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Решение задач по курсу математической логики

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1 Формулы логики предикатов

Упражнение 1.1

1. $\Sigma = \{L^2\}$ $\quad L(x,y) - x$ *любит* y

   $\forall x\ L(x,x) \quad \to \quad \exists y\ \exists z\ L(y,z)$

2. $\Sigma = \{P^1, M^1, S^1, C^2, a, b\}$ $\quad \left\{ \begin{array}{lll} P(x) & - & x \text{ - задача} \\ M(x) & - & x \text{ - математик} \\ S(x) & - & \text{задача } x \text{ - разрешима} \\ C(x,y) & - & \text{математик } x \text{ может решить задача } y \\ a & - & \text{константа «Я»} \\ b & - & \text{константа «эта задача»} \end{array} \right\}$

   $(\forall x (P(x) \& S(x) \ \to\ \exists y (M(y)\ \&\ C(y,x)))) \ \& \ (M(a) \& \neg C(a,b)) \ \to\ S(b)$

3. $\Sigma = \{C^3, a\}$ $\quad \left\{ \begin{array}{lll} C(x,y,t) & - & x \text{ может обмануть } y \text{ в момент времени } t \\ a & - & \text{Константа «Вы»} \end{array} \right\}$

   $(\exists t\ \forall x\ C(a,x,t))\ \&\ (\exists x\ \forall t\ (Ca,x,t))\ \&\ \neg(\forall x\ \forall t\ C(a,x,t))$

Упражнение 1.2

1. $\exists x\ (\forall y\ (B(y)\ \&\ C(y)\ \&\ U(x,y)))\ \&\ S(x)$
2. $\forall x\ \forall y\ (B(x)\ \&\ S(x)\ \&\ W(y)\ \&\ C(y)\ \to\ \neg U(y,x))$
3. $\forall x\ (B(x)\ \to\ (S(x)\ \& (\forall y\ (W(y)\ \&\ C(y)\ \to\ U(y,x)))) \vee (C(x)\ \&\ (\exists\ y\ (S(y)\ \&\ U(x,y)))))$
4. $\forall x\ \forall y\ (B(x)\ \&\ C(x)\ \&\ W(y)\ \&\ S(y)\ \to \neg(U(x,y) \vee U(y,x)))$
5. $(\forall x\ (S(x)\ \to\ B(x)))\ \to\ (\forall y\ (C(y)\ \to\ \neg W(y)))$
6. $\forall x\ (\neg(C(x)\ \&\ W(x)\ \&\ (\exists z\ (S(z)\ \&\ U(x,z))))\ \to\ B(x)\ \&\ (\forall z\ (W(x)\ \to\ U(z,x))))$

Упражнение 1.3

1. $\forall x\ \forall y\ (P(x)\ \&\ P(y)\ \&\ \neg E(x,y)\ \to$
   $\exists k\ (L(k)\ \&\ B(x,k)\ \&\ B(y,k)\ \&\ (\forall s\ (L(s)\ \&\ B(x,s)\ \&\ B(y,s)\ \to\ E(k,s)))))$
2. $\forall i\ (P(i)\ \&\ L(x)\ \&\ L(y)\ \&\ B(i,x)\ \to\ \neg B(i,y)) \quad [= Par(x,y)]$
3. $\forall x\ (L(x)\ \to\ \forall y\ (P(y)\ \&\ \neg B(y,x)\ \to\ \exists k\ (L(k)\ \&\ B(y,k)\ \&$
   $Par(x,k)\ \&\ \forall s\ (L(s)\ \&\ B(y,s)\ \&\ Par(x,s)\ \to\ E(k,s)))))$

Упражнение 1.4

1. $Z(x) = \forall y\ S(y,x,y)$
2. $O(x) = \forall y\ P(y,x,y)$
3. $T(x) = \exists k\ \forall y\ (P(y,k,y)\ \&\ S(k,k,x))$
4. $\exists y\ (Z(y)\ \&\ S(x,y,n))$
5. $\exists y\ \exists z\ (T(y)\ \&\ P(z,y,x))$
6. $(\forall k\ \forall l\ (P(k,l,x)\ \to (O(k) \vee O(l))))\ \&\ \neg(O(x) \vee Z(x)))$

1. $E(x,y) = \exists k\ ((\forall y\ S(y,k,y))\ \&\ S(x,k,y))$
2. $L(x,y) = \exists k\ ((\exists x\ \neg S(k,x,k))\ \&\ S(x,k,y))$
3. $F(x,y) = \exists k\ P(y,k,x)$

2 Вывод семантических таблиц

Упражнение 2.1

1. $\exists x\ P(x)\ \&\ \exists x \neg\ P(x)$

   - *Выполнима*
     $D_I = \{0, 1\},\ \overline{\boldsymbol{P}}(0) = \boldsymbol{true}, \overline{\boldsymbol{P}}(1) = \boldsymbol{false}$
   - *Не общезначима*
     $D_I = \{0\},\ \overline{\boldsymbol{P}}(0) = \boldsymbol{true}$

2. $\exists x\ P(x)\ \vee\ \exists x \neg\ P(x)$

   - *Общезначима*

$$\langle \emptyset \mid \exists x\ P(x)\ \vee\ \exists x \neg\ P(x)\rangle$$

$$\Big\downarrow R\vee$$

$$\langle \emptyset \mid \exists x\ P(x),\ \exists x \neg\ P(x)\rangle$$

$$\Big\downarrow R\exists$$

$$\langle \emptyset \mid \exists x\ P(x),\ \exists x \neg\ P(x), P(c_1)\rangle$$

$$\Big\downarrow R\exists$$

$$\langle \emptyset \mid \exists x\ P(x),\ \exists x \neg\ P(x),\ P(c_1),\ \neg P(c_1)\rangle$$

$$\Big\downarrow R\neg$$

$$\left\langle \underline{P(c_1)} \mid \exists x\ P(x),\ \exists x \neg\ P(x),\ \underline{P(c_1)}\right\rangle$$

Закрытая таблица

3. $\exists x\ \forall y\ (P(x)\ \&\ \neg P(y))$

   - *Невыполнима*
     Докажем невыполнимость путем доказательства общезначимости отрицания

$$\langle \exists x\ \forall y\ (P(x)\ \&\ \neg P(y)) | \emptyset\rangle$$

$$\Big\downarrow L\exists$$

$$\langle \forall y\ (P(c_1)\ \&\ \neg P(y)) | \emptyset\rangle$$

$$\Big\downarrow L\forall$$

$$\langle \forall y\ (P(c_1)\ \&\ \neg P(y)), P(c_1)\ \&\ \neg P(c_1) | \emptyset\rangle$$

$$\Big\downarrow L\&$$

$$\langle \forall y\ (P(c_1)\ \&\ \neg P(y)), P(c_1),\ \neg P(c_1) | \emptyset\rangle$$

$$\Big\downarrow L\neg$$

$$\left\langle \forall y\ (P(c_1)\ \&\ \neg P(y)), \underline{P(c_1)} | \underline{P(c_1)}\right\rangle$$

Закрытая таблица

4. $P(x)\ \rightarrow\ \forall x\ P(x)$

   - *Выполнима*
     $D_I = \{0\}, \overline{\boldsymbol{P}}(0) = \boldsymbol{true}$
   - *Не общезначима*
     $D_I = \{0, 1\}, \overline{\boldsymbol{P}}(0) = \boldsymbol{true}, \overline{\boldsymbol{P}}(1) = \boldsymbol{false}$

5. $\forall x\ P(x)\ \rightarrow\ P(x)$

- *Общезначима (очевидно)*

6. $\forall y\ \exists x\ R(x,y)\ \rightarrow\ \exists x\ \forall y\ R(x,y)$

   - *Выполнима*
     $D_I = \{0\},\ \overline{\boldsymbol{R}}(0,0) = \boldsymbol{true}$
   - *Не общезначима*
     $D_I = N,\ \overline{\boldsymbol{R}}(x,y) = \boldsymbol{x > y}$

7. $(\forall x\ P(x)\ \rightarrow\ \forall x\ Q(x))\ \rightarrow\ \forall x\ (P(x)\ \rightarrow\ Q(x))$

   - *Выполнима*
     $D_I = N,\ \overline{\boldsymbol{P}}(x) = \overline{\boldsymbol{Q}}(x)$
   - *Не общезначима*
     $D_I = N, \overline{\boldsymbol{P}}(x) = (\boldsymbol{x\ mod\ 2\ ==\ 0}),\ \overline{\boldsymbol{Q}}(x) = (\boldsymbol{x\ mod\ 4\ ==\ 0})$

Упражнение 2.2

1. $\exists x\ P(x)\ \rightarrow\ \neg\forall x\ \neg P(x)$

$$T_\phi = \langle \emptyset \mid \exists x\ P(x)\ \rightarrow\ \neg\forall x\ \neg P(x)\rangle$$
$$\Big\downarrow R\rightarrow$$
$$T_1 = \langle \exists x\ P(x) \mid \neg\forall x\ \neg P(x)\rangle$$
$$\Big\downarrow L\exists$$
$$T_2 = \langle P(c_1) \mid \neg\forall x\ \neg P(x)\rangle$$
$$\Big\downarrow R\neg$$
$$T_3 = \langle P(c_1),\ \forall x\ \neg P(x) \mid \emptyset\rangle$$
$$\Big\downarrow L\forall$$
$$T_4 = \langle P(c_1),\ \forall x\ \neg P(x),\ \neg P(c_1) \mid \emptyset\rangle$$
$$\Big\downarrow L\neg$$
$$T_5 = \Big\langle \underline{P(c_1)},\ \forall x\ \neg P(x) \mid \underline{P(c_1)}\Big\rangle$$

Закрытая таблица

2. $\exists x\forall y\ R(x,y)\ \rightarrow\ \forall y\exists x\ R(x,y)$

$$T_\phi = \langle \emptyset \mid \exists x\forall y\ R(x,y)\ \rightarrow\ \forall y\exists x\ R(x,y)\rangle$$
$$\Big\downarrow R\rightarrow$$
$$T_\phi = \langle \exists x\forall y\ R(x,y) \mid \forall y\exists x\ R(x,y)\rangle$$
$$\Big\downarrow L\exists$$
$$T_\phi = \langle \forall y\ R(c_1,y) \mid \forall y\exists x\ R(x,y)\rangle$$
$$\Big\downarrow R\forall$$
$$T_\phi = \langle \forall y\ R(c_1,y) \mid \exists x\ R(x,c_2)\rangle$$
$$\Big\downarrow R\exists$$
$$T_\phi = \langle \forall y\ R(c_1,y) \mid \exists x\ R(x,c_2),\ R(c_1,c_2)\rangle$$
$$\Big\downarrow L\forall$$
$$T_\phi = \Big\langle \forall y\ R(c_1,y)\ ,\underline{R(c_1,c_2)} \mid \exists x\ R(x,c_2),\ \underline{R(c_1,c_2)}\Big\rangle$$

