [X, C])), not(elem(G, [C, X])),
        no_edges(X, L, G).

%% Для данного множества чисел построить максимальное его подмножество,
%% свободное от сумм

sum_free(L, X) :- sublist(L, X), not(sum_non_free_set(X)), m_length(X, N),
        not(exists_gt_sum_free(L, N)).

sum_non_free_set(X) :- elem(X, A), elem(X, B), elem(X, C),
        A \= B,  S is A + B, C = S.

exists_gt_sum_free(L, N) :- sublist(L, X), not(sum_non_free_set(X)),
        m_length(X, M), M > N.

%% Для данных текстов L1 и L2 построить текст L3, состоящий из тех слов L1, которые
%% содержат хотя бы одну букву, не встречающуюся ни в одном слове из L2

mk_text(L1, L2, L3) :- build_letters(L2, Lett), filter_text(L1, Lett, L3).

build_letters([], []).
build_letters([C|L], R) :- build_letters(L, TR), m_concat(TR, C, R).

filter_text([], _, []).
filter_text([C|L1], Lett, [C|L3]) :- has_uniq_letter(C, Lett), !,
        filter_text(L1, Lett, L3).

filter_text([_|L1], Lett, L3) :- filter_text(L1, Lett, L3).

has_uniq_letter(C, Lett) :- elem(C, X), not(elem(Lett, X)).
```