Закрытая таблица

3. $\forall x\ (P(x)\ \to \exists y\ R(x,f(y)))\ \to\ (\exists x\ \neg P(x)\ \vee\ \forall x \exists z\ R(x,z))$

$$T_\phi = \langle \emptyset \mid \forall x\ (P(x)\ \to \exists y\ R(x,f(y)))\ \to\ (\exists x\ \neg P(x)\ \vee\ \forall x \exists z\ R(x,z)) \rangle$$

$$\big\downarrow R\to$$

$$T_1 = \langle \forall x\ (P(x)\ \to\ \exists y\ R(x,f(y))) \mid \exists x\ \neg P(x)\ \vee\ \forall x \exists z\ R(x,z) \rangle$$

$$\big\downarrow R\vee$$

$$T_2 = \langle \forall x\ (P(x)\ \to\ \exists y\ R(x,f(y))) \mid \exists x\ \neg P(x),\ \forall x \exists z\ R(x,z) \rangle$$

$$\big\downarrow R\forall$$

$$T_3 = \langle \forall x\ (P(x)\ \to\ \exists y\ R(x,f(y))) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z) \rangle$$

$$\big\downarrow L\forall$$

$$T_4 = \left\langle \overbrace{\forall x\ (P(x)\ \to\ \exists y\ R(x,f(y)))}^{\phi_1},\ P(c_1)\ \to\ \exists y\ R(c_1,f(y)) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z) \right\rangle$$

$$L\to \big\downarrow \qquad \big\downarrow L\to$$

$$T_{4.1} = \langle \phi_1,\ \exists y\ R(c_1,f(y)) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z) \rangle \qquad T_{4.2} = \langle \phi_1,\ \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z),\ P(c_1) \rangle$$

$$L\exists \big\downarrow \qquad \big\downarrow R\exists$$

$$T_{5.1} = \langle \phi_1,\ R(c_1,f(c_2)) \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z) \rangle \qquad T_{5.2} = \langle \phi_1,\ \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z),\ P(c_1),\ \neg P(c_1) \rangle$$

$$R\exists \big\downarrow \qquad \big\downarrow R/$$

$$T_{5.1} = \left\langle \phi_1,\ \underline{R(c_1,f(c_2))} \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z), \underline{R(c_1,f(c_2))} \right\rangle \qquad T_{5.2} = \left\langle \phi_1,\ \underline{P(c_1)} \mid \exists x\ \neg P(x),\ \exists z\ R(c_1,z),\ \underline{P(c_1)} \right\rangle$$

Закрытые таблицы

4. $\forall x \exists y \forall z (P(x,y)\ \to\ P(y,z))$

$$T_\phi = \langle \emptyset \mid \forall x \exists y \forall z (P(x,y)\ \to\ P(y,z)) \rangle$$

$$\big\downarrow R\forall$$

$$T_1 = \langle \emptyset \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)) \rangle$$

$$\big\downarrow R\exists$$

$$T_2 = \langle \emptyset \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)), \forall z (P(c_1,c_1)\ \to\ P(c_1,z)) \rangle$$

$$\big\downarrow R\forall$$

$$T_3 = \langle \emptyset \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)), P(c_1,c_1)\ \to\ P(c_1,c_2) \rangle$$

$$\big\downarrow R\to$$

$$T_4 = \langle P(c_1,c_1) \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ P(c_1,c_2) \rangle$$

$$\big\downarrow R\exists$$

$$T_5 = \langle P(c_1,c_1) \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ P(c_1,c_2), \forall z (P(c_1,c_2)\ \to\ P(c_2,z)) \rangle$$

$$\big\downarrow R\forall$$

$$T_6 = \langle P(c_1,c_1) \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ P(c_1,c_2), P(c_1,c_2)\ \to\ P(c_2,c_3) \rangle$$

$$\big\downarrow R\to$$

$$T_7 = \left\langle P(c_1,c_1),\ \underline{P(c_1,c_2)} \mid \exists y \forall z (P(c_1,y)\ \to\ P(y,z)),\ \underline{P(c_1,c_2)},\ P(c_2,c_3) \right\rangle$$

Закрытая таблица

5. $\exists x \forall y \exists z (P(x,y) \;\to\; P(y,z))$

$$T_\phi = \left\langle \emptyset \mid \underbrace{\exists x \forall y \exists z (P(x,y) \;\to\; P(y,z))}_{\phi_1} \right\rangle$$

$$\Big\downarrow R\exists$$

$$T_1 = \langle \emptyset \mid \phi_1,\ \forall y \exists z (P(c_1,y) \;\to\; P(y,z)) \rangle$$

$$\Big\downarrow R\forall$$

$$T_2 = \left\langle \emptyset \mid \phi_1,\ \underbrace{\exists z (P(c_1,c_2) \;\to\; P(c_2,z))}_{\phi_2} \right\rangle$$

$$\Big\downarrow R\exists$$

$$T_3 = \langle \emptyset \mid \phi_1,\ \phi_2,\ P(c_1,c_2) \;\to\; P(c_2,c_1),\ P(c_1,c_2) \;\to\; P(c_2,c_2) \rangle$$

$$\Big\downarrow R\to$$

$$T_4 = \langle P(c_1,c_2) \mid \phi_1,\ \phi_2,\ P(c_2,c_1),\ P(c_2,c_2) \rangle$$

$$\Big\downarrow R\exists \quad (\phi_1)$$

$$T_5 = \langle P(c_1,c_2) \mid \phi_1,\ \phi_2,\ P(c_2,c_1),\ P(c_2,c_2),\ \forall y \exists z (P(c_2,y) \;\to\; P(y,z)) \rangle$$

$$\Big\downarrow R\forall$$

$$T_6 = \left\langle P(c_1,c_2) \mid \phi_1,\ \phi_2,\ P(c_2,c_1),\ P(c_2,c_2),\ \underbrace{\exists z (P(c_2,c_3) \;\to\; P(c_3,z))}_{\phi_3} \right\rangle$$

$$\Big\downarrow R\exists$$

$$T_7 = \langle P(c_1,c_2) \mid \phi_1,\ \phi_2,\ \phi_3,\ P(c_2,c_1),\ P(c_2,c_2),\ P(c_2,c_3) \;\to\; P(c_3,c_3) \rangle$$

$$\Big\downarrow R\to$$

$$T_8 = \langle P(c_1,c_2),\ P(c_2,c_3) \mid \phi_1,\ \phi_2,\ \phi_3,\ P(c_2,c_1),\ P(c_2,c_2),\ P(c_3,c_3) \rangle$$

$$\Big\downarrow R\exists \quad (\phi_2)$$

$$T_9 = \langle P(c_1,c_2),\ P(c_2,c_3) \mid \phi_1,\ \phi_2,\ \phi_3,\ P(c_2,c_1),\ P(c_2,c_2),\ P(c_3,c_3),\ P(c_1,c_2) \;\to\; P(c_2,c_3) \rangle$$

$$\Big\downarrow R\to$$

$$T_{10} = \left\langle P(c_1,c_2),\ \underline{P(c_2,c_3)} \mid \phi_1,\ \phi_2,\ \phi_3,\ P(c_2,c_1),\ P(c_2,c_2),\ P(c_3,c_3),\ \underline{P(c_2,c_3)} \right\rangle$$

Закрытая таблица

6. $\forall x\ (P(x)\ \&\ R(x)) \;\to\; (\forall x\ P(x)\ \&\ \forall x\ R(x))$

$$T_\phi = \langle \emptyset \mid \forall x\ (P(x)\ \&\ R(x)) \ \rightarrow\ (\forall x\ P(x)\ \&\ \forall x\ R(x))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \forall x\ (P(x)\ \&\ R(x)) \mid \forall x\ P(x)\ \&\ \forall x\ R(x)\rangle$$

$$\Big\downarrow R\& \qquad \Big\downarrow R\&$$

$$T_{2.1} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid \forall x\ R(x)\rangle \quad T_{2.2} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid \forall x\ P(x)\rangle$$

$$\Big\downarrow R\forall \qquad \Big\downarrow R\forall$$

$$T_{2.1} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid R(c_1)\rangle \quad T_{2.2} = \langle \forall x\ (P(x)\ \&\ R(x)) \mid P(c_1)\rangle$$

$$\Big\downarrow L\forall \qquad \Big\downarrow L\forall$$

$$T_{3.1} = \langle \forall x\ (P(x)\ \&\ R(x)),\ P(c_1)\ \&\ R(c_1) \mid R(c_1)\rangle \quad T_{3.2} = \langle \forall x\ (P(x)\ \&\ R(x)),\ P(c_1)\ \&\ R(c_1) \mid P(c_1)\rangle$$

$$\Big\downarrow L\& \qquad \Big\downarrow L\&$$

$$T_{3.1} = \Big\langle \forall x\ (P(x)\ \&\ R(x)),\ P(c_1),\ \underline{R(c_1)} \mid \underline{R(c_1)}\Big\rangle \quad T_{3.2} = \Big\langle \forall x\ (P(x)\ \&\ R(x)),\ \underline{P(c_1)},\ R(c_1) \mid \underline{P(c_1)}\Big\rangle$$

Закрытые таблицы

7. $(\forall x\ P(x)\ \&\ \forall x\ R(x)) \ \rightarrow\ \forall x\ (P(x)\ \&\ R(x))$

$$T_\phi = \langle \emptyset \mid (\forall x\ P(x)\ \&\ \forall x\ R(x)) \ \rightarrow\ \forall x\ (P(x)\ \&\ R(x))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \forall x\ P(x)\ \&\ \forall x\ R(x)) \mid \forall x\ (P(x)\ \&\ R(x))\rangle$$

$$\Big\downarrow L\&$$

$$T_2 = \langle \forall x\ P(x),\ \forall x\ R(x)) \mid \forall x\ (P(x)\ \&\ R(x))\rangle$$

$$\Big\downarrow R\forall$$

$$T_3 = \langle \forall x\ P(x),\ \forall x\ R(x)) \mid P(c_1)\ \&\ R(c_1)\rangle$$

$$\Big\downarrow L\forall$$

$$T_4 = \langle \forall x\ P(x),\ \forall x\ R(x),\ P(c_1),\ R(c_1) \mid P(c_1)\ \&\ R(c_1)\rangle$$

$$\Big\downarrow R\& \qquad \Big\downarrow R\&$$

$$T_{4.1} = \Big\langle \forall x\ P(x),\ \forall x\ R(x),\ P(c_1),\ \underline{R(c_1)} \mid \underline{R(c_1)}\Big\rangle \quad T_{4.2} = \Big\langle \forall x\ P(x),\ \forall x\ R(x),\ \underline{P(c_1)},\ R(c_1) \mid \underline{P(c_1)}\Big\rangle$$

Закрытые таблицы

8. $\exists x\ (P(x)\ \vee\ R(x)) \ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ R(x))$

$$T_\phi = \langle \emptyset \;|\exists x\ (P(x)\ \lor\ R(x))\ \rightarrow\ (\exists x\ P(x)\ \lor\ \exists x\ R(x))\ \rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \exists x\ (P(x)\ \lor\ R(x))\ |\ \exists x\ P(x)\ \lor\ \exists x\ R(x)\ \rangle$$

$$\Big\downarrow L\exists$$

$$T_2 = \langle P(c_1)\ \lor\ R(c_1)\ |\ \exists x\ P(x)\ \lor\ \exists x\ R(x)\ \rangle$$

$$\Big\downarrow R\lor$$

$$T_3 = \langle P(c_1)\ \lor\ R(c_1)\ |\ \exists x\ P(x),\ \exists x\ R(x)\ \rangle$$

$$\Big\downarrow R\exists$$

$$T_4 = \langle P(c_1)\ \lor\ R(c_1)\ |\ \exists x\ P(x),\ \exists x\ R(x),\ P(c_1),\ R(c_1)\ \rangle$$

$$\Big\downarrow L\lor \qquad \Big\downarrow L\lor$$

$$T_{5.1} = \left\langle \underline{R(c_1)}\ |\ \exists x\ P(x),\ \exists x\ R(x),\ P(c_1),\ \underline{R(c_1)}\ \right\rangle \quad T_{5.2} = \left\langle \underline{P(c_1)}\ |\ \exists x\ P(x),\ \exists x\ R(x),\ \underline{P(c_1)},\ R(c_1)\ \right\rangle$$

Закрытые таблицы

9. $(\exists x\ P(x)\ \lor\ \exists x\ R(x))\ \rightarrow\ \exists x\ (P(x)\ \lor\ R(x))$

$$T_\phi = \langle \emptyset\ |\ (\exists x\ (P(x)\ \lor\ \exists x\ R(x))\ \rightarrow\ \exists x\ (P(x)\ \lor\ R(x))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \exists x\ (P(x)\ \lor\ \exists x\ R(x)\ |\ \exists x\ (P(x)\ \lor\ R(x))\rangle$$

$$\Big\downarrow L\lor \qquad \Big\downarrow L\lor$$

$$T_{2.1} = \langle \exists x\ R(x)\ |\ \exists x\ (P(x)\ \lor\ R(x))\rangle \quad T_{2.2} = \langle \exists x\ P(x)\ |\ \exists x\ (P(x)\ \lor\ R(x))\rangle$$

$$\Big\downarrow L\exists \qquad \Big\downarrow L\exists$$

$$T_{3.1} = \langle R(c_1)\ |\ \exists x\ (P(x)\ \lor\ R(x))\rangle \quad T_{3.2} = \langle P(c_1)\ |\ \exists x\ (P(x)\ \lor\ R(x))\rangle$$

$$\Big\downarrow R\exists \qquad \Big\downarrow R\exists$$

$$T_{4.1} = \langle R(c_1)\ |\ \exists x\ (P(x)\ \lor\ R(x)),\ P(c_1)\ \lor\ R(c_1)\rangle \quad T_{4.2} = \langle P(c_1)\ |\ \exists x\ (P(x)\ \lor\ R(x)),\ P(c_1)\ \lor\ R(c_1)\rangle$$

$$\Big\downarrow R\lor \qquad \Big\downarrow R\lor$$

$$T_{5.1} = \left\langle \underline{R(c_1)}\ |\ \exists x\ (P(x)\ \lor\ R(x)),\ P(c_1),\ \underline{R(c_1)}\right\rangle \quad T_{5.2} = \left\langle \underline{P(c_1)}\ |\ \exists x\ (P(x)\ \lor\ R(x)),\ \underline{P(c_1)},\ R(c_1)\right\rangle$$

Закрытые таблицы

10. $(\forall x\ P(x)\ \lor\ R(y))\ \rightarrow\ \forall x\ (P(x)\ \lor\ R(y))$

$$T_\phi = \langle \emptyset \mid (\forall x\ P(x)\ \vee\ R(y))\ \rightarrow\ \forall x\ (P(x)\ \vee\ R(y))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \forall x\ P(x)\ \vee\ R(y) \mid \forall x\ (P(x)\ \vee\ R(y))\rangle$$

$$\Big\downarrow R\forall$$

$$T_2 = \langle \forall x\ P(x)\ \vee\ R(y) \mid P(c_1)\ \vee\ R(y)\rangle$$

$$\Big\downarrow R\vee$$

$$T_3 = \langle \forall x\ P(x)\ \vee\ R(y) \mid P(c_1),\ R(y)\rangle$$

$$\Big\downarrow R\vee \qquad\qquad \Big\downarrow R\vee$$

$$T_{4.1} = \langle \forall x\ P(x) \mid P(c_1),\ R(y)\rangle \quad T_{4.2} = \Big\langle \underline{R(y)} \mid P(c_1),\ \underline{R(y)}\Big\rangle$$

$$\Big\downarrow R\forall$$

$$T_{5.1} = \Big\langle \forall x\ P(x),\ \underline{P(c_1)} \mid \underline{P(c_1)},\ R(y)\Big\rangle$$

Закрытые таблицы

11. $\forall x\ (P(x)\ \vee\ R(y))\ \rightarrow\ (\forall x\ P(x)\ \vee\ R(y))$

$$T_\phi = \langle \emptyset \mid \forall x\ (P(x)\ \vee\ R(y))\ \rightarrow\ (\forall x\ P(x)\ \vee\ R(y))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \forall x\ (P(x)\ \vee\ R(y)) \mid (\forall x\ P(x)\ \vee\ R(y))\rangle$$

$$\Big\downarrow R\vee$$

$$T_2 = \langle \forall x\ (P(x)\ \vee\ R(y)) \mid \forall x\ P(x),\ R(y)\rangle$$

$$\Big\downarrow R\forall$$

$$T_3 = \langle \forall x\ (P(x)\ \vee\ R(y)) \mid P(c_1),\ R(y))\rangle$$

$$\Big\downarrow L\forall$$

$$T_4 = \langle \forall x\ (P(x)\ \vee\ R(y)),\ P(c_1)\ \vee\ R(y) \mid P(c_1),\ R(y))\rangle$$

$$\Big\downarrow R\vee \qquad\qquad \Big\downarrow R\vee$$

$$T_{5.1} = \Big\langle \forall x\ (P(x)\ \vee\ R(y)),\ \underline{P(c_1)} \mid \underline{P(c_1)},\ R(y)\Big\rangle \quad T_{5.2} = \Big\langle \forall x\ (P(x)\ \vee\ R(y)),\ \underline{R(y)} \mid P(c_1),\ \underline{R(y)}\Big\rangle$$

Закрытые таблицы

12. $\exists y \forall x\ Q(x,y)\ \rightarrow\ \forall x \exists y\ Q(x,y)$

$$T_\phi = \langle \emptyset \mid \exists y \forall x\ Q(x,y)\ \rightarrow\ \forall x \exists y\ Q(x,y)\rangle$$

$$\Big\downarrow R\rightarrow$$

$$T_1 = \langle \exists y \forall x\ Q(x,y) \mid \forall x \exists y\ Q(x,y)\rangle$$

$$\Big\downarrow R\forall$$

$$T_2 = \langle \exists y \forall x\ Q(x,y) \mid \exists y\ Q(c_1,y)\rangle$$

$$\Big\downarrow L\exists$$

$$T_3 = \langle \forall x\ Q(x,c_2) \mid \exists y\ Q(c_1,y)\rangle$$

$$\Big\downarrow L\forall$$

$$T_4 = \langle \forall x\ Q(x,c_2),\ Q(c_1,c_2) \mid \exists y\ Q(c_1,y)\rangle$$

$$\Big\downarrow R\exists$$

$$T_4 = \Big\langle \forall x\ Q(x,c_2),\ \underline{Q(c_1,c_2)} \mid \exists y\ Q(c_1,y),\ \underline{Q(c_1,c_2)}\Big\rangle$$

Закрытая таблица

Упражнение 2.3

1. $\forall x\ (P(x)\ \vee\ Q(x))\ \rightarrow\ (\forall x\ P(x)\ \vee\ \forall x\ Q(x))$
   Вывод не будет успешным так как формула не общезначима.
   $D_I = N, \overline{\boldsymbol{P}}(x) = (\boldsymbol{x\ mod\ 2\ ==\ 0}),\ \overline{\boldsymbol{Q}}(x) = (\boldsymbol{x\ mod\ 2\ ==\ 1})$

2. $\exists x\ (P(x)\ \vee\ Q(x))\ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ Q(x))$
   Построим вывод

$$\langle \emptyset | \exists x\ (P(x)\ \vee\ Q(x))\ \rightarrow\ (\exists x\ P(x)\ \vee\ \exists x\ Q(x))\rangle$$

$$\Big\downarrow R\rightarrow$$

$$\langle \exists x\ (P(x)\ \vee\ Q(x)) | \exists x\ P(x)\ \vee\ \exists x\ Q(x)\rangle$$

$$\Big\downarrow \exists$$

$$\langle P(c_1)\ \vee\ Q(c_1) | \exists x\ P(x)\ \vee\ \exists x\ Q(x)\rangle$$

$$\Big\downarrow R\vee$$

$$\langle P(c_1)\ \vee\ Q(c_1) | \exists x\ P(x),\ \exists x\ Q(x)\rangle$$

$$\Big\downarrow R\exists$$

$$\langle P(c_1)\ \vee\ Q(c_1) | \exists x\ P(x),\ \exists x\ Q(x),\ P(c_1),\ Q(c_1)\rangle$$

$$L\vee \Big\downarrow \quad \Big\downarrow L\vee$$

$$\Big\langle \underline{P(c_1)} | \exists x\ P(x),\ \exists x\ Q(x),\ \underline{P(c_1)},\ Q(c_1)\Big\rangle \Big\langle \underline{Q(c_1)} | \exists x\ P(x),\ \exists x\ Q(x),\ P(c_1),\ \underline{Q(c_1)}\Big\rangle$$

Упражнение 2.4 *Пусть такая формула существует. Рассмотрим ее на интерпретации, область которой содержит три элемента. На данной интерпретации формула истинна. То есть для любой подстановки она истинна. Следовательно существует подстановка, состоящая из 1 или 2 объектов, на которой формула так же истинна. Следовательно формула истинна на интерпретации, область которой содержит только эти 2 объекта, следовательно такой формулы нет.*

Упражнение 2.5 *Данная задача была решена некорректно.*

3 Нормальные формы и унификация

3.1 Преведение к ССФ

1. Переименование переменных
   $\models \ \substack{\exists\\ \forall} \ x \ F(x) \ \equiv \ \substack{\exists\\ \forall} \ y \ F(y)$

2. Уничтожение импликаций $\models \ (A \ \to \ B) \ \equiv \ (\neg A \ \vee \ B)$

3. Отрицания

   (a) $\models \ \neg(A \substack{\&\\ \vee} B) \ \equiv \ (\neg A \substack{\vee\\ \&} \neg B)$

   (b) $\models \ (\neg \substack{\exists\\ \forall} x F(x)) \equiv (\substack{\forall\\ \exists} x \neg F(x))$

   (c) $\models \ \neg\neg A \ \equiv \ A$

4. Вынос кванторов
   $\models \ \substack{\exists\\ \forall} x F(x) \ \substack{\&\\ \vee} \ B \ \equiv \ \substack{\exists\\ \forall} x (F(x) \ \substack{\&\\ \vee} \ B)$

5. $\models \ A \,\&\, B \ \vee \ C \ \equiv (A \vee C) \ \& \ (B \vee C)$

3.2 Нахождение НОУ

$$P(t_1, t_2, \ldots t_n) \quad = \quad P(s_1, s_2, \ldots s_n) \quad \to \quad \begin{cases} t_1 & = & s_1 \\ t_2 & = & s_2 \\ \ldots & \ldots & \ldots \\ t_n & = & s_n \end{cases}$$

1. $\{ \ f(t_1, t_2, \ldots t_n) \quad = \quad f(s_1, s_2, \ldots s_n) \quad \to \quad \begin{cases} t_1 & = & s_1 \\ t_2 & = & s_2 \\ \ldots & \ldots & \ldots \\ t_n & = & s_n \end{cases}$

2. $\{ \ f(t_1, t_2, \ldots t_n) \quad = \quad f(s_1, s_2, \ldots s_k) \quad \to$ НОУ не существует

3. $t_i = x_i \quad \to \quad x_1 = t_i \quad (t_i \neq x_i)$

4. $t_i = t_i \quad \to \quad \emptyset$

5. $x_i = t_i \quad (x_i \notin Var_{t_i}, \ \exists k \ x_i \in Var_{t_k}) \quad \to$ Во все t_k подствить вместо x_i t_i

6. $x_i = t_i \quad (x_i \in Var_{t_i}) \quad \to$ НОУ не существует

3.3 Задачи

Упражнение 3.1

1. $\exists x \forall y \ P(x, y) \ \& \ \forall x \exists y \ P(y, x)$

$\exists x \forall y \ P(x, y) \ \& \ \forall x \exists y \ P(y, x) \ \xrightarrow{1} \ \exists x_1 \forall y_1 \ P(x_1, y_1) \ \& \ \forall x_2 \exists y_2 \ P(y_2, x_2)$
$\exists x_1 \forall y_1 \ P(x_1, y_1) \ \& \ \forall x_2 \exists y_2 \ P(y_2, x_2) \ \xrightarrow{4} \ \exists x_1 \forall y_1 \forall x_2 \exists y_2 \ P(x_1, y_1) \ \& \ P(y_2, x_2)$

2. $\forall x ((\exists y \ P(y, x) \ \to \ \exists y \ P(x, y)) \ \to \ Q(x)) \ \to \ \exists x \ Q(x)$

$\forall x ((\exists y \ P(y, x) \ \to \ \exists y \ P(x, y)) \ \to \ Q(x)) \ \to \ \exists x \ Q(x) \ \xrightarrow{1}$
$\forall x_1 ((\exists y_1 \ P(y_1, x_1) \ \to \ \exists y_2 \ P(x_1, y_2)) \ \to \ Q(x_1)) \ \to \ \exists x_2 \ Q(x_2) \ \xrightarrow{2,3}$
$\exists x_1 \ ((\forall y_1 \ \neg P(y_1, x_1) \ \vee \ \exists y_2 \ P(x_1, y_2)) \ \& \ \neg Q(x_1)) \ \vee \ \exists x_2 \ Q(x_2) \ \xrightarrow{4}$
$\exists x_1 \forall y_1 \exists y_2 \forall x_2 \ ((\neg P(y_1, x_1) \ \vee \ P(x_1, y_2)) \ \& \ \neg Q(x_1) \ \vee \ Q(x_2)) \ \xrightarrow{5}$
$\exists x_1 \forall y_1 \exists y_2 \forall x_2 \ ((\neg P(y_1, x_1) \ \vee \ P(x_1, y_2) \ \vee \ Q(x_2)) \ \& \ (\neg Q(x_1) \ \vee \ Q(x_2)))$

3. $\neg\forall y\ (\exists x\ P(x,y)\ \rightarrow\ \forall u\ (R(y,u)\ \rightarrow\ \forall z\ (P(z,u)\ \vee\ \neg R(z,y))))$

$\neg\forall y\ (\exists x\ P(x,y)\ \rightarrow\ \forall u\ (R(y,u)\ \rightarrow\ \forall z\ (P(z,u)\ \vee\ \neg R(z,y)))) \xrightarrow{2,3}$
$\exists y\ (\exists x\ P(x)\ \&\ (\exists u(R(z,u)\ \&\ \forall z\ (P(z,u)\ \vee\ \neg R(z,y))))) \xrightarrow{4}$
$\exists y\exists x\exists u\forall z\ (P(x,y)\ \&\ R(y,u)\ \&\ (P(z,u)\ \vee\ R(z,y)))$

4. $\exists x\exists y\ (P(x,y)\ \rightarrow\ R(x))\ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y)\ \vee R(x))$

$\exists x\exists y\ (P(x,y)\ \rightarrow\ R(x))\ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y)\ \vee R(x)) \xrightarrow{1}$
$\exists x_1\exists y_1\ (P(x_1,y_1)\ \rightarrow\ R(x_1))\ \rightarrow\ \forall x_2\ (\neg\exists y_2\ P(x_2,y_2)\ \vee R(x_2)) \xrightarrow{2,3}$
$\forall x_1\forall x_2\ (P(x_1,y_2)\ \&\ \neg R(x_1))\ \vee\ \forall x_2(\forall y_2\ \neg P(x_2,y_2)\ \vee\ R(x_2)) \xrightarrow{4}$
$\forall x_1\forall x_2\forall x_2\forall y_2\ (P(x_1,y_2)\ \&\ \neg R(x_1)\ \vee\ \neg P(x_2,y_2)\ \vee\ R(x_2)) \xrightarrow{5}$
$\forall x_1\forall x_2\forall x_2\forall y_2\ ((P(x_1,y_2)\ \vee\ \neg P(x_2,y_2)\ \vee\ R(x_2))\ \&\ (\neg R(x_1)\ \vee\ \neg P(x_2,y_2)\ \vee\ R(x_2)))$

5. $\exists x\forall y\ (P(x,y)\ \rightarrow\ (\neg P(y,x)\ \rightarrow\ (P(x,x)\ \rightarrow\ P(y,y))\ \&(P(y,y)\ \rightarrow\ P(x,x))))$

$\exists x\forall y\ (P(x,y)\ \rightarrow\ (\neg P(y,x)\ \rightarrow\ (P(x,x)\ \rightarrow\ P(y,y))\ \&(P(y,y)\ \rightarrow\ P(x,x)))) \xrightarrow{2,3}$
$\exists x\forall y\ (\neg P(x,y)\ \vee\ P(y,x)\ \vee(\neg P(x,x)\ \vee\ P(y,y))\ \&\ (\neg P(y,y)\ \vee\ P(x,x))) \xrightarrow{5}$
$\exists x\forall y\ ((\neg P(x,y)\ \vee\ P(y,x)\ \vee\ \neg P(x,x)\ \vee\ P(y,y))\ \&\ (\neg P(x,y)\ \vee\ P(y,x)\ \vee\ \neg P(y,y)\ \vee\ P(x,x)))$

6. $\exists x\ (\forall x\ P(x,x)\ \vee\ \exists x\ \neg R(x))\ \rightarrow\ \exists x\ (R(x)\ \rightarrow\ \exists y\ P(f(x),y))$

$\exists x\ (\forall x\ P(x,x)\ \vee\ \exists x\ \neg R(x))\ \rightarrow\ \exists x\ (R(x)\ \rightarrow\ \exists y\ P(f(x),y)) \xrightarrow{1}$
$\exists k\ (\forall x_1\ P(x_1,x_1)\ \vee\ \exists x_2\ \neg R(x_2))\ \rightarrow\ \exists x_3\ (R(x_3)\ \rightarrow\ \exists y\ P(f(x_3),y)) \xrightarrow{2,3}$
$\exists k\ (\exists x_1\ \neg P(x_1,x_1)\ \&\ \forall x_2\ \neg R(x_2)\ \vee\ \exists x_3(\neg R(x_3)\ \vee \exists y\ R(f(x_3),y))) \xrightarrow{4}$
$\exists k\exists x_1\forall x_2\exists x_3\exists y\ (\neg P(x_1,x_1)\ \&\ \neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y)) \xrightarrow{5}$
$\exists k\exists x_1\forall x_2\exists x_3\exists y\ ((\neg P(x_1,x_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y)))$

Упражнение 3.2

1. $\forall x\exists y\forall z\exists u\ R(x,y,z,u)$

$\forall x\exists y\forall z\exists u\ R(x,y,z,u) \longrightarrow$
$\forall x\forall z\exists u\ R(x,f(x),z,u) \longrightarrow$
$\forall x\forall z\ R(x,f(x),z,g(x,z))$

2. $\neg\forall x\ (\exists y\ R(x,y)\ \rightarrow\ \forall z\ P(z,x)) \xrightarrow{2,3}$
$\exists z\ (\exists y\ R(x,y)\ \&\exists z\ \neg P(z,x)) \xrightarrow{4}$
$\exists x\exists y\exists z\ (R(x,y)\ \&\ \neg P(z,x)) \longrightarrow$
$\exists y\exists z\ (R(c_1,y)\ \&\ \neg P(z,c_1)) \longrightarrow$
$\exists z\ (R(c_1,c_2)\ \&\ \neg P(z,c_1)) \longrightarrow$
$R(c_1,c_2)\ \&\ \neg P(_3,c_1)$

3. $\neg\forall y\ (\exists x\ P(x,y)\ \rightarrow\ \forall u\ (R(y,u)\ \rightarrow\ \forall z\ (P(z,u)\ \vee\ \neg R(z,y))))$

$\neg\forall y\ (\exists x\ P(x,y)\ \rightarrow\ \forall u\ (R(y,u)\ \rightarrow\ \forall z\ (P(z,u)\ \vee\ \neg R(z,y)))) \xrightarrow{2,3}$
$\exists y\ (\exists x\ P(x)\ \&\ (\exists u(R(z,u)\ \&\ \forall z\ (P(z,u)\ \vee\ \neg R(z,y))))) \xrightarrow{4}$
$\exists y\exists x\exists u\forall z\ (P(x,y)\ \&\ R(y,u)\ \&\ (P(z,u)\ \vee\ R(z,y))) \longrightarrow$
$\exists x\exists u\forall z\ (P(x,c_1)\ \&\ R(c_1,u)\ \&\ (P(z,u)\ \vee\ R(z,c_1))) \longrightarrow$
$\exists u\forall z\ (P(c_2,c_1)\ \&\ R(c_1,u)\ \&\ (P(z,u)\ \vee\ R(z,c_1))) \longrightarrow$
$\forall z\ (P(c_2,c_1)\ \&\ R(c_1,c_3)\ \&\ (P(z,c_3)\ \vee\ R(z,c_1)))$

4. $\exists x\exists y\ (P(x,y)\ \rightarrow\ R(x))\ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y)\ \vee R(x))$

$\exists x\exists y\ (P(x,y)\ \rightarrow\ R(x))\ \rightarrow\ \forall x\ (\neg\exists y\ P(x,y)\ \vee R(x)) \xrightarrow{1}$

$\exists x_1 \exists y_1\ (P(x_1,y_1)\ \rightarrow\ R(x_1))\ \rightarrow\ \forall x_2\ (\neg \exists y_2\ P(x_2,y_2)\ \vee R(x_2)) \xrightarrow{2,3}$
$\forall x_1 \forall x_2\ (P(x_1,y_2)\ \&\ \neg R(x_1))\ \vee\ \forall x_2(\forall y_2\ \neg P(x_2,y_2)\ \vee\ R(x_2)) \xrightarrow{4}$
$\forall x_1 \forall x_2 \forall x_2 \forall y_2\ (P(x_1,y_2)\ \&\ \neg R(x_1)\ \vee\ \neg P(x_2,y_2)\ \vee\ R(x_2)) \xrightarrow{5}$
$\forall x_1 \forall x_2 \forall x_2 \forall y_2\ ((P(x_1,y_2)\ \vee\ \neg P(x_2,y_2)\ \vee\ R(x_2))\ \&\ (\neg R(x_1)\ \vee\ \neg P(x_2,y_2)\ \vee\ R(x_2)))$

5. $\exists x \forall y\ (P(x,y)\ \rightarrow\ (\neg P(y,x)\ \rightarrow\ (P(x,x)\ \rightarrow\ P(y,y))\ \&(P(y,y)\ \rightarrow\ P(x,x))))$

$\exists x \forall y\ (P(x,y)\ \rightarrow\ (\neg P(y,x)\ \rightarrow\ (P(x,x)\ \rightarrow\ P(y,y))\ \&(P(y,y)\ \rightarrow\ P(x,x)))) \xrightarrow{2,3}$
$\exists x \forall y\ (\neg P(x,y)\ \vee\ P(y,x)\ \vee (\neg P(x,x)\ \vee\ P(y,y))\ \&\ (\neg P(y,y)\ \vee\ P(x,x))) \xrightarrow{5}$
$\exists x \forall y\ ((\neg P(x,y)\ \vee\ P(y,x)\ \vee\ \neg P(x,x)\ \vee\ P(y,y))\ \&\ (\neg P(x,y)\ \vee\ P(y,x)\ \vee\ \neg P(y,y)\ \vee\ P(x,x))) \longrightarrow$
$\forall y\ ((\neg P(c,y)\ \vee\ P(y,c)\ \vee\ \neg P(c,c)\ \vee\ P(y,y))\ \&\ (\neg P(c,y)\ \vee\ P(y,c)\ \vee\ \neg P(y,y)\ \vee\ P(c,c)))$

6. $\exists x\ (\forall x\ P(x,x)\ \vee\ \exists x\ \neg R(x))\ \rightarrow\ \exists x\ (R(x)\ \rightarrow\ \exists y\ P(f(x),y))$

$\exists x\ (\forall x\ P(x,x)\ \vee\ \exists x\ \neg R(x))\ \rightarrow\ \exists x\ (R(x)\ \rightarrow\ \exists y\ P(f(x),y)) \xrightarrow{1}$
$\exists k\ (\forall x_1\ P(x_1,x_1)\ \vee\ \exists x_2\ \neg R(x_2))\ \rightarrow\ \exists x_3\ (R(x_3)\ \rightarrow\ \exists y\ P(f(x_3),y)) \xrightarrow{2,3}$
$\exists k\ (\exists x_1\ \neg P(x_1,x_1)\ \&\ \forall x_2\ \neg R(x_2)\ \vee\ \exists x_3(\neg R(x_3)\ \vee \exists y\ R(f(x_3),y))) \xrightarrow{4}$
$\exists k \exists x_1 \forall x_2 \exists x_3 \exists y\ (\neg P(x_1,x_1)\ \&\ \neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y)) \xrightarrow{5}$
$\exists k \exists x_1 \forall x_2 \exists x_3 \exists y\ ((\neg P(x_1,x_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))) \longrightarrow$
$\exists x_1 \forall x_2 \exists x_3 \exists y\ ((\neg P(x_1,x_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))) \longrightarrow$
$\forall x_2 \exists x_3 \exists y\ ((\neg P(c_1,c_1)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(x_3)\ \vee\ R(f(x_3),y))) \longrightarrow$
$\forall x_2 \exists y\ ((\neg P(c_1,c_1)\ \vee\ \neg R(g(x_2))\ \vee\ R(g(x_2),y))\ \&\ (\neg R(x_2)\ \vee\ \neg R(g(x_2))\ \vee\ R(f(g(x_2)),y))) \longrightarrow$
$\forall x_2\ ((\neg P(c_1,c_1)\ \vee\ \neg R(g(x_2))\ \vee\ R(g(x_2),h(x_2))\ \&\ (\neg R(x_2)\ \vee\ \neg R(g(x_2))\ \vee\ R(f(g(x_2)),h(x_2))))$

Упражнение 3.3 *Данная задача не рассматривалась на семинарах. Если будет время, ее решение будет добавленно. Если пришлете мне решение данной задачи, оно появится тут скорее ;-).*

Упражнение 3.4

1. $P(c,X,f(X)) \qquad P(c,Y,Y)$

$$\begin{cases} c = c \\ X = Y \\ f(X) = Y \end{cases} \xrightarrow{4} \begin{cases} X = Y \\ f(X) = Y \end{cases} \xrightarrow{3}$$

$$\begin{cases} X = Y \\ Y = f(X) \end{cases} \xrightarrow{5} \begin{cases} X = Y \\ \underline{Y} = \underline{f(Y)} \end{cases} \quad \text{НОУ Нет}$$

2. $P(f(X,Y),Z,h(Z,Y)) \qquad P(f(Y,X),g(Y),V)$

$$\begin{cases} f(X,Y) = f(Y,X) \\ Z = g(Y) \\ h(Z,Y) = V \end{cases} \xrightarrow{1} \begin{cases} X = Y \\ Y = X \\ Z = g(Y) \\ h(Z,Y) = V \end{cases} \xrightarrow{1}$$

$$\begin{cases} X = Y \\ Y = Y \\ Z = g(Y) \\ h(Z,Y) = V \end{cases} \xrightarrow{4} \begin{cases} X = Y \\ Z = g(Y) \\ h(Z,Y) = V \end{cases} \xrightarrow{3}$$

$$\begin{cases} X = Y \\ Z = g(Y) \\ V = h(Z,Y) \end{cases} \xrightarrow{3} \begin{cases} X = Y \\ Z = g(Y) \\ V = h(g(Y),Y) \end{cases} \quad \text{НОУ построен}$$

3. $R(Z, f(X,b,Z)) \qquad R(h(X), f(g(a),Y,Z))$

$$\left\{\begin{array}{ccc} Z & = & h(X) \\ f(X,b,Z) & = & f(g(a),Y,Z) \end{array}\right. \xrightarrow{1} \left\{\begin{array}{ccc} Z & = & h(X) \\ X & = & g(a) \\ b & = & Y \\ Z & = & Z \end{array}\right. \xrightarrow{4}$$

$$\left\{\begin{array}{ccc} Z & = & h(X) \\ X & = & g(a) \\ b & = & Y \end{array}\right. \xrightarrow{3} \left\{\begin{array}{ccc} Z & = & h(X) \\ X & = & g(a) \\ Y & = & b \end{array}\right. \xrightarrow{5}$$

$$\left\{\begin{array}{ccc} Z & = & h(g(a)) \\ X & = & g(a) \\ Y & = & b \end{array}\right. \quad \textit{НОУ построен}$$

4. $P(X, f(Y), h(Z,X)) \qquad P(f(Y), X, h(f(Y), f(Z)))$

$$\left\{\begin{array}{ccc} X & = & f(Y) \\ f(Y) & = & X \\ h(Z,X) & = & h(f(Y), f(Z)) \end{array}\right. \xrightarrow{1} \left\{\begin{array}{ccc} X & = & f(Y) \\ f(Y) & = & X \\ Z & = & f(Y) \\ X & = & f(Z) \end{array}\right. \xrightarrow{5}$$

$$\left\{\begin{array}{ccc} X & = & f(Y) \\ f(Y) & = & f(Y) \\ Z & = & f(Y) \\ f(Y) & = & f(Z) \end{array}\right. \xrightarrow{4} \left\{\begin{array}{ccc} X & = & f(Y) \\ Z & = & f(Y) \\ f(Y) & = & f(Z) \end{array}\right. \xrightarrow{1}$$

$$\left\{\begin{array}{ccc} X & = & f(Y) \\ Z & = & f(Y) \\ Y & = & Z \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} X & = & f(Y) \\ \underline{Z} & = & \underline{f(Z)} \\ Y & = & Z \end{array}\right. \quad \textit{НОУ Нет}$$

5. $P(X_1, X_2, X_3.X_4) \qquad P(f(c,c), f(X_1,X_1), f(X_2,X_2), f(X_3,X_3))$

$$\left\{\begin{array}{ccc} X_1 & = & f(c,c) \\ X_2 & = & f(X_1,X_1) \\ X_3 & = & f(X_2,X_2) \\ X_4 & = & f(X_3,X_3) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} X_1 & = & f(c,c) \\ X_2 & = & f(f(c,c),f(c,c)) \\ X_3 & = & f(X_2,X_2) \\ X_4 & = & f(X_3,X_3) \end{array}\right. \xrightarrow{5}$$

$$\left\{\begin{array}{ccc} X_1 & = & f(c,c) \\ X_2 & = & f(f(c,c),f(c,c)) \\ X_3 & = & f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c))) \\ X_4 & = & f(X_3,X_3) \end{array}\right. \xrightarrow{5}$$

$$\left\{\begin{array}{ccc} X_1 & = & f(c,c) \\ X_2 & = & f(f(c,c),f(c,c)) \\ X_3 & = & f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c))) \\ X_4 & = & f(f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c))),f(f(f(c,c),f(c,c)),f(f(c,c),f(c,c)))) \end{array}\right. \quad \textit{НОУ построен}$$

4 Метод резолюций

Упражнение 4.1

1. $\neg P(f(x,y),z,h(z,y)) \ \vee \ R(z,v), \ Q(x) \ \vee \ P(f(y,x),g(y),v)$

   $D_1 = \neg P(f(x,y),z,h(z,y)) \ \vee \ R(z,v)$

   $D_2 = Q(x) \ \vee \ P(f(y,x),g(y),v)$

   НОУ$(P(f(y,x),g(y),v), \neg P(f(x,y),z,h(z,y)))$

   $$\left\{\begin{array}{ccc} f(y,x) & = & f(x,y) \\ g(y) & = & z \\ v = & = & h(z,y) \end{array}\right. \xrightarrow{3} \left\{\begin{array}{ccc} f(y,x) & = & f(x,y) \\ z & = & g(y) \\ v = & = & h(z,y) \end{array}\right. \xrightarrow{1}$$

   $$\left\{\begin{array}{ccc} y & = & x \\ x & = & y \\ z & = & g(y) \\ v = & = & h(z,y) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} y & = & x \\ x & = & x \\ z & = & g(x) \\ v = & = & h(z,x) \end{array}\right. \xrightarrow{4}$$

   $$\left\{\begin{array}{ccc} y & = & x \\ z & = & g(x) \\ v = & = & h(z,x) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} y & = & x \\ z & = & g(x) \\ v = & = & h(g(x),x) \end{array}\right.$$

   $\Theta = \{y/x, z/g(x), v/h(g(x),x)\}$

   $D_3 \quad \underset{\Theta}{\overset{D_1,D_2}{=}} \quad R(g(x),h(g(x),x)) \ \vee \ Q(x)$

2. $P(x,y,h(y,x)) \ \vee \ R(y,f(x)), \ \neg P(x,f(x),h(x,y)) \ \vee \ P(y,g(x),h(y,y))$

   $D_1 = P(x_1,y_1,h(y_1,x_1)) \ \vee \ R(y_1,f(x_1))$

   $D_2 = \neg P(x_2,f(x_2),h(x_2,y_2)) \ \vee \ P(y_2,g(x_2),h(y_2,y_2))$

   НОУ$(P(x_1,y_1,h(y_1,x_1)), \vee P(y_2,g(x_2),h(y_2,y_2)))$

   $$\left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ h(y_1,x_1) & = & h(y_2,y_2) \end{array}\right. \xrightarrow{1} \left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ y_1 & = & y_2 \\ x_1 & = & y_2 \end{array}\right. \xrightarrow{3}$$

   $$\left\{\begin{array}{ccc} y_2 & = & x_1 \\ y_1 & = & g(x_2) \\ y_1 & = & y_2 \\ x_1 & = & y_2 \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} y_2 & = & x_1 \\ y_1 & = & g(x_2) \\ y_1 & = & x_1 \\ x_1 & = & x_1 \end{array}\right. \xrightarrow{4}$$

   $$\left\{\begin{array}{ccc} y_2 & = & x_1 \\ y_1 & = & g(x_2) \\ y_1 & = & y_2 \end{array}\right. \xrightarrow{3} \left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ y_2 & = & y_1 \end{array}\right. \xrightarrow{5}$$

   $$\left\{\begin{array}{ccc} x_1 & = & y_2 \\ y_1 & = & g(x_2) \\ y_2 & = & g(x_2) \end{array}\right. \xrightarrow{5} \left\{\begin{array}{ccc} x_1 & = & g(g_2) \\ y_1 & = & g(x_2) \\ y_2 & = & g(x_2) \end{array}\right.$$

   $\Theta = \{x_1/g(x_2), y_1/g(x_2), y_2/g(x_2)\}$

   $D_3 \quad \underset{\Theta}{\overset{D_1,D_2}{=}} \quad R(g(x_2),f(g(x_2))) \ \vee \ \neg P(x_2,f(x_2),,h(x_2,g(x_2)))$

Упражнение 4.2

1. $S = \{D_1, D_2, D_3, D_4, D_5\}$

   $D_1 = P(X_1, f(X_1))$
   $D_2 = R(Y_2, Z_2) \ \vee \ \neg P(Y_2, f(a))$
   $D_3 = \vee R(c, X_3)$
   $D_4 = R(X_4, Y_4) \ \vee \ R(Z_4, f(Z_4)) \ \vee \ \neg P(Z_4, Y_4)$
   $D_5 = P(X_5, X_5)$

   $D_6 \quad \underset{\{X_1/a, Y_2/a\}}{\overset{D_1, D_2}{=}} \quad R(a, Z_6)$

   $D_7 \quad \underset{\{X_4/Z_4, Y_4/f(Z_4)\}}{\overset{D_4}{=}} \quad R(Z_7, f(Z_7)) \ \vee \ \neg P(Z_7, f(Z_7))$

   $D_8 \quad \underset{\{X_3/f(c), Z_7/c\}}{\overset{D_7, D_3}{=}} \quad \neg P(c, f(c))$

   $D_9 \quad \underset{\{X_1/c\}}{\overset{D_1, D_8}{=}} \quad \Box$

2. $S = \{D_1, D_2.D_3, D_4, D_5, D_6, D_7\}$

   $D_1 = E(x_1) \ \vee \ V(y_1) \ \vee \ C(f(x_1))$
   $D_2 = E(x_2) \ \vee \ S(x_2, f(x_2))$
   $D_3 = \neg E(a)$
   $D_4 = P(a)$
   $D_5 = P(f(x_5)) \ \vee \ \neg S(y_5, z_5)$
   $D_6 = \neg P(x_6) \ \vee \ \neg V(g(x_6)) \neg \ \vee V(y_6)$
   $D_7 = \neg P(x_7) \ \vee \ \neg C(y_7)$

   $D_8 \quad \underset{\{y_6/g(x)\}}{\overset{D_6}{=}} \quad \neg P(x_8) \ \vee \ \neg V(g(x_8))$

   $D_9 \quad \underset{\{x_7/a\}}{\overset{D_4, D_7}{=}} \quad \neg C(y_9)$

   $D_{10} \quad \underset{\{x_8/a\}}{\overset{D_8, D_4}{=}} \quad \neg V(g(a))$

   $D_{11} \quad \underset{\{y_9/f(x_1)\}}{\overset{D_1, D_9}{=}} \quad E(x_{11}) \ \vee \ V(y_{11})$

   $D_{12} \quad \underset{\{y_{11}/g(a))\}}{\overset{D_{11}, D_{10}}{=}} \quad E(x_{12})$

   $D_{13} \quad \underset{\{x_{12}/a\}}{\overset{D_{12}, D_3}{=}} \quad \Box$

3. $S = \{D_1, D_2, D_3, D_4\}$

   $D_1 = P(y_1, f(x_1))$
   $D_2 = \neg Q(y_2) \ \vee \ \neg Q(z_2) \ \vee \ \neg P(y, f(z)) \ \vee \ Q(v)$
   $D_3 = Q(b)$
   $D_4 = \neg Q(a)$

   $D_5 \quad \underset{\{x_1/z_2, y_1/y_2\}}{\overset{D_1, D_2}{=}} \quad \neg Q(y_5) \ \vee \ \neg Q(z_5) \ \vee \ Q(v_5)$

   $D_6 \quad \underset{\{z_5/y_5\}}{\overset{D_5}{=}} \quad \neg Q(y_6) \ \vee \ Q(v_6)$

   $D_7 \quad \underset{\{v_6/a\}}{\overset{D_6, D_4}{=}} \quad \neg Q(y_7)$

   $D_8 \quad \underset{\{y_7/b\}}{\overset{D_7, D_3}{=}} \quad \Box$

Упражнение 4.3

1. $\exists x\ P(x)\ \rightarrow\ \neg\forall x\ \neg P(x)$

   $\phi_0 = \neg(\exists x\ P(x)\ \rightarrow\ \neg\forall y\ \neg P(x))$
   $\phi_{01} = \exists x\ P(x)\ \&\ \forall y\ \neg P(y)$
   $\phi_{02} = \exists x\forall y\ P(x)\ \&\ \neg P(y)$
   $\phi_1 = \forall y\ P(c)\ \&\ \neg P(y)$

   $S = \{P(c), \neg P(y)\}$

   $D_1 = P(c)$
   $D_2 = \neg P(y)$
   $D_3 \overset{D_1,D_2}{\underset{\{y/c\}}{=}} \square$

2. $\exists x\forall y R(x,y)\ \rightarrow\ \forall y\exists x R(x,y)$
   $\phi_0 = \neg(\exists x_1\forall y_1 R(x_1,y_1)\ \rightarrow\ \forall y_2\exists x_2 R(x_2,y_2))$
   $\phi_{01} = \exists x_1\forall y_1 R(x_1,y_1)\ \&\ \exists y_2\forall x_2\neg R(x_2,y_2)$
   $\phi_{02} = \exists x_1\forall y_1\exists y_2\forall x_2\ R(x_1,y_1)\ \&\ \neg R(x_2,y_2)$
   $\phi_1 = \forall y_1\forall x_2\ R(c,y_1)\ \&\ \neg R(x_2,f(y_1))$
   $S = \{R(c,y_1), \neg R(x_2,f(y_1))\}$

   $D_1 = R(c,y_1)$
   $D_2 = \neg R(x_2,f(y_2))$ *переименование переменных*
   $D_3 \overset{D_1,D_2}{\underset{\{x_2/c,y_1/f(y_2)\}}{=}} \square$

3. $\forall x(P(x)\ \rightarrow\ \exists y\ R(x,f(y)))\ \rightarrow\ (\exists x\ \neg P(x)\ \vee\ \forall x\exists z R(x,z))$

   $\phi_0 = \neg(\forall x_1(P(x_1)\ \rightarrow\ \exists y_1\ R(x,_1 f(y_1)))\ \rightarrow\ (\exists x_2\ \neg P(x_2)\ \vee\ \forall x_3\exists z_1 R(x_3,z_1)))$
   $\phi_{01} = \forall x_1(\neg P(x_1)\ \vee\ \exists y_1\ R(x_1,f(y_1)))\ \&\ \forall x_2\ P(x_2)\ \&\ \exists x_3\forall z_1\neg R(x_3,z_1)$
   $\phi_{02} = \forall x_1\exists y_1\forall x_2\exists x_3\forall z_1(\neg P(x_1)\ \vee\ R(x_1,f(y_1)))\ \&\ P(x_2)\ \&\ \neg R(x_3,z_1)$
   $\phi_1 = \forall x_1\forall x_2\forall z_1(\neg P(x_1)\ \vee\ R(x_1,f(g(x_1))))\ \&\ P(x_2)\ \&\ \neg R(h(x_1,x_2),z_1)$
   $S = \{\neg P(x_1)\ \vee\ R(x_1,f(g(x_1))), P(x_2), \neg R(h(x_1,x_2),z_1)\}$

   $D_1 = \neg P(x_1)\ \vee\ R(x_1,f(g(x_1)))$
   $D_2 = P(x_2)$
   $D_3 = \neg R(h(x_{31},x_{32}),z_3)$
   $D_4 \overset{D_1,D_2}{\underset{\{x_1/x_2\}}{=}} R(x_4,f(g(x_4))$
   $D_5 \overset{D_3,D_4}{\underset{\{x_4/h(x_{31},x_{32}),z_3/f(g(h(x_{31},x_{32})))\}}{=}} \square$

4. $\forall x\exists y\forall z(P(x,y)\ \rightarrow\ P(y,z))$

   $\phi_0 = \neg(\forall x\exists y\forall z(P(x,y)\ \rightarrow\ P(y,z)))$
   $\phi_{01} = \exists x\forall y\exists z(P(x,y)\ \&\ \neg P(y,z)))$
   $\phi_1 = \forall y(P(c,y)\ \&\ \neg P(y,f(y)))$
   $S = \{P(c,y), \neg P(y,f(y))\}$

   $D_1 = P(c,y_1)$
   $D_2 = \neg P(y_2,f(y_2))$
   $D_3 \overset{D_1,D_2}{\underset{\{y_2/c,y_1/f(c)\}}{=}} \square$

5. $\exists x \forall y \exists z (P(x,y) \;\rightarrow\; P(y,z))$

$\phi_0 = \neg(\exists x \forall y \exists z (P(x,y) \;\rightarrow\; P(y,z)))$
$\phi_{01} = \forall x \exists y \forall z (P(x,y) \;\&\; \neg P(y,z)))$
$\phi_{02} = \forall x \forall z (P(x,f(x)) \;\&\; \neg P(y,z)))$
$S = \{P(x,f(x)), \neg P(y,z)\}$

$D_1 = P(x_1, f(x_1))$
$D_2 = \neg P(y_2, z_2)$
$D_3 \underset{\{y_2/x_1, z_2/f(x_1)\}}{\overset{D_1,D_2}{=}} \square$

6. $\exists x \forall y (\forall z (P(y,z) \;\rightarrow\; P(x,z)) \;\rightarrow\; (P(x,x) \;\rightarrow\; PP(y,z)))$

$\phi_0 = \neg(\exists x \forall y (\forall z (P(y,z) \;\rightarrow\; P(x,z)) \;\rightarrow\; (P(x,x) \;\rightarrow\; PP(y,z))))$
$\phi_{01} = \forall x \exists y ((\forall z (\neg P(y,z) \;\vee\; P(x,z))) \;\&\; P(x,x) \;\&\; \neg P(y,z))$
$\phi_{02} = \forall x \exists y \forall z ((\neg P(y,z) \;\vee\; P(x,z)) \;\&\; P(x,x) \;\&\; \neg P(y,z))$
$S = \{\neg P(y,z) \;\vee\; P(x,z), P(x,x), \neg P(y,z)\}$

$D_1 = \neg P(y_1, z_1) \;\vee\; P(x_1, z_1)$
$D_2 = P(x_2, x_2)$
$D_3 = \neg P(y_3, z_3)$
$D_4 \underset{\{y_1/x_2, z_1/x_2\}}{\overset{D_1,D_2}{=}} P(x_{31}, x_{32})$
$D_5 \underset{\{y_3/x_{31}, z_3/x_{32}\}}{\overset{D_4,D_3}{=}} \square$

Упражнение 4.4 *Данная задача не рассматривалась на семинарах. Если будет время, ее решение будет добавленно. Если пришлете мне решение данной задачи, оно появится тут скорее ;-).*

5 Хорновские логические программы. Декларативные и операционные семантики.

Упражнение 5.1

1. $parent(X, Y) \ \leftarrow \ father(X, Y).$
   $parent(X, Y) \ \leftarrow \ mather(X, Y).$

2. $grandfather(X, Y) \ \leftarrow \ father(X, Z), parent(Z, Y).$

3. $to\_be\_a\_father(X) \ \leftarrow \ father(X, Z).$

4. $brother(X, Y) \ \leftarrow \ parent(Z, X), man(X), parent(Z, Y), Z \neq Y.$

5. $offspring(X, Y) \ \leftarrow \ parent(Y, X).$
   $offspring(X, Y) \ \leftarrow \ parent(Z, X), offspring(X, Z).$

Упражнение 5.2

1. $list(X)$

   $list(nil) \ \leftarrow;$
   $list(X.Y) \ \leftarrow \ list(Y).$

2. $elem(X, Y)$

   $elem(X, X.Y) \ \leftarrow;$
   $elem(X, Z.Y) \ \leftarrow \ elem(X, Y);$

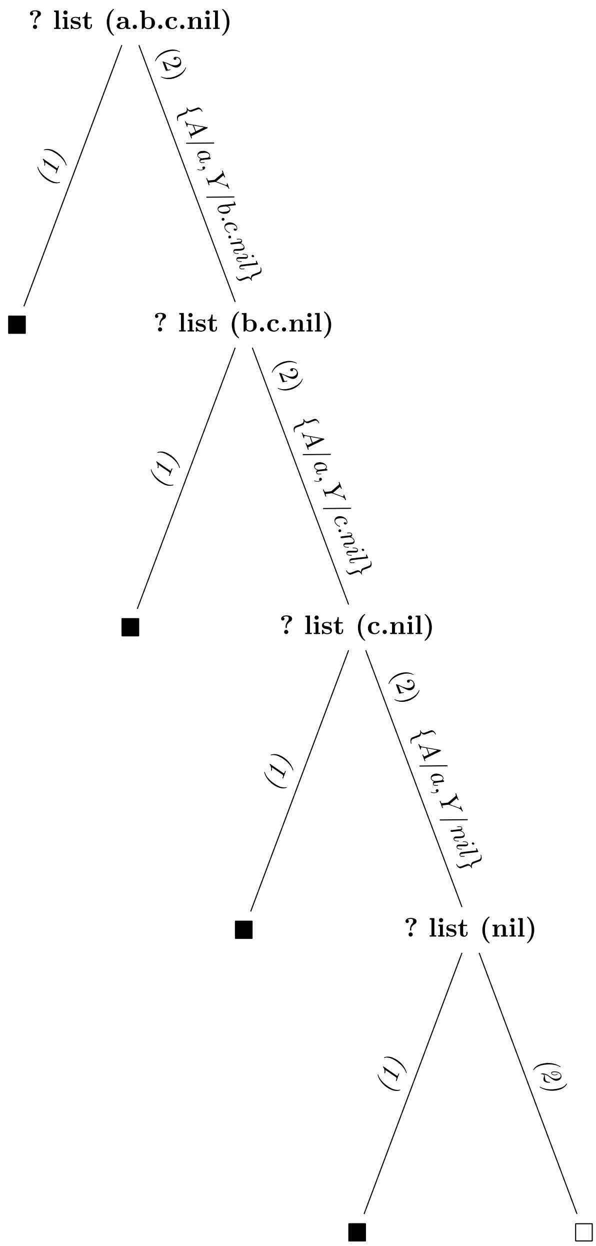
1. *True.*
2. X *- любой атом.*
3. *False.*
4. $X = a, X = b, X = c.$
5. X *- любой список, содержащий атом* a.

Упражнение 5.3

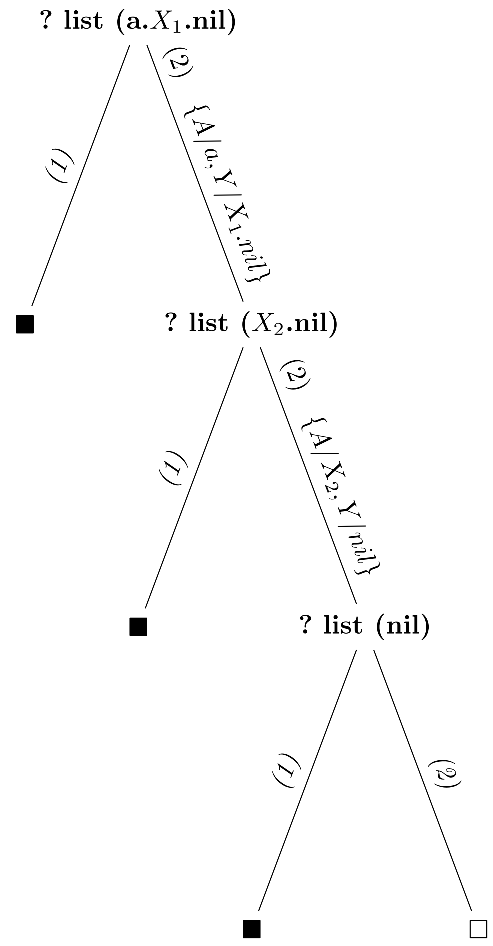
1. **list(nil).**
2. **list(A.Y)** $\leftarrow$ **list(Y).**

1. **elem(X, X.Y).**
2. **elem(X, Z.Y)** $\leftarrow$ **elem(X, Y).**

1. **? list(a.b.c.nil)**

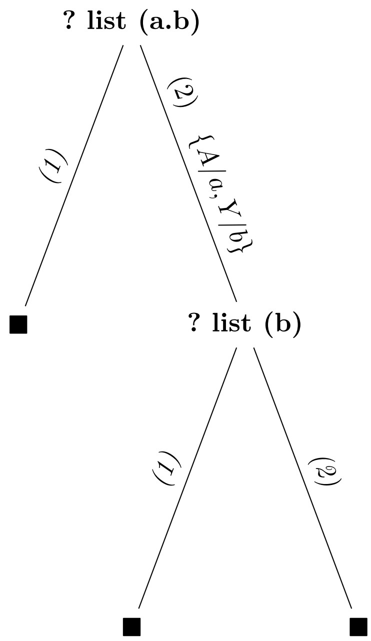


2. **? list(a.X.nil)**

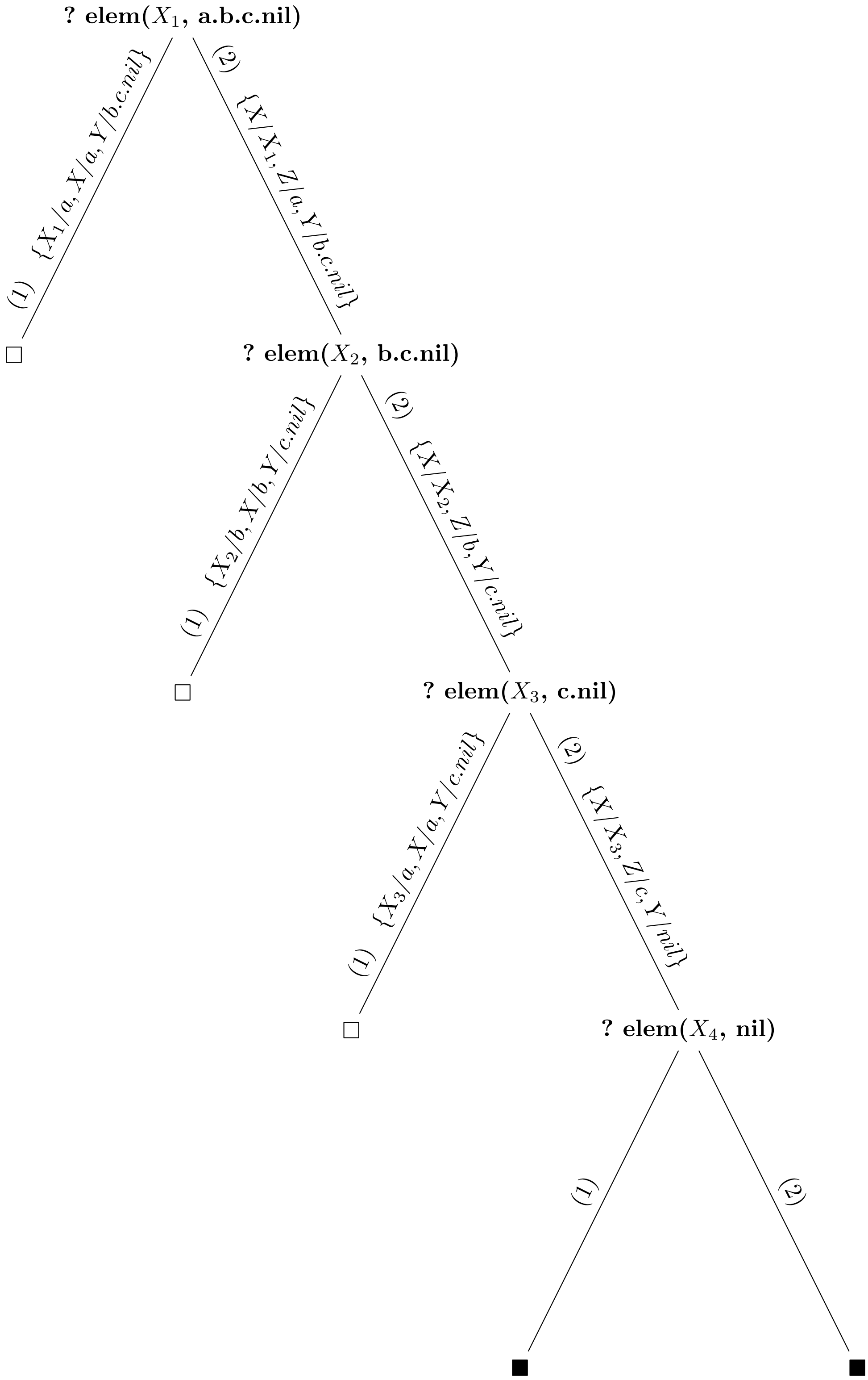


$X = A$

3. **? list(a.b)**

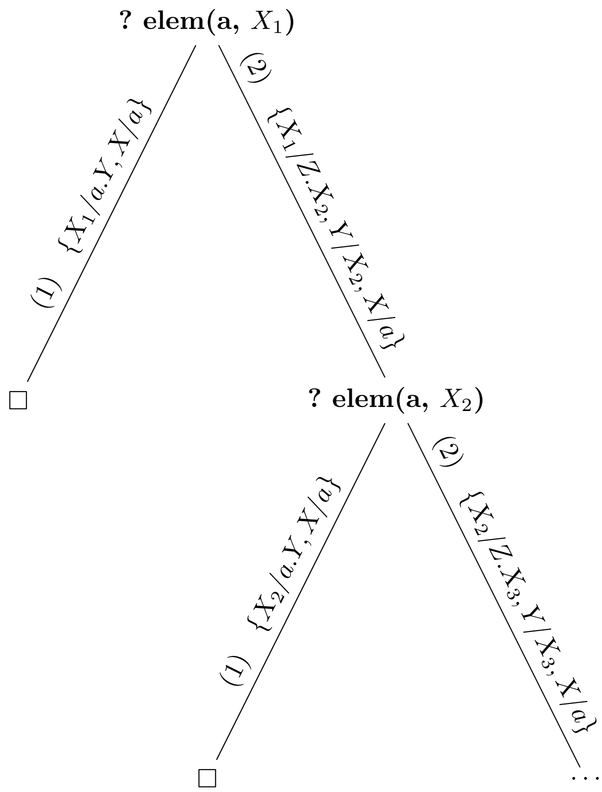


4. **? elem(X, a.b.c.nil)**



$X = a,\ X = b,\ X = c$

5. **? elem(a,X)**

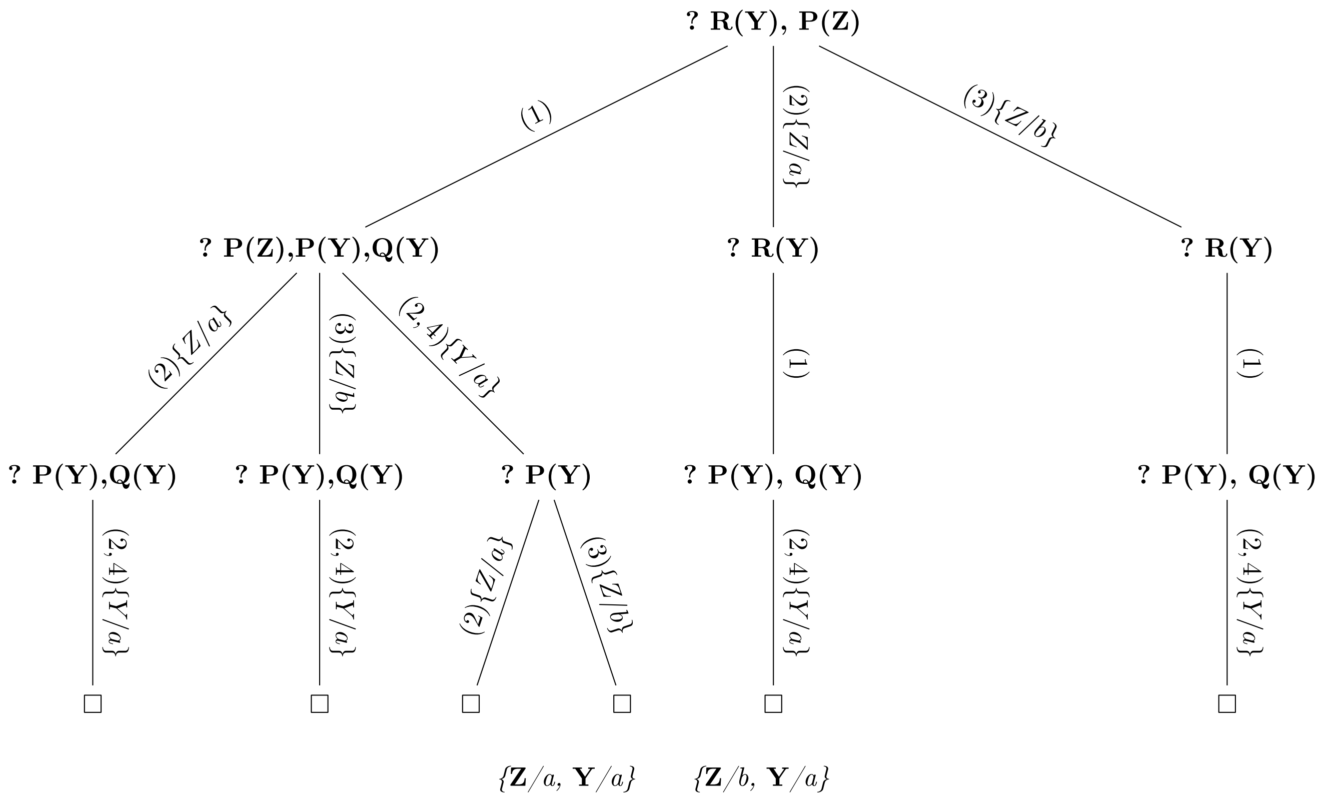


? elem(a, X_1)
(1) $\{X_1/a.Y, X/a\}$
(2) $\{X_1/Z.X_2, Y/X_2, X/a\}$
□
? elem(a, X_2)
(1) $\{X_2/a.Y, X/a\}$
(2) $\{X_2/Z.X_3, Y/X_3, X/a\}$
□
…

Упражнение 5.4

1. **R(Y) ← P(Y),Q(Y);**
2. **P(a) ← ;**
3. **P(b) ← ;**
4. **Q(a) ← ;**
5. **Q(f(x)) ← Q(X)**

? R(Y), P(Z)



Упражнение 5.5

```
%Sub
elem(X, [X|_]).
elem(X, [_|Y]) :- elem(X, Y).

%1
head([X| _], X).

%2
tail([_| Tail], Z) :-tail(Tail, Z).
tail([_| B], B).

%3
prefix([Head| Tail_1], [Head| Tail_2]) :- prefix(Tail_1, Tail_2).
prefix(_, []).

%4
sublist(List, Sublist) :- prefix(List, Sublist).
sublist([_| Tail], Sublist) :- sublist(Tail, Sublist).

%5
less([_|Tail_1], [_|Tail_2]) :- less(Tail_1, Tail_2).
less([], [_|_]).

%6
subset([], _).
subset([Head| Tail], Y) :- elem(Head, Y),subset(Tail, Y).

%7
concat(X, [], X).
concat([Head|Tail_1], [Head|Tail_2], X) :- concat(Tail_1, Tail_2, X).

%8

reverse(X, Y) :- reverse_loop([], X, Y).
reverse_loop(Rev, [], Rev).
reverse_loop(Rev, [Head| Tail], Goal) :- reverse_loop([Head| Rev], Tail, Goal).

%9
period(X, Y) :- loop_period(X, Y, Y).
loop_period([], [], _).
loop_period(Main, [], Base) :- loop_period(Main, Base, Base).
loop_period([Head| Main], [Head| Curr], Base) :- loop_period(Main,Curr, Base).
```

Упражнение 5.6

```
%1
main_less([], [], 1).
main_less([], [_], _).
main_less([_|Tail_X], [_|Tail_Y], -1) :- main_less(Tail_X, Tail_Y, -1).
main_less([_|Tail_X], [_|Tail_Y], 1) :- main_less(Tail_X, Tail_Y, 1).
main_less([A|Tail_X], [A|Tail_Y], 0) :- main_less(Tail_X, Tail_Y, 0).
main_less([0|Tail_X], [1|Tail_Y], 0) :- main_less(Tail_X, Tail_Y, 1).
main_less([1|Tail_X], [0|Tail_Y], 0) :- main_less(Tail_X, Tail_Y, -1).
less([0|Tail_X], Y) :- less(Tail_X, Y).
less(X, [0|Tail_Y]) :- less(X, Tail_Y).
less([], [1|_]).
less([1|Tail_X], [1|Tail_Y]) :- main_less(Tail_X, Tail_Y, 0).
%2 Z = X + Y
sum(X,Y,Z) :- reverse(X, R_X), reverse(Y, R_Y), reverse(Z, R_Z), r_sum(R_X, R_Y, R_Z, 0)
r_sum([A|Tail_X], [A|Tail_Y], [B|Tail_Z], B) :- r_sum(Tail_X, Tail_Y, Tail_Z, A).
r_sum([_|Tail_X], [_|Tail_Y], [1|Tail_Z], 0) :- r_sum(Tail_X, Tail_Y, Tail_Z, 0).
r_sum([_|Tail_X], [_|Tail_Y], [0|Tail_Z], 1) :- r_sum(Tail_X, Tail_Y, Tail_Z, 1).
r_sum([], [1|Tail_Y], [0|Tail_Z], 1) :- r_sum([], Tail_Y, Tail_Z, 1).
r_sum([], [0|Tail_Y], [1|Tail_Z], 1) :- r_sum([], Tail_Y, Tail_Z, 0).
r_sum([1|Tail_X], [], [0|Tail_Z], 1) :- r_sum(Tail_X, [], Tail_Z, 1).
r_sum([0|Tail_X], [], [1|Tail_Z], 1) :- r_sum(Tail_X, [], Tail_Z, 0).
r_sum([], N, N, 0).
r_sum(N, [], N, 0).
```

6 Не решенные задачи

1. 2.5
2. 3.3
3. 4.4